1. Calcule as seguintes integrais:

(a) **(1,5)** $\int \ln(x^2 - 4x + 5)\mathrm{d}x$

(b) **(1,5)** $\int \frac{x^5}{\sqrt{(9-x^2)^3}}\mathrm{d}x$

(a) $\int \underbrace{\ln(x^2-4x+5)}_{u}\underbrace{dx}_{dv} = x\ln(x^2-4x+5) - \int \frac{x(2x-4)}{x^2-4x+5}dx$

Utilizar integração por partes:

$u = \ln(x^2-4x+5)$

$du = \frac{2x-4}{x^2-4x+5}dx$

$dv = dx$

$v = x$

$= x\ln(x^2-4x+5) - 2\int \frac{x^2-2x}{x^2-4x+5}dx$

Calcular $\int \frac{x^2-2x}{x^2-4x+5}dx \overset{(*)}{=} \int dx + \int \frac{2x-5}{x^2-4x+5}dx$

(*) Como o grau do numerador é igual ao grau do denominador, temos que efetuar a divisão

$$\begin{array}{r|l} \cancel{x^2} - 2x & x^2-4x+5 \\ \hline -\cancel{x^2}+4x-5 & 1 \\ \hline 2x-5 & \end{array}$$

Calcular $\int \frac{2x-5}{x^2-4x+5}dx = \int \frac{2x-5}{(x-2)^2+1}dx \underset{\substack{y=x-2\\ dy=dx\\ x=y+2}}{=} \int \frac{2(y+2)-5}{y^2+1}dy$

$= \int \frac{2y}{y^2+1}dy - \int \frac{dy}{y^2+1} = \ln(y^2+1) - \operatorname{arctg} y + C$

$= \ln(x^2-4x+5) - \operatorname{arctg}(x-2) + C'$

Assim

$$\int \ln(x^2-2x+5)dx = x\ln(x^2-4x+5) - 2\left[x + \ln(x^2-4x+5) - \operatorname{arctg}(x-2)\right] + k$$

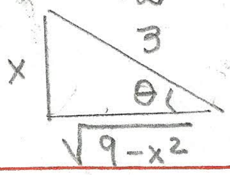
(b) $\displaystyle\int \frac{x^5}{\sqrt{(9-x^2)^3}}\,dx \overset{(*)}{=} \int \frac{3^5 \operatorname{sen}^5\theta \; 3\cos\theta \, d\theta}{3^3 \cos^3\theta}$

(*) Substituição Trigonométrica

$x = 3 \operatorname{sen}\theta \qquad -\pi/2 < \theta < \pi/2$

$dx = 3\cos\theta \, d\theta$

$\sqrt{9-x^2} = 3\cos\theta$



$$= 3^3 \int \frac{\operatorname{sen}^5\theta}{\cos^2\theta}\,d\theta = 3^3 \int \frac{(1-\cos^2\theta)^2}{\cos^2\theta}\operatorname{sen}\theta \, d\theta$$

$$= -3^3 \int \frac{(1-u^2)^2}{u^2}\,du = -3^3 \int \left(\frac{1}{u^2} - 2 + u^2\right) du$$

$u = \cos\theta$

$du = -\operatorname{sen}\theta \, d\theta$

$$= -3^3 \left[-\frac{1}{u} - 2u + \frac{u^3}{3}\right] + C$$

$$= -3^3 \left[-\frac{1}{\cos\theta} - 2\cos\theta + \frac{\cos^3\theta}{3}\right] + C$$

$$\boxed{= -3^3 \left[-\frac{3}{\sqrt{9-x^2}} - \frac{2\sqrt{9-x^2}}{3} + \frac{\left(\sqrt{9-x^2}\right)^3}{3^4}\right] + C}$$