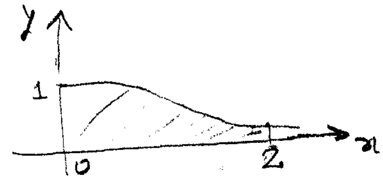
Prova A

4-) (2,5 pontos) Calcule o volume do sólido obtido pela rotação em torno do eixo Ox do conjunto $R = \{(x, y) \in \mathbb{R}^2 \mid 0 \leqslant x \leqslant 2,\ 0 \leqslant y \leqslant \frac{1}{1+x^2}\}$.



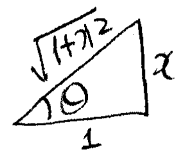
$$\text{vol.} = \pi \int_0^2 \left(\frac{1}{1+x^2}\right)^2 dx$$

Calculemos $I = \int \left(\frac{1}{1+x^2}\right)^2 dx$, com a substituição $\begin{cases} x = tg\theta, & -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \\ dx = sec^2\theta\, d\theta \end{cases}$

fica $I = \int \frac{sec^2\theta\, d\theta}{(1+tg^2\theta)^2} = \int \frac{d\theta}{sec^2\theta} = \int cos^2\theta\, d\theta =$

$$= \frac{1}{2}\int (1+\cos 2\theta)\, d\theta = \frac{1}{2}\left(\theta + \frac{sen 2\theta}{2}\right) + C = \frac{1}{2}(\theta + sen\theta \cos\theta) + C$$

e voltando para a variável x:



$x = tg\theta$

$\therefore \theta = arctg\, x$

$\therefore \begin{cases} sen\theta = \frac{x}{\sqrt{1+x^2}} \\ \cos\theta = \frac{1}{\sqrt{1+x^2}} \end{cases}$

$$\therefore \text{vol} = \frac{\pi}{2}\left[arctg 2 + \frac{x}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+x^2}}\right]_0^2 = \frac{\pi}{2}\left(arctg 2 + \frac{2}{5}\right).$$

(Para a prova B, substituir 2 por 3, na integral inicial, obtendo-se: $\text{vol.} = \frac{\pi}{2}\left(arctg 3 + \frac{3}{10}\right)$)