2. **(2,0)** Calcule a integral

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{\sec^2(x)}{(\mathrm{tg}(x)-1)(\mathrm{tg}(x)-2)^2}dx.$$

Fazendo a substituição $t = \mathrm{tg}\, x$, temos $dt = \sec^2 x\, dx$,

$x = -\pi/3 \Rightarrow t = -\sqrt{3}$ e $x = \pi/6 \Rightarrow t = 1/\sqrt{3}$.

Assim, a integral fica $\displaystyle\int_{-\sqrt{3}}^{1/\sqrt{3}} \frac{1}{(t-1)(t-2)^2}\,dt$

Agora, $\displaystyle\frac{1}{(t-1)(t-2)^2} = \frac{A}{t-1} + \frac{B}{t-2} + \frac{C}{(t-2)^2} = \frac{A(t-2)^2 + B(t-1)(t-2) + C(t-1)}{(t-1)(t-2)^2}$

Resolvendo o sistema, obtemos $A = 1$, $B = -1$ e $C = 1$.

Portanto,

$$\int_{-\sqrt{3}}^{1/\sqrt{3}} \frac{1}{(t-1)(t-2)^2}\,dt = \int_{-\sqrt{3}}^{1/\sqrt{3}} \left(\frac{1}{t-1} - \frac{1}{t-2} + \frac{1}{(t-2)^2}\right)dt = \left(\ln|t-1| - \ln|t-2| - \frac{1}{t-2}\right)\Bigg|_{-\sqrt{3}}^{1/\sqrt{3}}$$

$$= \ln\left(1-\frac{1}{\sqrt{3}}\right) - \ln\left(2-\frac{1}{\sqrt{3}}\right) - \frac{1}{\frac{1}{\sqrt{3}}-2} - \ln(1+\sqrt{3}) + \ln(2+\sqrt{3}) - \frac{1}{\sqrt{3}+2}$$

3. **(1,5)** Seja $F(x) = \int_0^{\sqrt{x}} (x+t^2)e^{t^2}\,dt$, $x > 0$. Determine $F'(x)$ e $F''(x)$.

$F(x) = x\displaystyle\int_0^{\sqrt{x}} e^{t^2}\,dt + \int_0^{\sqrt{x}} t^2 e^{t^2}\,dt$. As funções e^{t^2} e $t^2e^{t^2}$ são contínuas. Pelo TFC,

$$F'(x) = \int_0^{\sqrt{x}} e^{t^2}\,dt + x.e^{x}.\frac{1}{2\sqrt{x}} + xe^{x}.\frac{1}{2\sqrt{x}} = \int_0^{\sqrt{x}} e^{t^2}\,dt + \sqrt{x}e^{x}$$

e

$$F''(x) = e^{x}.\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}e^{x} + \sqrt{x}e^{x} = e^{x}\left(\frac{1}{\sqrt{x}} + \sqrt{x}\right)$$