(1,5) **Questão 2.** Calcule, caso exista, o seguinte limite: $\displaystyle\lim_{x\to\frac{\pi}{2}^-}\left(\frac{3\pi}{2}-3x\right)^{\cos x}$.

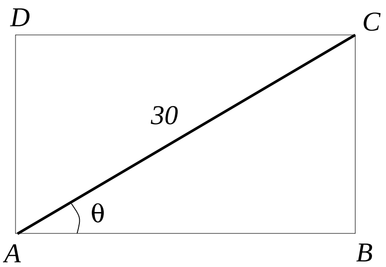
Temos que $\displaystyle\lim_{x\to\frac{\pi}{2}^-}\left(\tfrac{3\pi}{2}-3x\right)^{\cos x}=\lim_{x\to\frac{\pi}{2}^-}\mathrm{e}^{\ln\left(\frac{3\pi}{2}-3x\right)^{\cos x}}=\lim_{x\to\frac{\pi}{2}^-}\mathrm{e}^{\cos x\ln\left(\frac{3\pi}{2}-3x\right)}=(*)$

Como $\displaystyle\lim_{x\to\frac{\pi}{2}^-}\cos x\ln\left(\tfrac{3\pi}{2}-3x\right)=\lim_{x\to\frac{\pi}{2}^-}\frac{\ln\left(\frac{3\pi}{2}-3x\right)}{\frac{1}{\cos x}}=\lim_{x\to\frac{\pi}{2}^-}\frac{\ln\left(\frac{3\pi}{2}-3x\right)}{\sec x}\underset{L'H}{\overset{\frac{\infty}{\infty}}{=}}\lim_{x\to\frac{\pi}{2}^-}\frac{\frac{-3}{\frac{3\pi}{2}-3x}}{\sec x\,\mathrm{tg}x}=$

$$=\lim_{x\to\frac{\pi}{2}^-}\frac{\frac{-1}{\frac{\pi}{2}-x}}{\frac{\mathrm{sen}x}{\cos^2 x}}=\lim_{x\to\frac{\pi}{2}^-}\frac{\cos^2 x}{(x-\frac{\pi}{2})\,\mathrm{sen}x}\underset{L'H}{\overset{\frac{0}{0}}{=}}\lim_{x\to\frac{\pi}{2}^-}\frac{-2\cos x\,\mathrm{sen}x}{\mathrm{sen}x+(x-\frac{\pi}{2})\cos x}=0.$$

Portanto, $(*)=\mathrm{e}^0=1$.

(2,0) **Questão 3.** A superfície lateral de um cilindro circular reto é obtida unindo-se os lados $AD$ e $BC$ de um retângulo $ABCD$. Dentre todos os retângulos de diagonal $40cm$, determine o ângulo $\theta$ (ver figura) que permite construir um cilindro de volume máximo.




Sejam $r$ o raio da base e $h$ a altura do cilindro.

$m(\overline{AB})=30\cos\theta$ e $m(\overline{BC})=30\,\mathrm{sen}\theta$

$m(\overline{AB})=2\pi r=30\cos\theta\Rightarrow r=\frac{30}{2\pi}\cos\theta$

$h=m(\overline{BC})=30\,\mathrm{sen}\theta$

$V=\pi r^2h=\pi\left(\frac{30}{2\pi}\cos\theta\right)^2 30\,\mathrm{sen}\theta,\ \ 0<\theta<\frac{\pi}{2}$

$V(\theta)=\frac{30^3}{4\pi}\,\mathrm{sen}\theta\cos^2\theta$

$V'(\theta)=\frac{30^3}{4\pi}(\cos^3\theta-2\,\mathrm{sen}^2\theta\cos\theta)\Rightarrow V'(\theta)=\frac{30^3}{4\pi}\cos\theta(\cos^2\theta-2\,\mathrm{sen}^2\theta)$

$V'(\theta)=0\Leftrightarrow\cos\theta=0$ ou $1-\ \mathrm{sen}^2\theta-2\,\mathrm{sen}^2\theta\Leftrightarrow\theta=\frac{\pi}{2}$ ou $\theta=\ \mathrm{arcsen}(\frac{1}{\sqrt{3}})$

| | | |
|---|---|---|
| $\nearrow$ | $\searrow$ | $V$ |
| $+$ | $-$ | $1-3\,\mathrm{sen}^2\theta$ |
| $+$ | $-$ | $V'$ |
| $0$ | $\mathrm{arcsen}(\frac{1}{\sqrt{3}})$ | $\frac{\pi}{2}$ |

Portanto, para se ter o volume máximo, temos que ter $\theta=\ \mathrm{arcsen}(\frac{1}{\sqrt{3}})$.