3. **(2,0)** Calcule a área da região limitada compreendida entre os gráficos de $f(x) = x^2\sqrt{1-x^2}$ e $g(x) = x^2(x-1)$.

$f(x) = x^2\sqrt{1-x^2}$

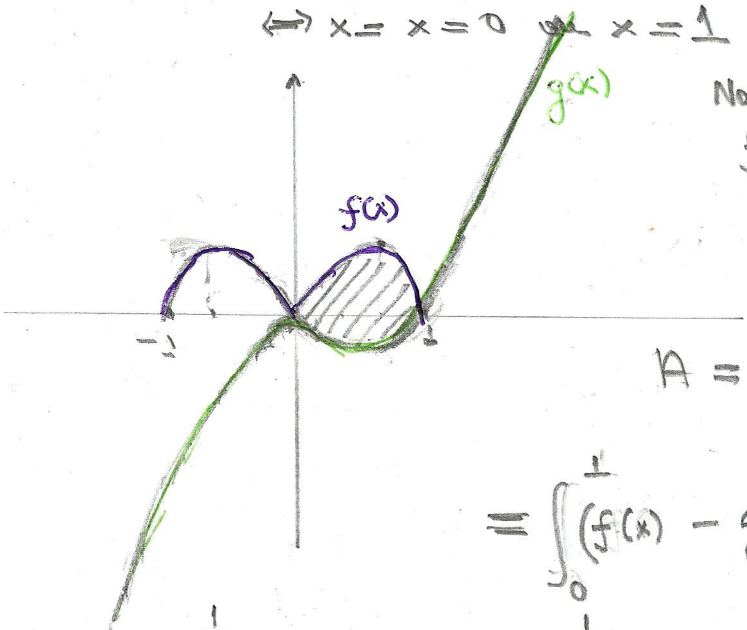
$g(x) = x^2(x-1)$

$f(x) = g(x) \iff x^2\sqrt{1-x^2} = x^2(x-1)$

$\iff x = 0$ ou $\sqrt{1-x^2} = x - 1$

$\sqrt{1-x^2} = x-1 \iff 1 - x^2 = x^2 - 2x + 1 \iff 2x^2 - 2x = 0$

$\iff \cancel{x=}\ x = 0$ ou $x = 1$



No intervalo $[0,1]$ temos que $f(x) \geq 0$ e $g(x) \leq 0$.

A área ~~A é~~

$$A = \int_0^1 |f(x) - g(x)|\,dx$$

$$= \int_0^1 (f(x) - g(x))\,dx$$

$$= \int_0^1 x^2\sqrt{1-x^2}\,dx - \int_0^1 (x^3 - x^2)\,dx$$

$$\int_0^1 (x^3 - x^2)\,dx = \frac{x^4}{4} - \frac{x^3}{3}\Big|_0^1 = \frac{1}{4} - \frac{1}{3} = -\frac{1}{12}$$

$$\int_0^1 x^2\sqrt{1-x^2}\,dx = \int_0^{\pi/2} \operatorname{sen}^2\theta \cos^2\theta\,d\theta = \int_0^{\pi/2} \frac{\operatorname{sen}^2(2\theta)}{4}\,d\theta$$

$x = \operatorname{sen}\theta$, $\theta \in [-\pi/2, \pi/2]$

$dx = \cos\theta\,d\theta$

$x = 0 \Rightarrow \theta = 0$

$x = 1 \Rightarrow \theta = \pi/2$

$$= \frac{1}{4}\int_0^{\pi/2} \frac{1}{2}(1 - \cos 4\theta)\,d\theta$$

$$= \frac{1}{8}\left[\theta - \frac{\operatorname{sen} 4\theta}{4}\right]_0^{\pi/2} = \frac{1}{16}\pi - \frac{1}{32}\operatorname{sen} 4(\pi/2)$$

$$= \frac{\pi}{16} \quad //$$

Logo $A = \dfrac{\pi}{16} + \dfrac{1}{12}$