Questão 1. Calcule as seguintes integrais:

a) $$\int_{-1}^{1} \frac{x+3}{x^2+2x+5}\,dx = \int_{-1}^{1} \frac{x+3}{(x+1)^2+4}\,dx = \frac{1}{4}\int_{-1}^{1} \frac{x+3}{(\frac{x+1}{2})^2+1}\,dx \overset{(*)}{=} \frac{1}{4}\int_{0}^{1} \frac{2+2u}{u^2+1}\,2du$$

$$(*)\ \boxed{x = -1+2u \Rightarrow dx = 2du \quad \text{e} \quad x+3 = 2+2u}$$

$$= \frac{1}{2}\int_0^1 \left[\frac{2}{u^2+1} + \frac{2u}{u^2+1}\right]\,du = \int_0^1 \frac{1}{u^2+1}\,du + \frac{1}{2}\int_0^1 \frac{2u}{u^2+1}\,du = \operatorname{arctg}(u)\Big|_0^1 + \frac{1}{2}\int_0^1 \frac{2u}{u^2+1}\,du \overset{(**)}{=}$$

$$(**)\ \boxed{v = u^2+1 \quad \Rightarrow dv = 2u\,du}$$

$$= (\operatorname{arctg}(1) - \operatorname{arctg}(0)) + \frac{1}{2}\int_1^2 \frac{1}{v}\,dv = \frac{\pi}{4} + \frac{1}{2}\ln|v|\Big|_1^2 = \frac{\pi}{4} + \frac{1}{2}(\ln|2| - \ln|1|) = \frac{\pi}{4} + \frac{\ln 2}{2}\cdot$$

b) $$\int \frac{\operatorname{arctg}(5x)}{x^3}dx \overset{(*)}{=} -\frac{\operatorname{arctg}(5x)}{2x^2} + \frac{5}{2}\int \frac{1}{x^2(1+25x^2)dx} = -\frac{\operatorname{arctg}(5x)}{2x^2} + \frac{5}{2}\int \left[\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{1+25x^2}\right]dx \overset{(**)}{=}$$

$$(*)\ \boxed{\begin{array}{ll} u = \operatorname{arctg}(5x) & du = \frac{5}{1+25x^2}dx \\ dv = \frac{1}{x^3}dx & v = -\frac{1}{2x^2} \end{array}} \qquad (**)\ \boxed{A=0 \quad B=1 \quad C=0 \quad D=-25}$$

$$= -\frac{\operatorname{arctg}(5x)}{2x^2} + \frac{5}{2}\int \left[\frac{1}{x^2} + \frac{-25}{1+25x^2}\right]dx \overset{(***)}{=} -\frac{\operatorname{arctg}(5x)}{2x^2} - \frac{5}{2x} + \frac{5}{2}\int \frac{-5}{1+u^2}du$$

$$(***)\ \boxed{u = 5x \qquad du = 5\,dx}$$

$$= -\frac{\operatorname{arctg}(5x)}{2x^2} - \frac{5}{2x} - \frac{25}{2}\operatorname{arctg}(u) + k = -\frac{\operatorname{arctg}(5x)}{2x^2} - \frac{5}{2x} - \frac{25}{2}\operatorname{arctg}(5x) + k.$$

Questão 2.

a) Esboce o gráfico de $f(x) = (x^2 + 2)\,e^{-x^2}$.

$D_f = \mathbb{R}$ e f é contínua $\Rightarrow$ não existem assíntotas verticais.

$f'(x) = (-2x^3 - 2x)e^{-x^2} = -2x(x^2 + 1)e^{-x^2} \;\Rightarrow x = 0$ é o único ponto crítico de f.

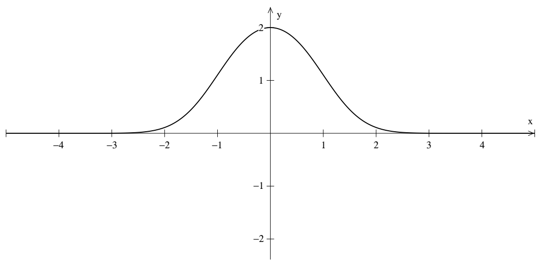
$f''(x) = (4x^4 - 2x^2 - 2)e^{-x^2} = 0 \quad \Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = -1$ ou $x = 1$.

$\nearrow$		$\searrow$	f
$+$	$\vert$	$-$	f'
	0		

$\cup$		$\cap$		$\cup$	f
$+$	$\vert$	$-$	$\vert$	$+$	f''
	-1		1		

$\Rightarrow x = -1$ e $x = 1$ são pontos de inflexão.

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to+\infty} f(x) = 0 \Rightarrow \quad y = 0$ é uma assíntota horizontal $\;\Rightarrow \nexists$ assíntotas oblíquas.

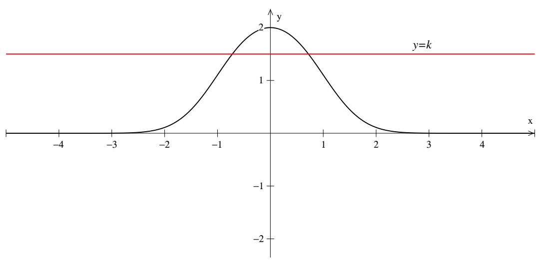


b) Para que valores de k a curva $y = \sqrt{\ln(x^2 + 2) - \ln(k)}$ corta a reta $y = x$?

Se x é um ponto de intersecção das duas curvas, temos:

$x = \sqrt{\ln(x^2+2) - \ln(k)} \Rightarrow x^2 = \ln(x^2+2) - \ln(k) \Rightarrow x^2 = \ln(\frac{x^2+2}{k}) \Rightarrow e^{x^2} = \frac{x^2+2}{k} \Rightarrow \boxed{k = (x^2+2)\,e^{-x^2}}$.

Ou seja, $0 < k \leq (c^2 + 2)\,e^{-c^2}$, onde c é o ponto de máximo de $(x^2 + 2)\,e^{-x^2} = f(x)$.



Pelo item (a) temos que $c = 0$, portanto $0 < k \leq f(0) = 2$.