5. Calcule as seguintes integrais:

(a) **(1,0)** $\displaystyle\int \frac{x\ln x}{\sqrt{x^2-1}}\mathrm{d}x$,

(b) **(1,0)** $\displaystyle\int \frac{\mathrm{e}^{2x}}{1+\mathrm{e}^{4x}}\mathrm{d}x$.

(a) por partes

$u = \ln x \Rightarrow du = \frac{1}{x}\,dx$

$dv = \frac{x}{\sqrt{x^2-1}}\,dx \Rightarrow v = \sqrt{x^2-1}$

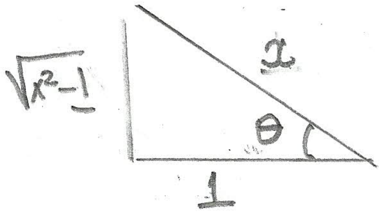
$$\int \frac{x\ln x}{\sqrt{x^2-1}}\,dx = (\ln x)\sqrt{x^2-1} - \int \frac{\sqrt{x^2-1}}{x}\,dx$$

$$\int \frac{\sqrt{x^2-1}}{x}\,dx = \int \frac{\operatorname{tg}\theta \sec\theta \operatorname{tg}\theta\, d\theta}{\sec\theta} =$$

$x = \sec\theta$, $\theta \in [0, \pi/2[\cup [\pi, 3\pi/2[$

$dx = \sec\theta \operatorname{tg}\theta\, d\theta$

$\sqrt{x^2-1} = |\operatorname{tg}\theta| = \operatorname{tg}\theta$



$$= \int \operatorname{tg}^2\theta\, d\theta = \int (\sec^2\theta - 1)\, d\theta = \operatorname{tg}\theta - \theta + C$$

$$= \sqrt{x^2-1} - \operatorname{arc\,sec}(x) + C,$$

Logo $\displaystyle\int \frac{x\ln x}{\sqrt{x^2-1}}\,dx = \ln x\sqrt{x^2-1} - \sqrt{x^2-1} + \operatorname{arc\,sec}(x) + C$

(b) $y = e^{2x}$ $dy = 2e^{2x}\,dx$

$$\int \frac{e^{2x}\,dx}{1+e^{4x}} = \frac{1}{2}\int \frac{dy}{1+y^2} = \frac{1}{2}\operatorname{arctg}(y) + C$$

$$= \frac{1}{2}\operatorname{arctg}(e^{2x}) + C$$