1. (a) **(1,0)** Seja $f(x) = \cos(\sqrt[3]{x^2})$. Calcule $f'(x)$ nos pontos $x \in \mathbb{R}$ nos quais ela existe.

(b) **(1,0)** Calcule $\lim\limits_{x \to 0^+} \operatorname{tg} x^{\frac{1}{\ln x}}$

(a) $f'(x) = -\operatorname{sen}(\sqrt[3]{x^2}) \cdot \frac{2}{3} x^{-1/3} \quad \forall x \neq 0$

Para $x = 0$

$$f'(0) = \lim_{x \to 0} \frac{\cos\sqrt[3]{x^2} - 1}{x} = \lim_{x \to 0} \frac{\cos\sqrt[3]{x^2} - 1}{x} \cdot \frac{(\cos\sqrt[3]{x^2} + 1)}{(\cos\sqrt[3]{x^2} + 1)}$$

$$= \lim_{x \to 0} \frac{-\operatorname{sen}^2\sqrt[3]{x^2}}{x(\cos\sqrt[3]{x^2} + 1)} = \lim_{x \to 0} -\Big(\underbrace{\frac{\operatorname{sen}\sqrt[3]{x^2}}{x^{2/3}}}_{(*)\ \to 1}\Big)^2 \frac{x^{1/3} \to 0}{\underbrace{\cos\sqrt[3]{x^2} + 1}_{\to 1}}$$

$= -1 \cdot 0 \quad = 0$

$$\boxed{(*) \lim_{x \to 0} \frac{\operatorname{sen}\sqrt[3]{x^2}}{x^{2/3}} = \lim_{u \to 0} \frac{\operatorname{sen} u}{u} = 1}$$

$u = x^{2/3}$

$x \to 0 \Rightarrow u \to 0$

↳ limite fundamental

(b) $\operatorname{tg} x^{1/\ln x} = e^{\frac{1}{\ln x} \ln \operatorname{tg} x}$

Como a função $f(x) = e^x$ é contínua, se $\lim\limits_{x \to 0^+} \frac{\ln \operatorname{tg} x}{\ln x} = L$ então $\lim\limits_{x \to 0^+} e^{\frac{\ln \operatorname{tg} x}{\ln x}} = e^L$.

Vamos então calcular L.

$$\lim_{x \to 0^+} \frac{\overset{\to -\infty}{\ln \operatorname{tg} x}}{\underset{\to -\infty}{\ln x}} \underset{L'H}{\overset{\left(\frac{\infty}{\infty}\right)}{=}} \lim_{x \to 0^+} \frac{\frac{1}{\operatorname{tg} x} \cdot \sec^2 x}{1/x} = \lim_{x \to 0^+} \frac{x \cancel{\cos x}}{\operatorname{sen} x} \cdot \frac{1}{\cos^{\cancel{2}} x}$$

$$= \lim_{x \to 0^+} \underbrace{\frac{x}{\operatorname{sen} x}}_{\to 1 \text{ (limite fundamental)}} \cdot \frac{1}{\underbrace{\cos x}_{\to 1}} = 1.$$

Logo $\lim\limits_{x \to 0^+} \operatorname{tg} x^{1/\ln x} = e^1 = e$.