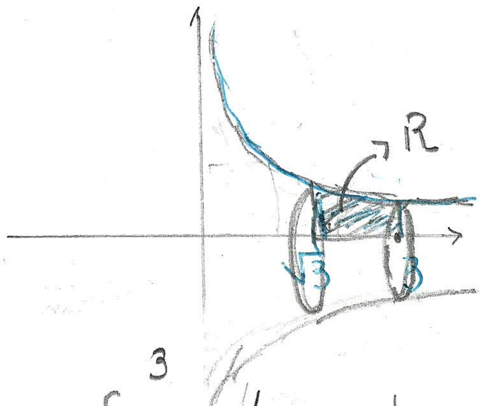
Questão 5. (2,0 pontos) Seja R a região compreendida entre o gráfico de $f(x) = \dfrac{1}{x\sqrt[4]{x^2+9}}$, o eixo Ox e as retas $x = \sqrt{3}$ e $x = 3$. Determine o volume do sólido obtido pela rotação de R ao redor do eixo Ox.



$f(x) > 0 \quad \forall\, x > 0$

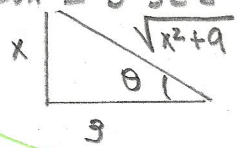
$\left(\lim_{x \to 0^+} f(x) = +\infty \quad e \quad \lim_{x \to +\infty} f(x) = 0 \right)$

$$V = \pi \int_{\sqrt{3}}^{3} \frac{1}{x^2\sqrt{x^2+9}}\, dx$$

$$\int_{\sqrt{3}}^{3} \frac{1}{x^2\sqrt{x^2+9}}\, dx =$$

$x = 3\,tg\,\theta, \quad \theta \in \left]-\pi/2, \pi/2\right[$

$dx = 3\sec^2\theta\, d\theta$



$x = 3 \Rightarrow 3\,tg\,\theta = 3 \Rightarrow tg\,\theta = 1 \Rightarrow \theta = \pi/4$

$x = \sqrt{3} \Rightarrow \sqrt{3} = 3\,tg\,\theta \Rightarrow tg\,\theta = \sqrt{3}/3 \Rightarrow \theta = \pi/6$

$$= \int_{\pi/6}^{\pi/4} \frac{1}{9\,tg^2\theta \cdot 3\sec\theta}\, 3\sec^2\theta\, d\theta$$

$$\frac{1}{9}\int_{\pi/6}^{\pi/4} \frac{\sec\theta}{tg^2\theta}\, d\theta = \frac{1}{9}\int_{\pi/6}^{\pi/4} \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{sen^2\theta}\, d\theta$$

$$= \frac{1}{9}\int_{\pi/6}^{\pi/4} \frac{\cos\theta}{sen^2\theta}\, d\theta = \frac{1}{9}\int_{1/2}^{\sqrt{2}/2} \frac{1}{u^2}\, du$$

$u = sen\,\theta$

$du = \cos\theta\, d\theta$

$\theta = \pi/6 \Rightarrow u = 1/2$

$\theta = \pi/4 \Rightarrow u = \sqrt{2}/2$

$$= \frac{1}{9}\left[-\frac{1}{u}\right]_{1/2}^{\sqrt{2}/2} = \frac{1}{9}\left[-\sqrt{2} + 2\right]$$

Logo $V = \dfrac{\pi}{9}\left[2 - \sqrt{2}\right]$