Questão 1. (3,0 pontos) Calcule as seguintes integrais indefinidas

a) $\int x^5 \cos(x^3+3)dx$

b) $\int \frac{3e^{2x}+7e^x}{(e^x-2)(e^{2x}+9)}dx$

a) $\int x^5 \cos(x^3+3)\,dx = \int (t-3)\cos t \frac{dt}{3} = \frac{1}{3}\int (t-3)\cos t\,dt = \frac{1}{3}\left((t-3)\operatorname{sen} t - \int \operatorname{sen} t\,dt \right)$

$(t=x^3+3 \Rightarrow dt = 3x^2dx)$

$u = t-3 \rightarrow du = dt$

$dv = \cos t\,dt \rightarrow v = \operatorname{sen} t$

$= \frac{1}{3}\left((t-3)\operatorname{sen} t + \cos t \right) + C. = \frac{1}{3}\left(x^3 \operatorname{sen}(x^3+3) + \cos(x^3+3) \right) + C$

b) $\int \frac{3e^{2x}+7e^x}{(e^x-2)(e^{2x}+9)}dx = \int \frac{3t+7}{(t-2)(t^2+9)}dt$

$t = e^x \Rightarrow dt = e^x dx$

$\frac{3t+7}{(t-2)(t^2+9)} = \frac{A}{t-2} + \frac{Bt+C}{t^2+9} = \frac{A(t^2+9)+(Bt+C)(t-2)}{(t-2)(t^2+9)}$

$t=2 \Rightarrow 13 = 13A \Rightarrow A = 1.$

$t=0 \Rightarrow 7 = 9 - 2C \Rightarrow C = 1.$

$t=1 \Rightarrow 10 = 10 - B - 1 \Rightarrow B = -1.$

$\int \frac{1}{t-2}dt = \ln|t-2| + C_1$

$\int \frac{t}{t^2+9}dt = \frac{1}{2}\int \frac{2t}{t^2+9}dt = \frac{1}{2}\ln(t^2+9) + C_2.$

$\int \frac{1}{t^2+9}dt = \frac{1}{9}\int \frac{1}{(\frac{t}{3})^2+1}dt = \frac{1}{9}\int \frac{1}{u^2+1}\cdot 3\,du = \frac{1}{3}\operatorname{arctg} u + C_3 = \frac{1}{3}\operatorname{arctg}\left(\frac{t}{3}\right) + C_3.$

$u = \frac{t}{3} \Rightarrow du = \frac{dt}{3}$

logo,

$\int \frac{3t+7}{(t-2)(t^2+9)}dt = \ln|t-2| - \frac{1}{2}\ln(t^2+9) + \frac{1}{3}\operatorname{arctg}\left(\frac{t}{3}\right) + C$

$= \ln|e^x-2| - \frac{1}{2}\ln(e^{2x}+9) + \frac{1}{3}\operatorname{arctg}\left(\frac{e^x}{3}\right) + C.$