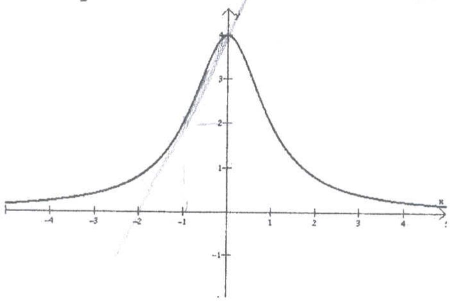
Questão 2: (Os dois itens são independentes.)

B

(2,0) a) Calcule a área da região compreendida entre o gráfico de $f(x) = \dfrac{4}{x^2+1}$ (figura abaixo) e sua reta tangente no ponto de abscissa $x = -1$.

$$f'(x) = -\frac{8x}{(x^2+1)^2}$$



(1,0) b) Calcule, caso existam, $\lim\limits_{x \to +\infty} (\ln x)^{1/(x^3-1)}$ e $\lim\limits_{x \to 1^+} (\ln x)^{1/(x^3-1)}$.

a) Reta tangente: $\dfrac{y - f(-1)}{x+1} = f'(-1) = \dfrac{8}{4} = 2$

$y - 2 = 2x + 2 \quad \therefore \quad y = 2x + 4$

$$A = \int_{-1}^{0} \left[\frac{4}{x^2+1} - (2x+4)\right] dx = \left(4 \cdot \operatorname{arctg} x - x^2 - 4x\right)\Big|_{-1}^{0} =$$

$$= -\left(4 \operatorname{arctg}(-1) - 1 + 4\right) = 4 \operatorname{arctg} 1 - 3 = \pi - 3$$

b) $(\ln x)^{1/(x^3-1)} = e^{\frac{\ln(\ln x)}{x^3-1}}$

$\rightarrow \lim\limits_{x \to +\infty} \dfrac{\ln(\ln x)}{x^3 - 1} \underset{\text{L'H. } \frac{\infty}{\infty}}{=} \lim\limits_{x \to +\infty} \dfrac{1}{(\ln x)\, x \cdot 2x^2} = 0$

$\rightarrow \lim\limits_{x \to 1^+} \dfrac{\ln(\ln x)}{x^3 - 1} = -\infty$,

pois $\lim\limits_{x \to 1^+} \ln(\ln x) = \lim\limits_{u \to 0^+} \ln u = -\infty \quad (u = \ln x)$

e $\lim\limits_{x \to 1} (x^3 - 1) = 0$ e $x^3 - 1 > 0$ se $x > 1$

Daí:

$\lim\limits_{x \to +\infty} (\ln x)^{\frac{1}{x^3-1}} = e^0 = 1$ e $\lim\limits_{x \to 1^+} \dfrac{\ln(\ln x)}{x^3-1} = \lim\limits_{y \to -\infty} e^y = 0$