**Questão 1)** (Valor: 3.0) Calcule o limite ou explique por que não existe.

a) $\lim_{x\to 2} \dfrac{\sqrt{x^3+x-9}-\sqrt{x^3-7}}{\sqrt{x+2}-2}$

b) $\lim_{x\to 1} \dfrac{\sqrt{x^3+x^2-5x+3}}{x^2-1}$

c) $\lim_{x\to +\infty} \dfrac{\sqrt{x}\cos x + 2x^2 \operatorname{sen}\left(\frac{1}{x}\right)}{x-\sqrt{1+x^2}}$

**Resolução:**

$$
\begin{aligned}
\text{a) } \lim_{x\to 2} \frac{\sqrt{x^3+x-9}-\sqrt{x^3-7}}{\sqrt{x+2}-2} &= \lim_{x\to 2} \frac{(\sqrt{x^3+x-9}-\sqrt{x^3-7})(\sqrt{x+2}+2)}{(\sqrt{x+2}-2)(\sqrt{x+2}+2)} \\
&= \lim_{x\to 2} \frac{(\sqrt{x^3+x-9}-\sqrt{x^3-7})(\sqrt{x+2}+2)}{x-2} \\
&= \lim_{x\to 2} \frac{(\sqrt{x^3+x-9}-\sqrt{x^3-7})(\sqrt{x^3+x-9}+\sqrt{x^3-7})(\sqrt{x+2}+2)}{(x-2)(\sqrt{x^3+x-9}+\sqrt{x^3-7})} \\
&= \lim_{x\to 2} \frac{(x-2)(\sqrt{x+2}+2)}{(x-2)(\sqrt{x^3+x-9}+\sqrt{x^3-7})} \\
&= \lim_{x\to 2} \frac{(\sqrt{x+2}+2)}{(\sqrt{x^3+x-9}+\sqrt{x^3-7})} = \frac{4}{2} = 2
\end{aligned}
$$

b) $\lim_{x\to 1} \dfrac{\sqrt{x^3+x^2-5x+3}}{x^2-1} = \lim_{x\to 1} \dfrac{\sqrt{(x-1)^2(x+3)}}{x^2-1} = \lim_{x\to 1} \dfrac{|x-1|\sqrt{(x+3)}}{(x-1)(x+1)}$

$$\lim_{x\to 1+} \frac{|x-1|\sqrt{(x+3)}}{(x-1)(x+1)} = \lim_{x\to 1+} \frac{(x-1)\sqrt{(x+3)}}{(x-1)(x+1)} = \frac{2}{2} = 1$$

$$\lim_{x\to 1-} \frac{|x-1|\sqrt{(x+3)}}{(x-1)(x+1)} = \lim_{x\to 1-} \frac{-(x-1)\sqrt{(x+3)}}{(x-1)(x+1)} = \frac{-2}{2} = -1.$$

Logo, tal limite não existe.

c) $\lim_{x\to +\infty} \dfrac{\sqrt{x}\cos x + 2x^2 \operatorname{sen}\left(\frac{1}{x}\right)}{x-\sqrt{1+x^2}} = \lim_{x\to +\infty} \dfrac{x(\frac{\sqrt{x}}{x}\cos x + 2x \operatorname{sen}\left(\frac{1}{x}\right))}{x(1-\sqrt{\frac{1}{x^2}+1})} = \lim_{x\to +\infty} \dfrac{(\frac{\sqrt{x}}{x}\cos x + 2x \operatorname{sen}\left(\frac{1}{x}\right))}{(1-\sqrt{\frac{1}{x^2}+1})}.$

Como $\lim_{x\to +\infty} \dfrac{\sqrt{x}}{x} \overbrace{\cos x}^{\text{ltdo}} = 0$; $\lim_{x\to +\infty} 2x \operatorname{sen}\left(\dfrac{1}{x}\right) = 2\lim_{u\to 0} \dfrac{\operatorname{sen} u}{u} = 3$; $\lim_{x\to +\infty} (1-\sqrt{\dfrac{1}{x^2}+1}) = 0$ e $(1-\sqrt{\frac{1}{x^2}+1}) < 0$ para todo $x \neq 0$, temos que $\lim_{x\to +\infty} \dfrac{\sqrt{x}\cos x + 2x^2 \operatorname{sen}\left(\frac{1}{x}\right)}{x-\sqrt{1+x^2}} = -\infty.$