1. (Turma A)

$$a)\ \lim_{x\to-\infty} \frac{4x^2-\operatorname{sen}(x^2)}{2x^3\operatorname{sen}(\frac{1}{x})-5x^2} = \lim_{x\to-\infty} \frac{x^2\left[4-\frac{1}{x^2}\operatorname{sen}(x^2)\right]}{x^2\left[2x\operatorname{sen}(\frac{1}{x})-5\right]} = \lim_{x\to-\infty} \frac{4-\frac{1}{x^2}\operatorname{sen}(x^2)}{2x\operatorname{sen}(\frac{1}{x})-5} = -\frac{4}{3}$$

pois

- $\lim_{x\to-\infty} \frac{1}{x^2}\operatorname{sen}(x^2) = 0$ ($\operatorname{sen}(x^2)$ é limitada e $\lim_{x\to-\infty} \frac{1}{x^2} = 0$)
- $\lim_{x\to-\infty} x\operatorname{sen}(\frac{1}{x}) = \lim_{x\to-\infty} \frac{\operatorname{sen}(\frac{1}{x})}{\frac{1}{x}} = 1$ (limite trigonométrico fundamental)

$$b)\ \lim_{x\to3}\left(\frac{1}{x^2-3x}-\frac{\operatorname{sen}(x^2-2x-3)}{x^2-6x+9}\right) = \lim_{x\to3}\left(\frac{1}{x(x-3)}-\frac{\operatorname{sen}(x^2-2x-3)}{(x-3)^2}\frac{(x+1)}{(x+1)}\right)$$
$$= \lim_{x\to3}\frac{1}{x-3}\left(\frac{1}{x}-\frac{\operatorname{sen}(x^2-2x-3)}{x^2-2x-3}(x+1)\right)$$

O limite não existe porque

$$\lim_{x\to3^+}\frac{1}{x-3}\left(\frac{1}{x}-\frac{\operatorname{sen}(x^2-2x-3)}{x^2-2x-3}(x+1)\right) = -\infty$$
$$\lim_{x\to3^-}\frac{1}{x-3}\left(\frac{1}{x}-\frac{\operatorname{sen}(x^2-2x-3)}{x^2-2x-3}(x+1)\right) = +\infty$$

pois

- $\lim_{x\to3}\frac{1}{x}-\frac{\operatorname{sen}(x^2-2x-3)}{x^2-2x-3}(x+1) = \frac{1}{3}-1\cdot4 = -\frac{11}{4}$ (trigonométrico de novo)
- $\lim_{x\to3^+}\frac{1}{x-3} = +\infty$ e $\lim_{x\to3^-}\frac{1}{x-3} = -\infty$

$$c)\ \lim_{x\to+\infty} x(\sqrt{x^2+1}-\sqrt[4]{x^4+1}) = \lim_{x\to+\infty} x(\sqrt[4]{(x^2+1)^2}-\sqrt[4]{x^4+1})$$
$$= \lim_{x\to+\infty} x\left(\frac{(x^2+1)^2-(x^4+1)}{(\sqrt[4]{(x^2+1)^6}+\sqrt[4]{(x^2+1)^4(x^4+1)}+\sqrt[4]{(x^2+1)^2(x^4+1)^2}+\sqrt[4]{(x^4+1)^3})}\right)$$
$$= \lim_{x\to+\infty}\frac{2x^3}{(\sqrt[4]{(x^2+1)^6}+\sqrt[4]{(x^2+1)^4(x^4+1)}+\sqrt[4]{(x^2+1)^2(x^4+1)^2}+\sqrt[4]{(x^4+1)^3})}$$
$$= \lim_{x\to+\infty}\frac{2x^3}{x^3\left(\sqrt[4]{(1+\frac{1}{x^2})^6}+\sqrt[4]{(1+\frac{1}{x^2})^4}\sqrt[4]{(1+\frac{1}{x^4})}+\sqrt[4]{(1+\frac{1}{x^2})^2}\sqrt[4]{(1+\frac{1}{x^4})^2}+\sqrt[4]{(1+\frac{1}{x^4})^3}\right)}$$
$$= \lim_{x\to+\infty}\frac{2}{\left(\sqrt[4]{(1+\frac{1}{x^2})^6}+\sqrt[4]{(1+\frac{1}{x^2})^4}\sqrt[4]{(1+\frac{1}{x^4})}+\sqrt[4]{(1+\frac{1}{x^2})^2}\sqrt[4]{(1+\frac{1}{x^4})^2}+\sqrt[4]{(1+\frac{1}{x^4})^3}\right)}$$
$$= \frac{1}{2}$$

pois $\lim_{x\to+\infty}\frac{1}{x^2} = \lim_{x\to+\infty}\frac{1}{x^4} = 0$