Questão 2-A. (3,0 pontos) Calcule

a) $$\int x^2 \operatorname{arctg} x \, dx = \frac{x^3}{3}\operatorname{arctg}(x) - \frac{1}{3}\int \frac{x^3}{1+x^2}dx = \frac{x^3}{3}\operatorname{arctg}(x) - \frac{1}{6}\int \frac{u-1}{u}du =$$

$u = \operatorname{arctg}(x)$	$du = \frac{1}{1+x^2}dx$
$dv = x^2dx$	$v = \frac{x^3}{3}$

$u = 1 + x^2 \quad du = 2xdx$

$$= \frac{x^3}{3}\operatorname{arctg}(x) - \frac{1}{6}[u - \ln|u|] + k = \frac{x^3}{3}\operatorname{arctg}(x) - \frac{1+x^2}{6} + \frac{\ln(1+x^2)}{6} + k.$$

b) $$\int \frac{5e^{2x} - e^x}{e^{2x} - 1}dx = \int \frac{5u-1}{u^2-1}du = \int \frac{5u-1}{(u+1)(u-1)}du = \int \left[\frac{A}{(u+1)} + \frac{B}{(u-1)}\right]du =$$

$u = e^x \quad du = e^x dx$

$A = 3 \quad B = 2$

$$= \int \left[\frac{3}{(u+1)} + \frac{2}{(u-1)}\right]du = 3\ln|u+1| + 2\ln|u-1| + k = 3\ln|e^x+1| + 2\ln|e^x-1| + k.$$

c) $$\int x(\ln x)^2\,dx = \frac{x^2}{2}(\ln x)^2 - \int x\ln x dx = \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \int \frac{x}{2}dx =$$

$u = (\ln x)^2$	$du = \frac{2\ln x}{x}dx$
$dv = xdx$	$v = \frac{x^2}{2}$

$u = \ln x$	$du = \frac{1}{x}dx$
$dv = xdx$	$v = \frac{x^2}{2}$

$$= \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{x^2}{4} + k.$$