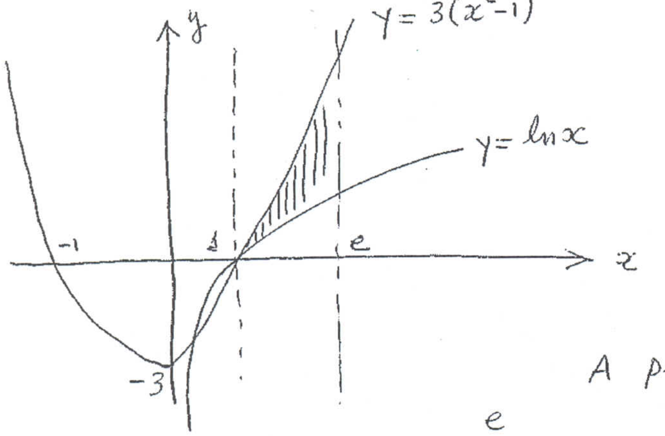


(a) Esboço da região R



(b) V_x volume do sólido obtido pela rotação de R em torno do eixo x

$$V_x = \pi \int_1^e f(x)^2 dx - \pi \int_1^e g(x)^2 dx$$

A primeira integral:

$$\pi \int_1^e \left(3(x^2-1)\right)^2 dx = 9\pi \int_1^e (x^4 - 2x^2 + 1)\, dx = 9\pi \left[\frac{x^5}{5} - \frac{2x^3}{3} + x\right]_1^e =$$

$$= 9\pi \left[\left(\frac{e^5}{5} - \frac{2e^3}{3} + e\right) - \left(\frac{1}{5} - \frac{2}{3} + 1\right)\right] = \frac{9\pi}{5} e^5 - 6\pi e^3 + 9\pi e - \frac{24}{5}\pi$$

A segunda integral será feita por partes:

$$\ln x \xrightarrow{D} \frac{1}{x}$$

$$\ln x \xrightarrow{I} x\ln x - x$$

$$\pi \int_1^e \ln^2 x\, dx = \pi \underbrace{\left[(\ln x)(x\ln x - x)\right]_1^e}_{\equiv 0} - \pi \int_1^e (\ln x - 1)\, dx$$

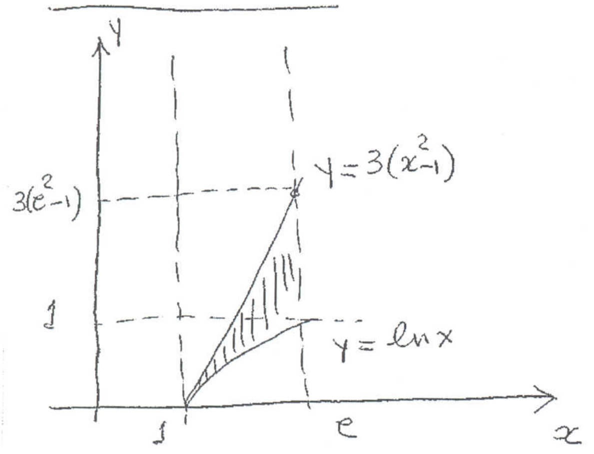
$$\pi \int_1^e \ln^2 x\, dx = -\pi \left[x\ln x - 2x\right]_1^e = -\pi\left[-e + 2\right] = \pi e - 2\pi$$

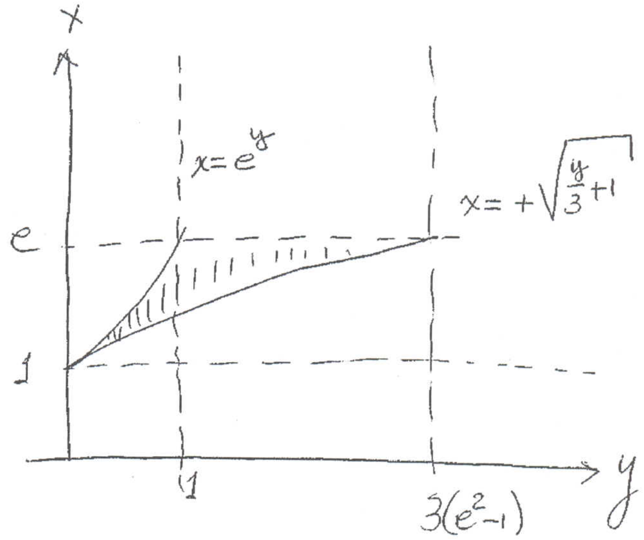
Assim, $V_x = \left(\frac{9\pi}{5} e^5 - 6\pi e^3 + 9\pi e - \frac{24}{5}\pi\right) - \left(\pi e - 2\pi\right)$

$$\boxed{V_x = \frac{9\pi}{5} e^5 - 6\pi e^3 + 8\pi e - \frac{12\pi}{5}}$$

(c) V_y = volume do sólido obtido pela rotação de R em torno do eixo y:

Primeira solução:





$$V_y = \pi\int_0^1 (e^y)^2\,dy - \pi\int_0^1 \left(\sqrt{\frac{y}{3}+1}\right)^2 dy + \pi e^2(3e^2-3-1) - \pi\int_1^{3(e^2-1)} \left(\sqrt{\frac{y}{3}+1}\right)^2 dy$$

$$V_y = \pi\left[\frac{e^{2y}}{2}\right]_0^1 - \pi\left[\frac{y^2}{6}+y\right]_0^1 + \pi e^2(3e^2-4) - \pi\left[\frac{y^2}{6}+y\right]_1^{3(e^2-1)}$$

$$V_y = \pi\left(\frac{e^2}{2}-\frac{1}{2}\right) - \pi\left(\frac{1}{6}+1\right) + \pi e^2(3e^2-4) - \pi\left[\frac{3(e^2-1)^2}{2} + 3(e^2-1) - \frac{1}{6} - 1\right]$$

$$\boxed{V_y = \frac{3\pi}{2}e^4 - \frac{7\pi}{2}e^2 + \pi}$$

Segunda solução: $V_y = 2\pi\int_1^e x\,3(x^2-1)\,dx - 2\pi\int_1^e x\ln x\,dx$

$$V_y = 2\pi\left[\frac{3x^4}{4} - \frac{3x^2}{2}\right]_1^e - 2\pi\int_1^e x\ln x\,dx$$

$$V_y = 2\pi\left[\left(\frac{3e^4}{4}-\frac{3e^2}{2}\right) - \left(\frac{3}{4}-\frac{3}{2}\right)\right] - 2\pi\int_1^e x\ln x\,dx$$

$$2\pi \int_1^e x \ln x \, dx = \left[\frac{x^2}{2} \ln x \right]_1^e \pi 2 - 2\pi \cdot \frac{1}{2} \int_1^e x \, dx$$

$$\ln x \xrightarrow{D} \frac{1}{x}$$

$$x \xrightarrow{I} \frac{x^2}{2}$$

$$2\pi \int_1^e x \ln x \, dx = 2\pi \left(\frac{e^2}{2} \right) - \pi \left[\frac{x^2}{2} \right]_1^e$$

$$= \pi e^2 - \pi \left(\frac{e^2}{2} - \frac{1}{2} \right) = \frac{\pi e^2}{2} + \frac{\pi}{2}$$

Assim,

$$V_y = 2\pi \left[\left(\frac{3e^4}{4} - \frac{3e^2}{2} \right) - \left(-\frac{3}{4} \right) \right] - \frac{\pi e^2}{2} - \frac{\pi}{2}$$

$$\boxed{V_y = \frac{3\pi}{2} e^4 - \frac{7\pi}{2} e^2 + \pi}$$