1. Calcule as seguintes integrais:

   (a) **(1,5)** $\int \ln(x^2+4x+5)\mathrm{d}x$

   (b) **(1,5)** $\int \frac{x^5}{\sqrt{(4-x^2)^3}}\mathrm{d}x$

(a) $\int \ln(x^2+4x+5)\,dx = x\ln(x^2+4x+5) - \int \frac{x(2x+4)}{x^2+4x+5}\,dx$

integração por partes

$$\boxed{\begin{array}{l} u = \ln(x^2+4x+5) \\ du = \frac{2x+4}{x^2+4x+5}\,dx \\ dv = dx \qquad v = x \end{array}}$$

Temos então que calcular $2\int \frac{x^2+2x}{x^2+4x+5}\,dx$. Como o grau do numerador é igual ao grau do denominador, efetuamos a divisão:

$$\begin{array}{l|l} x^2+2x & x^2+4x+5 \\ \underline{-x^2-4x} & 1 \\ -2x-5 & \end{array}$$

Logo $\int \frac{x^2+2x}{x^2+4x+5}\,dx = \int dx + \int \frac{-2x-5}{x^2+4x+5}\,dx$

$= x - \int \frac{2x+5}{x^2+4x+5}\,dx$.

Calcular então $\int \frac{2x+5}{x^2+4x+5}\,dx = \int \frac{2x+5\;dx}{(x+2)^2+1} = \int \frac{(2y-4+5)\,dy}{y^2+1}$

$$\boxed{\begin{array}{l} y = x+2 \\ dy = dx \\ x = y-2 \end{array}}$$

$= \int \frac{2y\,dy}{y^2+1} + \int \frac{dy}{y^2+1}$

$= \ln(y^2+1) + \operatorname{arctg} y + C = \ln(x^2+4x+5) + \operatorname{arctg}(x+2) + C$

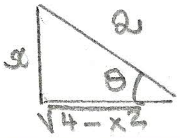
Portanto:

$$\boxed{\int \ln(x^2+4x+5)\,dx = x\ln(x^2+4x+5) + 2\left[x - \ln(x^2+4x+5) - \operatorname{arctg}(x+2)\right] + k, \quad k \in \mathbb{R}}$$

(b) $\displaystyle\int \frac{x^5\,dx}{\sqrt{(4-x^2)}^3} \overset{(*)}{=} \int \frac{2^5 \operatorname{sen}^5\theta \; 2\cos\theta \; d\theta}{2^3 \cos^3\theta}$

(*) Substituição Trigonométrica

$$\boxed{\begin{aligned} &* \quad x = 2\operatorname{sen}\theta \,, \quad \theta \in \left]-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right[\\ &dx = 2\cos\theta\, d\theta \\ &\sqrt{4-x^2} = 2\cos\theta \end{aligned}}$$



$$= 2^3 \int \frac{\operatorname{sen}^5\theta\, d\theta}{\cos^2\theta} = 2^3 \int \frac{(1-\cos^2\theta)^2}{\cos^2\theta} \operatorname{sen}\theta\, d\theta$$

$$= 2^3 \left[\int \frac{1}{\cos^2\theta} \operatorname{sen}\theta\, d\theta - 2\int \operatorname{sen}\theta\, d\theta + \int \cos^2\theta \operatorname{sen}\theta\, d\theta \right]$$

$$\underset{\boxed{\begin{aligned} &u = \cos\theta \\ &du = -\operatorname{sen}\theta\, d\theta \end{aligned}}}{=} -2^3 \left[\int \frac{du}{u^2} - 2\int du + \int u^2\, du \right]$$

$$= -2^3 \left[\frac{u^{-1}}{-1} - 2u + \frac{u^3}{3} \right] + C$$

$$= -2^3 \left[-\frac{1}{\cos\theta} - 2\cos\theta + \frac{\cos^3\theta}{3} \right] + C$$

$$= -2^3 \left[-\frac{2}{\sqrt{4-x^2}} - \frac{2\sqrt{4-x^2}}{2} + \frac{\left(\sqrt{4-x^2}\right)^3}{3.8} \right] + C$$

Assim:

$$\boxed{\int \frac{x^5}{\sqrt{(4-x^2)}^3}\, dx = \frac{16}{\sqrt{4-x^2}} + 8\sqrt{4-x^2} + \frac{\left(\sqrt{4-x^2}\right)^3}{3} + C}$$