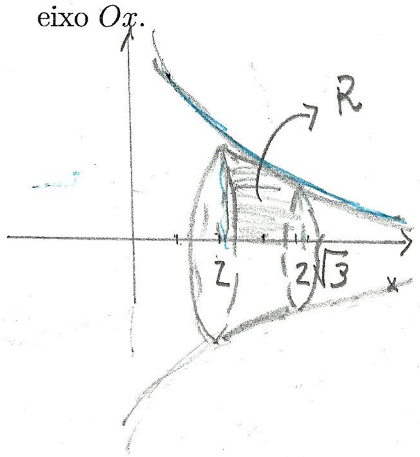
Questão 5. (2,0 pontos) Seja R a região compreendida entre o gráfico de $f(x) = \dfrac{1}{x\sqrt[4]{x^2+4}}$, o eixo Ox e as retas $x = 2$ e $x = 2\sqrt{3}$. Determine o volume do sólido obtido pela rotação de R ao redor do eixo Ox.

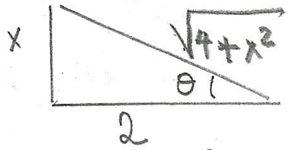


$f(x) > 0 \ \forall x > 0$

$\left(\lim\limits_{x \to 0^+} f(x) = +\infty \ \text{ e } \ \lim\limits_{x \to +\infty} f(x) = 0 \right)$

$$\boxed{V = \pi \int_2^{2\sqrt{3}} \frac{1}{x^2\sqrt{x^2+4}}\,dx}$$

$$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx \underset{\substack{x = 2\,tg\,\theta,\ \theta \in]-\pi/2,\,\pi/2[\\ dx = 2\sec^2\theta\,d\theta}}{=}$$



$$= \int \frac{2\sec^2\theta\,d\theta}{4\,tg^2\theta\, 2\sec\theta} = \frac{1}{4}\int \frac{\sec\theta\,d\theta}{tg^2\theta} = \frac{1}{4}\int \frac{\cos\theta\,d\theta}{sen^2\theta}$$

$$\underset{\substack{u = sen\,\theta \\ du = \cos\theta\,d\theta}}{=} \frac{1}{4}\int \frac{du}{u^2} = \frac{1}{4}\cdot\left(\frac{-1}{u}\right) + C = -\frac{1}{4\,sen\,\theta} + C$$

$$= -\frac{\sqrt{4+x^2}}{4x} + C$$

Logo $V = \pi \left[-\dfrac{\sqrt{4+x^2}}{4x} \right]_2^{2\sqrt{3}}$

$$= \pi\left[-\frac{4}{8\sqrt{3}} + \frac{2\sqrt{2}}{8} \right] = \pi\left[\frac{\sqrt{2}}{4} - \frac{\sqrt{3}}{6} \right]$$