**Questão 4.** Seja $F(x) = \int_0^{\sqrt[5]{x}} \sqrt[5]{x}\,\mathrm{sen}(t^4)dt$.

(1,0) a) Calcule $F'(x)$ para $x \neq 0$.

(1,5) b) Verifique se $F$ é derivável em $x_0 = 0$. Em caso afirmativo, calcule $F'(0)$.

Note que $F(x) = x^{1/5} \cdot \int_0^{x^{1/5}} \mathrm{sen}(t^4)dt$

a) $F'(x) = \frac{1}{5} x^{-4/5} \cdot \int_0^{x^{1/5}} \mathrm{sen}(t^4)dt + x^{1/5} \cdot \mathrm{sen}(x^{1/5})^4 \cdot \frac{1}{5} x^{-4/5}$

$= \frac{x^{-4/5}}{5} \cdot \left[ \int_0^{x^{1/5}} \mathrm{sen}(t^4)dt + x^{1/5}\,\mathrm{sen}(x^{4/5}) \right], \quad \underline{\underline{x \neq 0}}$

b) $\lim_{x \to 0} \frac{F(x) - F(0)}{x - 0} = \lim_{x \to 0} \frac{F(x)}{x} = \lim_{x \to 0} \frac{x^{1/5} \cdot \int_0^{x^{1/5}} \mathrm{sen}(t^4)dt}{x}$

$= \lim_{x \to 0} \frac{\int_0^{x^{1/5}} \mathrm{sen}(t^4)dt}{x^{4/5}} \overset{(*)}{=} \lim_{x \to 0} \frac{\mathrm{sen}(x^{4/5}) \cdot \frac{1}{5} x^{-4/5}}{\frac{4}{5} x^{-1/5}}$

$= \lim_{x \to 0} \frac{1}{4} \cdot \underbrace{\frac{\mathrm{sen}(x^{4/5})}{x^{4/5}}}_{\to 1} \cdot \underbrace{x^{1/5}}_{\to 0} = 0$

$\therefore F'(0) = 0$

( (*) Por L'Hospital)