

1) Calcule as seguintes integrais:

$$(1,5) \text{ a) } \int \frac{dx}{(x+1)\sqrt{x+5}} = \int \frac{2udu}{(u^2-4)u} = \int \frac{2du}{(u-2)(u+2)} = \int \left[ \frac{A}{u-2} + \frac{B}{u+2} \right] du = (*)$$

$\begin{aligned} u &= \sqrt{x+5} \\ x &= u^2 - 5 \\ x+1 &= u^2 - 4 \\ dx &= 2udu \end{aligned}$
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$\frac{2}{(u-2)(u+2)} = \frac{A}{u-2} + \frac{B}{u+2} = \frac{(A+B)u + 2A - 2B}{(u-2)(u+2)}$
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$\begin{cases} A+B=0 \\ 2A-2B=2 \end{cases} \Rightarrow \begin{cases} A=1/2 \\ B=-1/2 \end{cases}$
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$$(*) = \int \left[ \frac{1/2}{u-2} + \frac{-1/2}{u+2} \right] du = \frac{1}{2} \ln |u-2| - \frac{1}{2} \ln |u+2| + k =$$

$$= \frac{1}{2} \ln |\sqrt{x+5}-2| - \frac{1}{2} \ln |\sqrt{x+5}+2| + k$$

$$(1,5) \text{ b) } \int x \operatorname{arctg}(\sqrt{x^2-1}) dx = \frac{x^2}{2} \operatorname{arctg}(\sqrt{x^2-1}) - \frac{1}{2} \int \frac{x}{\sqrt{x^2-1}} dx =$$

$\begin{aligned} u &= \operatorname{arctg}(\sqrt{x^2-1}) & du &= \frac{1}{x\sqrt{x^2-1}} dx \\ dv &= x dx & v &= \frac{x^2}{2} \end{aligned}$	$\begin{aligned} u &= x^2 - 1 \\ du &= 2x dx \end{aligned}$
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$$= \frac{x^2}{2} \operatorname{arctg}(\sqrt{x^2-1}) - \frac{1}{2} \int \frac{1}{2\sqrt{u}} du = \frac{x^2}{2} \operatorname{arctg}(\sqrt{x^2-1}) - \frac{1}{2} \sqrt{u} + k =$$

$$= \frac{x^2}{2} \operatorname{arctg}(\sqrt{x^2-1}) - \frac{1}{2} \sqrt{x^2-1} + k.$$