

Questão 2-B. (3,0 pontos) Calcule

$$\text{a) } \int x^2 \operatorname{arctg} x \, dx = \frac{x^3}{3} \operatorname{arctg}(x) - \frac{1}{3} \int \frac{x^3}{1+x^2} dx = \frac{x^3}{3} \operatorname{arctg}(x) - \frac{1}{6} \int \frac{u-1}{u} du =$$

$$\boxed{\begin{array}{l} u = \operatorname{arctg}(x) \quad du = \frac{1}{1+x^2} dx \\ dv = x^2 dx \quad v = \frac{x^3}{3} \end{array}}$$

$$\boxed{u = 1 + x^2 \quad du = 2x dx}$$

$$= \frac{x^3}{3} \operatorname{arctg}(x) - \frac{1}{6} [u - \ln |u|] + k = \frac{x^3}{3} \operatorname{arctg}(x) - \frac{1+x^2}{6} + \frac{\ln(1+x^2)}{6} + k.$$

$$\text{b) } \int \frac{5e^{2x} + e^x}{e^{2x} - 1} dx = \int \frac{5u+1}{u^2-1} du = \int \frac{5u+1}{(u+1)(u-1)} du = \int \left[\frac{A}{(u+1)} + \frac{B}{(u-1)} \right] du =$$

$$\boxed{u = e^x \quad du = e^x dx}$$

$$\boxed{A = 2 \quad B = 3}$$

$$= \int \left[\frac{2}{(u+1)} + \frac{3}{(u-1)} \right] du = 2 \ln |u+1| + 3 \ln |u-1| + k = 2 \ln |e^x + 1| + 3 \ln |e^x - 1| + k.$$

$$\text{c) } \int x(\ln x)^2 dx = \frac{x^2}{2} (\ln x)^2 - \int x \ln x dx = \frac{x^2}{2} (\ln x)^2 - \frac{x^2}{2} \ln x + \int \frac{x}{2} dx =$$

$$\boxed{\begin{array}{l} u = (\ln x)^2 \quad du = \frac{2 \ln x}{x} dx \\ dv = x dx \quad v = \frac{x^2}{2} \end{array}}$$

$$\boxed{\begin{array}{l} u = \ln x \quad du = \frac{1}{x} dx \\ dv = x dx \quad v = \frac{x^2}{2} \end{array}}$$

$$= \frac{x^2}{2} (\ln x)^2 - \frac{x^2}{2} \ln x + \frac{x^2}{4} + k.$$