Questão 1) (Valor: 3.0) Calcule o limite ou explique por que não existe.

a) $\lim_{x\to 2} \dfrac{\sqrt{x+2}-2}{\sqrt{x^3+x-9}-\sqrt{x^3-7}}$

b) $\lim_{x\to 1} \dfrac{\sqrt{x^3+2x^2-7x+4}}{x^2-1}$

c) $\lim_{x\to +\infty} \dfrac{\sqrt{x}\,\cos x + 3x^2 \,\text{sen}\left(\frac{1}{x}\right)}{x-\sqrt{1+x^2}}$

Resolução:

$$
\begin{aligned}
\text{a) } \lim_{x\to 2} \frac{\sqrt{x+2}-2}{\sqrt{x^3+x-9}-\sqrt{x^3-7}} &= \lim_{x\to 2} \frac{(\sqrt{x+2}-2)(\sqrt{x+2}+2)}{(\sqrt{x^3+x-9}-\sqrt{x^3-7})(\sqrt{x+2}+2)} \\
&= \lim_{x\to 2} \frac{x-2}{(\sqrt{x^3+x-9}-\sqrt{x^3-7})(\sqrt{x+2}+2)} \\
&= \lim_{x\to 2} \frac{(x-2)(\sqrt{x^3+x-9}+\sqrt{x^3-7})}{(\sqrt{x^3+x-9}-\sqrt{x^3-7})(\sqrt{x^3+x-9}+\sqrt{x^3-7})(\sqrt{x+2}+2)} \\
&= \lim_{x\to 2} \frac{(x-2)(\sqrt{x^3+x-9}+\sqrt{x^3-7})}{(x-2)(\sqrt{x+2}+2)} \\
&= \lim_{x\to 2} \frac{(\sqrt{x^3+x-9}+\sqrt{x^3-7})}{(\sqrt{x+2}+2)} = \frac{2}{4} = \frac{1}{2}.
\end{aligned}
$$

$$
\text{b) } \lim_{x\to 1} \frac{\sqrt{x^3+2x^2-7x+4}}{x^2-1} = \lim_{x\to 1} \frac{\sqrt{(x-1)^2(x+4)}}{x^2-1} = \lim_{x\to 1} \frac{|x-1|\sqrt{(x+4)}}{(x-1)(x+1)}
$$

$$
\lim_{x\to 1+} \frac{|x-1|\sqrt{(x+4)}}{(x-1)(x+1)} = \lim_{x\to 1+} \frac{(x-1)\sqrt{(x+4)}}{(x-1)(x+1)} = \frac{\sqrt{5}}{2}.
$$

$$
\lim_{x\to 1-} \frac{|x-1|\sqrt{(x+3)}}{(x-1)(x+1)} = \lim_{x\to 1-} \frac{-(x-1)\sqrt{(x+4)}}{(x-1)(x+1)} = \frac{-\sqrt{5}}{2}.
$$

Logo, tal limite não existe.

$$
\text{c) } \lim_{x\to +\infty} \frac{\sqrt{x}\,\cos x + 3x^2\,\text{sen}\left(\frac{1}{x}\right)}{x-\sqrt{1+x^2}} = \lim_{x\to +\infty} \frac{x(\frac{\sqrt{x}}{x}\cos x + 3x\,\text{sen}\left(\frac{1}{x}\right))}{x(1-\sqrt{\frac{1}{x^2}+1})} = \lim_{x\to +\infty} \frac{(\frac{\sqrt{x}}{x}\cos x + 3x\,\text{sen}\left(\frac{1}{x}\right))}{(1-\sqrt{\frac{1}{x^2}+1})}.
$$

Como $\lim_{x\to +\infty} \frac{\sqrt{x}}{x} \overbrace{\cos x}^{\text{ltdo}} = 0$; $\lim_{x\to +\infty} 3x\,\text{sen}\left(\frac{1}{x}\right) = 3\lim_{u\to 0} \frac{\text{sen}\, u}{u} = 3$; $\lim_{x\to +\infty} (1-\sqrt{\frac{1}{x^2}+1}) = 0$ e $(1-\sqrt{\frac{1}{x^2}+1}) < 0$ para todo $x \neq 0$, temos que $\lim_{x\to +\infty} \frac{\sqrt{x}\,\cos x + 3x^2\,\text{sen}\left(\frac{1}{x}\right)}{x-\sqrt{1+x^2}} = -\infty$.