Questão 1. Calcule o limite ou justifique porque não existe:

a) (1,0 ponto) $\lim_{x\to+\infty} \dfrac{2x+5\,\text{sen}\,x}{x\sqrt{x}\,\text{sen}\left(\frac{3}{\sqrt{x}}\right)}$

b) (1,0 ponto) $\lim_{x\to 2} \left(\dfrac{1}{x^2-2x} - \dfrac{x+1}{x-2}\right)$

c) (1,0 ponto) $\lim_{x\to-\infty} \left(\sqrt{3x^6+2} - \sqrt{3x^6+2x^3-5}\right)$

a) $\lim_{x\to+\infty} \dfrac{2x+5\,\text{sen}x}{x\sqrt{x}\,\text{sen}\left(\frac{3}{\sqrt{x}}\right)} = \lim_{x\to+\infty} \dfrac{2x}{x\sqrt{x}\,\text{sen}\left(\frac{3}{\sqrt{x}}\right)} + \dfrac{5\,\text{sen}\,x}{x\sqrt{x}\,\text{sen}\left(\frac{3}{\sqrt{x}}\right)}$

$= \lim_{x\to+\infty} \dfrac{2}{\sqrt{x}\,\text{sen}\left(\frac{3}{\sqrt{x}}\right)} + \dfrac{1}{x}\text{sen}\,x \cdot \dfrac{5}{\sqrt{x}\,\text{sen}\left(\frac{3}{\sqrt{x}}\right)} = (*)$

Note que:

(i) $\lim_{x\to+\infty} \sqrt{x}\,\text{sen}\left(\frac{3}{\sqrt{x}}\right) = \lim_{x\to+\infty} \dfrac{3\,\text{sen}\left(\frac{3}{\sqrt{x}}\right)}{\frac{3}{\sqrt{x}}} = 3$ onde usamos o limite fundamental, uma vez que $\lim_{x\to+\infty} \dfrac{3}{\sqrt{x}} = 0$

(ii) $\lim_{x\to+\infty} \dfrac{1}{x}\text{sen}\,x = 0$ pelo corolário do Teorema do Confronto; pois $\lim_{x\to+\infty} \dfrac{1}{x} = 0$ & $|\text{sen}\,x| \leq 1$ (limitado).

Logo,

$(*) = \dfrac{2}{3} + 0 \cdot \dfrac{5}{3} = \boxed{\dfrac{2}{3}}$

b) $\lim\limits_{x\to 2}\left(\dfrac{1}{(x^2-2x)} - \dfrac{x+1}{x-2}\right) = \lim\limits_{x\to 2}\left(\dfrac{1-x^2-x}{x(x-2)}\right)$

Note que:

(i) $\lim\limits_{x\to 2} 1-x^2-x = -5$ (negativo)

(ii) $\lim\limits_{x\to 2^-} x(x-2) = 0^-$ (tende para zero por valores negativos)

(iii) $\lim\limits_{x\to 2^+} x(x-2) = 0^+$ (Tende para zero por valores positivos)

logo,

$$\left.\begin{aligned}&\lim_{x\to 2^-}\left(\frac{1}{x^2-2x} - \frac{x+1}{x-2}\right) = +\infty\\ &\lim_{x\to 2^+}\left(\frac{1}{x^2-2x} - \frac{x+1}{x-2}\right) = -\infty\end{aligned}\right\}$$

o limite não existe pois os limites laterais são diferentes

c) $\lim\limits_{x\to -\infty}\left(\sqrt{3x^6+2} - \sqrt{3x^6+2x^3-5}\right)$

$$= \lim_{x\to -\infty} \frac{\left(\sqrt{3x^6+2} - \sqrt{3x^6+2x^3-5}\right)\left(\sqrt{3x^6+2} + \sqrt{3x^6+2x^3-5}\right)}{\left(\sqrt{3x^6+2} + \sqrt{3x^6+2x^3-5}\right)} =$$

$$= \lim_{x\to -\infty} \frac{3x^6+2-3x^6-2x^3+5}{\sqrt{3x^6+2} + \sqrt{3x^6+2x^3-5}} = \lim_{x\to -\infty} \frac{x^3\left(-2+7/x^3\right)}{|x^3|\left(\sqrt{3+2/x^6} + \sqrt{3+2/x^3-5/x^6}\right)} = *$$

Note que

$$\left.\begin{aligned}&\lim_{x\to -\infty}\frac{x^3}{|x^3|} = -1\\ &\lim_{x\to -\infty}\frac{c}{x^n} = 0, \quad n \geq 1\end{aligned}\right\} \Rightarrow * = \frac{2}{2\sqrt{3}} = \boxed{\frac{1}{\sqrt{3}}}$$