Questão 2. (3 pontos) Seja $f(x) = \dfrac{2x+2}{x^2-4}\,\mathrm{sen}\Big((x+2)\sqrt{x^2+1}\Big)$.

a) Calcule $\lim\limits_{x\to -2} f(x)$.

b) Calcule $\lim\limits_{x\to +\infty} f(x)$.

c) Determine $f'(x)$ em todos os pontos de seu domínio.

(a) $$\lim_{x\to -2} f(x) = \left(\lim_{x\to -2} \frac{2(x+1)\sqrt{x^2+1}}{x-2}\right)\cdot\left(\lim_{x\to -2} \frac{\mathrm{sen}\left((x+2)\sqrt{x^2+1}\right)}{(x+2)\sqrt{x^2+1}}\right) =$$

$$= \frac{\sqrt{5}}{2}\cdot 1 = \frac{\sqrt{5}}{2}, \text{ pois } \lim_{x\to -2} (x+2)\sqrt{x^2+1} = 0$$

e $(x+2)\sqrt{x^2+1} \neq 0$ se $x \neq -2$

(b) $$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} \underbrace{\frac{\frac{2}{x}+\frac{2}{x^2}}{1-\frac{4}{x^2}}}_{\to 0}\cdot \underbrace{\mathrm{sen}\left((x+2)\sqrt{x^2+1}\right)}_{\text{limitada}} = 0$$

(c) $\forall x \in \mathbb{R}\setminus\{-2,2\}$,

$$f'(x) = \frac{d}{dx}\left(\frac{2x+2}{x^2-4}\right)\cdot \mathrm{sen}\left((x+2)\sqrt{x^2+1}\right) + \left(\frac{2x+2}{x^2-4}\right)\cdot\frac{d}{dx}\,\mathrm{sen}\left((x+2)\sqrt{x^2+1}\right) =$$

$$= \frac{2(x^2-4)-(2x+2)\cdot 2x}{(x^2-4)^2}\cdot \mathrm{sen}\left((x+2)\sqrt{x^2+1}\right) +$$

$$+\left(\frac{2x+2}{x^2-4}\right)\cdot \cos\left((x+2)\sqrt{x^2+1}\right)\cdot\left(\sqrt{x^2+1} + \frac{2x(x+2)}{2\sqrt{x^2+1}}\right)$$