Questão 1. (3,0 pontos) Calcule as seguintes integrais indefinidas

a) $\displaystyle\int x^5 \cos(x^3+2)dx$

b) $\displaystyle\int \frac{3e^{2x}+2e^x}{(e^x-2)(e^{2x}+4)}dx$

a) $\displaystyle\int x^5\cos(x^3+2)\,dx = \frac{1}{3}\int (t-2)\cos t\,dt = \frac{1}{3}\left((t-2)\operatorname{sen} t - \int \operatorname{sen} t\,dt\right)$

$(t = x^3+2 \Rightarrow dt = 3x^2dx)$ $\quad \left(\begin{array}{l} u = t-2 \rightarrow du = dt \\ dv = \cos t\,dt \rightarrow v = \operatorname{sen} t \end{array}\right)$

$\displaystyle = \frac{1}{3}\left((t-2)\operatorname{sen} t + \cos t\right) + C = \frac{1}{3}\left(x^3\operatorname{sen}(x^3+2) + \cos(x^3+2)\right) + C.$

b) $\displaystyle\int \frac{3e^{2x}+2e^x}{(e^x-2)(e^{2x}+4)}dx = \int \frac{3t+2}{(t-2)(t^2+4)}dt$

$t = e^x \rightarrow dt = e^x dx$

$\displaystyle\frac{3t+2}{(t-2)(t^2+4)} = \frac{A}{t-2} + \frac{Bt+C}{t^2+4} = \frac{A(t^2+4)+(Bt+C)(t-2)}{(t-2)(t^2+4)}$

$t = 2 \Rightarrow 8 = 8A \Rightarrow A = 1$

$t = 0 \Rightarrow 2 = 4-2C \Rightarrow C = 1$

$t = 1 \Rightarrow 5 = 5-B-1 \Rightarrow B = -1$

$\displaystyle\int \frac{1}{t-2}dt = \ln|t-2| + C_1$

$\displaystyle\int \frac{t}{t^2+4}dt = \frac{1}{2}\int \frac{2t}{t^2+4}dt = \frac{1}{2}\ln(t^2+4) + C_2.$

$\displaystyle\int \frac{1}{t^2+4}dt = \frac{1}{4}\int \frac{1}{\left(\frac{t}{2}\right)^2+1}dt = \frac{1}{4}\int \frac{1\cdot 2du}{u^2+1} = \frac{1}{2}\operatorname{arctg} u + C_3 = \frac{1}{2}\operatorname{arctg}\left(\frac{t}{2}\right) + C_3.$

$u = \frac{t}{2} \Rightarrow du = \frac{dt}{2}$

logo,

$\displaystyle\int \frac{3t+2}{(t-2)(t^2+4)}dt = \int \frac{1}{t-2}dt - \int \frac{t}{t^2+4}dt + \int \frac{1}{t^2+4}dt =$

$\displaystyle = \ln|t-2| - \frac{1}{2}\ln(t^2+4) + \frac{1}{2}\operatorname{arctg}\left(\frac{t}{2}\right) + C = \ln|e^x-2| - \frac{1}{2}\ln(e^{2x}+4) + \frac{1}{2}\operatorname{arctg}\left(\frac{e^x}{2}\right) + C$