3-) Calcule as seguintes integrais indefinidas:

(a) (1,5 pontos) $\int e^{2x} \operatorname{sen} x \, dx$

(b) (1,5 pontos) $\int \frac{dx}{x^3-8} \, dx$

(a) $\int e^{2x} \operatorname{sen} x \, dx \overset{(*)}{=} -e^{2x}\cos x + 2\int e^{2x}\cos x \, dx \overset{(**)}{=}$

(*) Usamos integração por partes:

$u = e^{2x} \quad du = 2e^{2x}dx$

$dv = \operatorname{sen} x \, dx \quad v = -\cos x$

(**) Usamos integração por partes em $\int e^{2x}\cos x \, dx$

fazendo agora $u = e^{2x} \quad dv = \cos x \, dx$

$du = 2e^{2x} \quad v = \operatorname{sen} x$

$\overset{(**)}{=} -e^{2x}\cos x + 2\left[e^{2x}\operatorname{sen} x - 2\int e^{2x}\operatorname{sen} x\right]$

Logo: $\int e^{2x}\operatorname{sen} x \, dx = -e^{2x}\cos x + 2e^{2x}\operatorname{sen} x - 4\int e^{2x}\operatorname{sen} x \, dx$

Portanto $\boxed{\int e^{2x}\operatorname{sen} x \, dx = \frac{1}{5}\left[2e^{2x}\operatorname{sen} x - e^{2x}\cos x\right] + K, \quad K \in \mathbb{R}}$

b) $\frac{1}{x^3-8} = \frac{A}{(x-2)} + \frac{Bx+C}{x^2+2x+4} = \frac{A(x^2+2x+4)+(Bx+C)(x-2)}{x^3-8}$

$= \frac{(A+B)x^2 + (2A-2B+C)x + (4A-2C)}{x^3-8}$

Logo: $\begin{cases} A+B=0 \\ 2A-2B+C=0 \\ 4A-2C=1 \end{cases}$ Resolvendo o sistema obtemos: $A = 1/12$, $B = -1/12$ e $C = -\frac{1}{3}$.

Assim $\int \frac{dx}{x^3-8} = \frac{1}{12}\int\frac{dx}{x-2} + \int \frac{-1/12\, x - 1/3}{x^2+2x+4}dx$

$= \frac{1}{12}\ln|x-2| - \frac{1}{3}\int \frac{1/4\, x + 1}{x^2+2x+4}dx$.

Cálculo de

$\int \frac{1/4\, x+1}{x^2+2x+4}dx = \int \frac{1/4\, x+1}{(x+1)^2+3}dx \overset{(*)}{=} \int \frac{1/4\,(u-1)+1}{u^2+3}du$

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(*) $u = x+1$, $du = dx$
$x = u-1$

$= \frac{1}{4}\int\frac{u}{u^2+3}du + \frac{3}{4}\int\frac{du}{u^2+3} = \frac{1}{8}\ln(u^2+3) + \frac{\sqrt{3}}{4}\operatorname{arctg}\left(\frac{u}{\sqrt{3}}\right) + K$

$= \frac{1}{8}\ln(x^2+2x+4) + \frac{\sqrt{3}}{4}\operatorname{arctg}\left(\frac{x+1}{\sqrt{3}}\right) + K$

Logo: $\boxed{\int\frac{dx}{x^3-8} = \frac{1}{12}\ln|x-2| - \frac{1}{24}\ln(x^2+2x+4) - \frac{1}{4\sqrt{3}}\operatorname{arctg}\left(\frac{x+1}{\sqrt{3}}\right) + C, \quad C \in \mathbb{R}}$