(1,5) **Questão 2.** Calcule, caso exista, o seguinte limite: $\lim\limits_{x\to\frac{\pi}{2}^-}\left(\dfrac{5\pi}{2}-5x\right)^{\cos x}$.

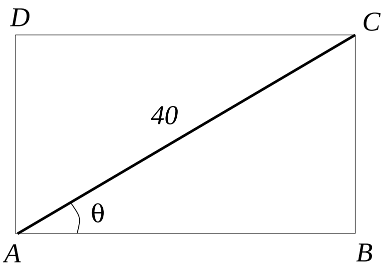
Temos que $\lim\limits_{x\to\frac{\pi}{2}^-}\left(\frac{5\pi}{2}-5x\right)^{\cos x} = \lim\limits_{x\to\frac{\pi}{2}^-} \mathrm{e}^{\ln\left(\frac{5\pi}{2}-5x\right)^{\cos x}} = \lim\limits_{x\to\frac{\pi}{2}^-} \mathrm{e}^{\cos x\ln\left(\frac{5\pi}{2}-5x\right)} = (*)$

Como $\lim\limits_{x\to\frac{\pi}{2}^-}\cos x\ln\left(\frac{5\pi}{2}-5x\right) = \lim\limits_{x\to\frac{\pi}{2}^-}\frac{\ln\left(\frac{5\pi}{2}-5x\right)}{\frac{1}{\cos x}} = \lim\limits_{x\to\frac{\pi}{2}^-}\frac{\ln\left(\frac{5\pi}{2}-5x\right)}{\sec x} \underset{L'H}{\overset{\frac{\infty}{\infty}}{=}} \lim\limits_{x\to\frac{\pi}{2}^-}\frac{\frac{-5}{\frac{5\pi}{2}-5x}}{\sec x\,\mathrm{tg}x} =$

$$= \lim_{x\to\frac{\pi}{2}^-}\frac{\frac{-1}{\frac{\pi}{2}-x}}{\frac{\mathrm{sen}x}{\cos^2 x}} = \lim_{x\to\frac{\pi}{2}^-}\frac{\cos^2 x}{(x-\frac{\pi}{2})\,\mathrm{sen}x} \underset{L'H}{\overset{\frac{0}{0}}{=}} \lim_{x\to\frac{\pi}{2}^-}\frac{-2\cos x\,\mathrm{sen}x}{\mathrm{sen}x+(x-\frac{\pi}{2})\cos x} = 0.$$

Portanto, $(*) = \mathrm{e}^0 = 1$.

(2,0) **Questão 3.** A superfície lateral de um cilindro circular reto é obtida unindo-se os lados AD e BC de um retângulo $ABCD$. Dentre todos os retângulos de diagonal $40cm$, determine o ângulo θ (ver figura) que permite construir um cilindro de volume máximo.



Sejam r o raio da base e h a altura do cilindro.

$m(\overline{AB}) = 40\cos\theta$ e $m(\overline{BC}) = 40\,\mathrm{sen}\theta$

$m(\overline{AB}) = 2\pi r = 40\cos\theta \Rightarrow r = \frac{40}{2\pi}\cos\theta$

$h = m(\overline{BC}) = 40\,\mathrm{sen}\theta$

$V = \pi r^2 h = \pi\left(\frac{40}{2\pi}\cos\theta\right)^2 40\,\mathrm{sen}\theta,\ \ 0<\theta<\frac{\pi}{2}$

$V(\theta) = \frac{40^3}{4\pi}\,\mathrm{sen}\theta\cos^2\theta$

$V'(\theta) = \frac{40^3}{4\pi}(\cos^3\theta - 2\,\mathrm{sen}^2\theta\cos\theta) \Rightarrow V'(\theta) = \frac{40^3}{4\pi}\cos\theta(\cos^2\theta - 2\,\mathrm{sen}^2\theta)$

$V'(\theta) = 0 \Leftrightarrow \cos\theta = 0$ ou $1-\ \mathrm{sen}^2\theta - 2\,\mathrm{sen}^2\theta \Leftrightarrow \theta = \frac{\pi}{2}$ ou $\theta =\ \mathrm{arcsen}(\frac{1}{\sqrt{3}})$

	$\nearrow$	$\searrow$	V
	$+$	$-$	$1-3\,\mathrm{sen}^2\theta$
	$+$	$-$	V'
0	$\mathrm{arcsen}(\frac{1}{\sqrt{3}})$	$\frac{\pi}{2}$	

Portanto, para se ter o volume máximo, temos que ter $\theta =\ \mathrm{arcsen}(\frac{1}{\sqrt{3}})$.