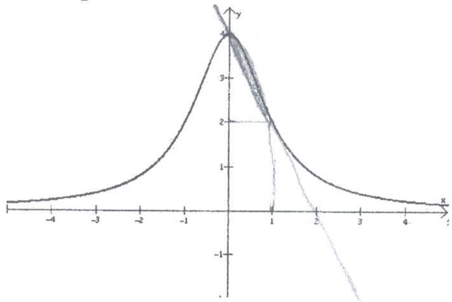
A

Questão 2: (Os dois itens são independentes.)

(2,0) a) Calcule a área da região compreendida entre o gráfico de $f(x) = \dfrac{4}{x^2+1}$ (figura abaixo) e sua reta tangente no ponto de abscissa $x = 1$.

$$f'(x) = -\frac{8x}{(x^2+1)^2}$$



(1,0) b) Calcule, caso existam, $\lim_{x\to+\infty} (\ln x)^{1/(x^2-1)}$ e $\lim_{x\to 1^+} (\ln x)^{1/(x^2-1)}$.

a) Reta tangente: $\dfrac{y - f(1)}{x - 1} = f'(1) \therefore \dfrac{y-2}{x-1} = -2 \therefore y - 2 = -2x +$

$$\boxed{y = -2x + 4}$$

$$A = \int_0^1 \left(\frac{4}{x^2+1} + 2x - 4\right) dx = \left(4\,\text{arctg}\, x + x^2 - 4x\right)\Big|_0^1 = \pi + 1 - 4 = \pi - 3$$

b) $(\ln x)^{\frac{1}{x^2-1}} = e^{\frac{\ln(\ln x)}{x^2-1}}$

$\rightarrow \lim\limits_{x\to+\infty} \dfrac{\ln(\ln x)}{x^2-1} \underset{\frac{\infty}{\infty}\ L'H}{=} \lim\limits_{x\to+\infty} \dfrac{1}{(\ln x)\cdot x \cdot 2x} = 0$

$\rightarrow \lim\limits_{x\to 1^+} \dfrac{\ln(\ln x)}{x^2-1} = -\infty,$

pois $\lim\limits_{x\to 1^+} \ln(\ln x) = \lim\limits_{u\to 0} \ln u = -\infty.$

$u = \ln x.$

e $\lim\limits_{x\to 1} (x^2-1) = 0$ e $x^2 - 1 > 0$ se $x > 1$.

Daí

$\lim\limits_{x\to+\infty} (\ln x)^{\frac{1}{x^2-1}} = e^0 = 1$ e

$\lim\limits_{x\to 1^+} (\ln x)^{\frac{1}{x^2-1}} = \lim\limits_{y\to-\infty} e^y = 0$