# MAT 2453 - Prova Substitutiva
## Cálculo Diferencial e Integral para Engenharia I
## 30.06.2009

**(B)**

Nome:________________________________________

No. USP:____________________RG:____________________

Turma:____________________Professor:____________________

Assinatura:________________________________________

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**Justifique todas as suas respostas. Boa prova!**

**1-)** (4,0 pontos) Considere $f : \mathbb{R} \to \mathbb{R}$ dada por $f(0) = 0$ e $f(x) = x^2 \ln|3x|$ se $x \neq 0$.

(a) Mostre que $f$ é derivável no zero e calcule $f'(0)$. Existe $f''(0)$?

(b) Analise o sinal de $f'$ e de $f''$.

(c) Esboce o gráfico de $f$, levando-se em conta os intervalos de crescimento e decrescimento, concavidade e limites em $\pm\infty$. Indique os pontos críticos e os pontos de inflexão.

(a) $$\underbrace{\lim_{x\to 0} \frac{f(x)-f(0)}{x-0}}_{f'(0)} = \lim_{x\to 0} \frac{x^2 \ln|3x|}{x} = \lim_{x\to 0} \frac{\ln|3x|}{\frac{1}{x}} \underset{\frac{\infty}{\infty}}{\overset{L'H}{=}} \lim_{x\to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = 0.$$

$$f'(x) = 2x\ln|3x| + x^2\frac{1}{x} = x\,(2\ln|3x|+1), \quad x \neq 0$$

$$\lim_{x\to 0} \frac{f'(x)-f'(0)}{x-0} = \lim_{x\to 0}(2\ln|3x|+1) = -\infty. \text{ Logo, } f''(0) \nexists.$$

(b) $f'(x) = 0 \Longleftrightarrow x = 0$ ou $\left(\ln|3x| = -\frac{1}{2} \Longleftrightarrow 3x = \pm e^{-\frac{1}{2}} \Longleftrightarrow x = \frac{\pm 1}{3\sqrt{e}}\right)$

$|3x| > e^{-\frac{1}{2}} \Longleftrightarrow 2\ln|3x|+1 > 0$

$f'<0$ ↘ $\quad -\frac{1}{3\sqrt{e}} \quad$ $f'>0$ ↗ $\quad 0 \quad$ $f'<0$ ↘ $\quad \frac{1}{3\sqrt{e}} \quad$ $f'>0$ ↗

$$f''(x) = 2\ln|3x| + 1 + x\cdot\frac{2}{x} = 2\ln|3x| + 3 = 0 \Longleftrightarrow x = \frac{\pm 1}{3e^{3/2}}$$

$f''>0$ $\cup$ $\quad -\frac{1}{3e^{3/2}} \quad$ $f''<0$ $\cap$ $\quad 0 \quad$ $f''<0$ $\cap$ $\quad \frac{1}{3e^{3/2}} \quad$ $f''>0$ $\cup$

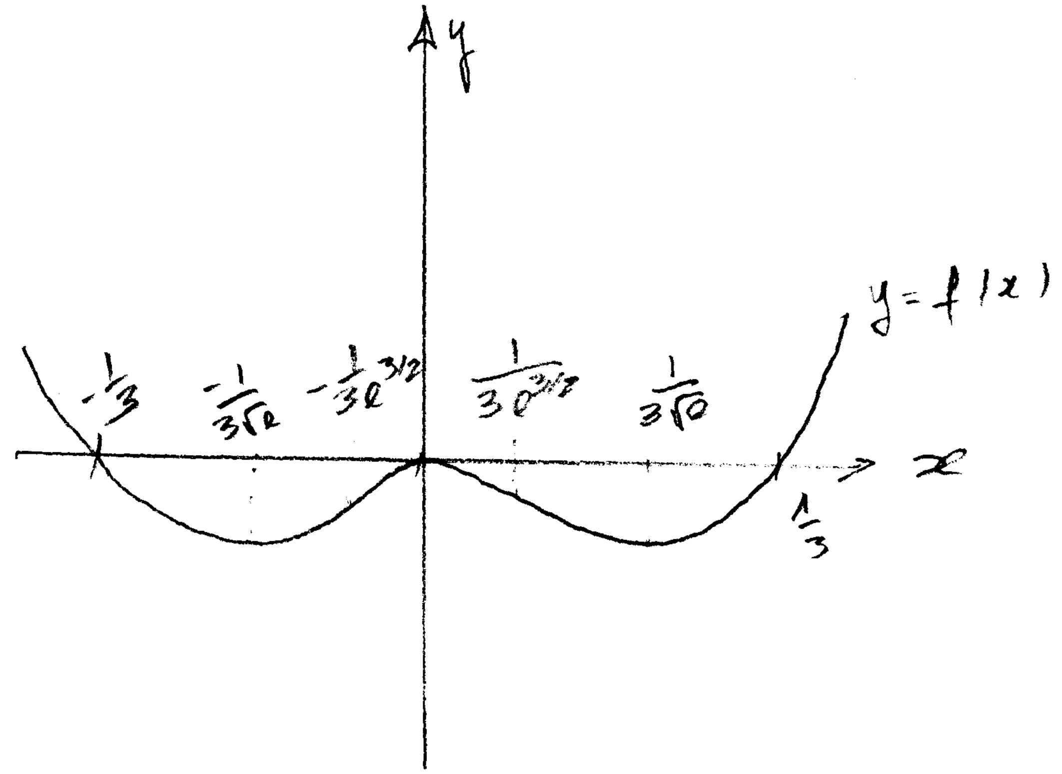
Pontos de mínimo local: $-\dfrac{1}{3\sqrt{e}}$ e $\dfrac{1}{3\sqrt{e}}$

Ponto de máximo local: $0$.

Pontos de inflexão: $-\dfrac{1}{3e^{3/2}}$ e $\dfrac{1}{3e^{3/2}}$

$$\lim_{x \to \pm\infty} x^2 \ln|3x| = +\infty.$$

$$f(x) = 0 \iff x = 0 \text{ ou } \left(|3x| = 1 \iff x = \pm\tfrac{1}{3}\right)$$



**2-)** (1,5 pontos) Dada $f : \mathbb{R} \to \mathbb{R}$ contínua, considere $g(x) = e^{-x^2} \int_0^x e^{t^2} f(t)\, dt$. Calcule $g'(0)$.

$$g'(x) = -2x e^{-x^2} \int_0^x e^{t^2} f(t)\, dt + e^{-x^2} e^{x^2} f(x)$$

$$g'(0) = 0 + f(0) = f(0)$$