

2. (2,0) Calcule a integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{6}} \frac{\sec^2(x)}{(\operatorname{tg}(x) - 2)(\operatorname{tg}(x) - 1)^2} dx.$$

Fazendo a substituição  $t = \operatorname{tg} x$ , temos  $dt = \sec^2 x dx$ ,

$$x = -\frac{\pi}{4} \Rightarrow t = -1 \text{ e } x = \frac{\pi}{6} \Rightarrow t = \frac{1}{\sqrt{3}}$$

Assim, a integral fica  $\int_{-1}^{\frac{1}{\sqrt{3}}} \frac{1}{(t-2)(t-1)^2} dt$ .

$$\text{Agora, } \frac{1}{(t-2)(t-1)^2} = \frac{A}{t-2} + \frac{B}{t-1} + \frac{C}{(t-1)^2} = \frac{A(t-1)^2 + B(t-1)(t-2) + C(t-2)}{(t-2)(t-1)^2}$$

Resolvendo o sistema, obtemos  $A=1$ ,  $B=-1$  e  $C=-1$ .

Portanto,

$$\begin{aligned} \int_{-1}^{\frac{1}{\sqrt{3}}} \frac{1}{(t-2)(t-1)^2} dt &= \int_{-1}^{\frac{1}{\sqrt{3}}} \left( \frac{1}{t-2} - \frac{1}{t-1} - \frac{1}{(t-1)^2} \right) dt = \left( \ln|t-2| - \ln|t-1| + \frac{1}{t-1} \right) \Big|_{-1}^{\frac{1}{\sqrt{3}}} \\ &= \ln\left(2 - \frac{1}{\sqrt{3}}\right) - \ln\left(1 - \frac{1}{\sqrt{3}}\right) + \frac{1}{\frac{1}{\sqrt{3}} - 1} - \ln 3 + \ln 2 - \frac{1}{2}. \end{aligned}$$

3. (1,5) Seja  $F(x) = \int_0^{\sqrt{x}} (x+t^2)e^{t^2} dt$ ,  $x > 0$ . Determine  $F'(x)$  e  $F''(x)$ .

$$F(x) = x \int_0^{\sqrt{x}} e^{t^2} dt + \int_0^{\sqrt{x}} t^2 e^{t^2} dt. \quad \text{As funções } e^{t^2} \text{ e } t^2 e^{t^2} \text{ são contínuas. Pelo TFC,}$$

$$F'(x) = \int_0^{\sqrt{x}} e^{t^2} dt + x e^x \cdot \frac{1}{2\sqrt{x}} + x e^x \cdot \frac{1}{2\sqrt{x}} = \int_0^{\sqrt{x}} e^{t^2} dt + \sqrt{x} e^x$$

e

$$F''(x) = e^x \cdot \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x}} e^x + \sqrt{x} e^x = e^x \left( \frac{1}{\sqrt{x}} + \sqrt{x} \right)$$