1. (Turma B)

$$a)\ \lim_{x\to-\infty} \frac{5x^2 - \operatorname{sen}(x^2)}{3x^3\operatorname{sen}(\frac{1}{x}) - 7x^2} = \lim_{x\to-\infty} \frac{x^2\left[5 - \frac{1}{x^2}\operatorname{sen}(x^2)\right]}{x^2\left[3x\operatorname{sen}(\frac{1}{x}) - 7\right]} = \lim_{x\to-\infty} \frac{5 - \frac{1}{x^2}\operatorname{sen}(x^2)}{3x\operatorname{sen}(\frac{1}{x}) - 7} = -\frac{5}{4}$$

pois

- $\lim_{x\to-\infty} \frac{1}{x^2}\operatorname{sen}(x^2) = 0$ ($\operatorname{sen}(x^2)$ é limitada e $\lim_{x\to-\infty} \frac{1}{x^2} = 0$)
- $\lim_{x\to-\infty} x\operatorname{sen}(\frac{1}{x}) = \lim_{x\to-\infty} \frac{\operatorname{sen}(\frac{1}{x})}{\frac{1}{x}} = 1$ (limite trigonométrico fundamental)

$$b)\ \lim_{x\to2}\left(\frac{1}{x^2-2x} - \frac{\operatorname{sen}(x^2-x-2)}{x^2-4x+4}\right) = \lim_{x\to2}\left(\frac{1}{x(x-2)} - \frac{\operatorname{sen}(x^2-x-2)}{(x-2)^2}\frac{(x+1)}{(x+1)}\right)$$

$$= \lim_{x\to2} \frac{1}{x-2}\left(\frac{1}{x} - \frac{\operatorname{sen}(x^2-x-2)}{x^2-x-2}(x+1)\right)$$

O limite não existe porque

$$\lim_{x\to2^+} \frac{1}{x-2}\left(\frac{1}{x} - \frac{\operatorname{sen}(x^2-x-2)}{x^2-x-2}(x+1)\right) = -\infty$$

$$\lim_{x\to2^-} \frac{1}{x-2}\left(\frac{1}{x} - \frac{\operatorname{sen}(x^2-x-2)}{x^2-x-2}(x+1)\right) = +\infty$$

pois

- $\lim_{x\to2} \frac{1}{x} - \frac{\operatorname{sen}(x^2-x-2)}{x^2-x-2}(x+1) = \frac{1}{2} - 1\cdot 3 = -\frac{5}{2}$ (trigonométrico de novo)
- $\lim_{x\to2^+} \frac{1}{x-2} = +\infty$ e $\lim_{x\to2^-} \frac{1}{x-2} = -\infty$

$$c)\ \lim_{x\to+\infty} x(\sqrt[4]{x^4+1} - \sqrt{x^2+1}) = -\lim_{x\to+\infty} x(\sqrt[4]{(x^2+1)^2} - \sqrt[4]{x^4+1})$$

$$= -\lim_{x\to+\infty} x\left(\frac{(x^2+1)^2 - (x^4+1)}{(\sqrt[4]{(x^2+1)^6} + \sqrt[4]{(x^2+1)^4(x^4+1)} + \sqrt[4]{(x^2+1)^2(x^4+1)^2} + \sqrt[4]{(x^4+1)^3})}\right)$$

$$= -\lim_{x\to+\infty} \frac{2x^3}{(\sqrt[4]{(x^2+1)^6} + \sqrt[4]{(x^2+1)^4(x^4+1)} + \sqrt[4]{(x^2+1)^2(x^4+1)^2} + \sqrt[4]{(x^4+1)^3})}$$

$$= -\lim_{x\to+\infty} \frac{2x^3}{x^3\left(\sqrt[4]{(1+\frac{1}{x^2})^6} + \sqrt[4]{(1+\frac{1}{x^2})^4}\sqrt[4]{(1+\frac{1}{x^4})} + \sqrt[4]{(1+\frac{1}{x^2})^2}\sqrt[4]{(1+\frac{1}{x^4})^2} + \sqrt[4]{(1+\frac{1}{x^4})^3}\right)}$$

$$= -\lim_{x\to+\infty} \frac{2}{\left(\sqrt[4]{(1+\frac{1}{x^2})^6} + \sqrt[4]{(1+\frac{1}{x^2})^4}\sqrt[4]{(1+\frac{1}{x^4})} + \sqrt[4]{(1+\frac{1}{x^2})^2}\sqrt[4]{(1+\frac{1}{x^4})^2} + \sqrt[4]{(1+\frac{1}{x^4})^3}\right)}$$

$$= -\frac{1}{2}$$

pois $\lim_{x\to+\infty} \frac{1}{x^2} = \lim_{x\to+\infty} \frac{1}{x^4} = 0$