

1) Calcule as seguintes integrais indefinidas:

$$(1,5) \text{ a) } \int \frac{dx}{(x+1)\sqrt{x+10}} = \int \frac{2udu}{(u^2-9)u} = \int \frac{2du}{(u-3)(u+3)} = \int \left[\frac{A}{u-3} + \frac{B}{u+3} \right] du = (*)$$

u	$=$	$\sqrt{x+10}$
x	$=$	$u^2 - 10$
$x+1$	$=$	$u^2 - 9$
dx	$=$	$2udu$

$\frac{2}{(u-3)(u+3)} = \frac{A}{u-3} + \frac{B}{u+3} = \frac{(A+B)u + 3A - 3B}{(u-3)(u+3)}$
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$\begin{cases} A+B=0 \\ 3A-3B=2 \end{cases} \Rightarrow \begin{cases} A=1/3 \\ B=-1/3 \end{cases}$
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$$(*) = \int \left[\frac{1/3}{u-3} + \frac{-1/3}{u+3} \right] du = \frac{1}{3} \ln |u-3| - \frac{1}{3} \ln |u+3| + k =$$

$$= \frac{1}{3} \ln |\sqrt{x+10} - 3| - \frac{1}{3} \ln |\sqrt{x+10} + 3| + k.$$

$$(1,5) \text{ b) } \int x \operatorname{arctg}(\sqrt{x^2-1}) dx = \frac{x^2}{2} \operatorname{arctg}(\sqrt{x^2-1}) - \frac{1}{2} \int \frac{x}{\sqrt{x^2-1}} dx =$$

$u = \operatorname{arctg}(\sqrt{x^2-1})$	$du = \frac{1}{x\sqrt{x^2-1}} dx$	$u = x^2 - 1$
$dv = x dx$	$v = \frac{x^2}{2}$	$du = 2x dx$

$$= \frac{x^2}{2} \operatorname{arctg}(\sqrt{x^2-1}) - \frac{1}{2} \int \frac{1}{2\sqrt{u}} du = \frac{x^2}{2} \operatorname{arctg}(\sqrt{x^2-1}) - \frac{1}{2} \sqrt{u} + k =$$

$$= \frac{x^2}{2} \operatorname{arctg}(\sqrt{x^2-1}) - \frac{1}{2} \sqrt{x^2-1} + k.$$