(2,0) **Questão 1.** Calcule os seguintes limites:

a) $\lim\limits_{x\to 0^+} (\operatorname{tg} x)^{\frac{2}{\ln(2x)}}$;

b) $\lim\limits_{x\to 0} \dfrac{\displaystyle\int_0^{\operatorname{sen} x} 3x\, e^{t^2}\, dt}{\displaystyle\int_{\cos x}^{1} e^{t^2}\, dt}$.

(a) $(\operatorname{tg} x)^{\frac{2}{\ln 2x}} = e^{\frac{2}{\ln(2x)} \ln \operatorname{tg} x} = e^{\frac{2 \ln \operatorname{tg} x}{\ln(2x)}}$

$\lim\limits_{x\to 0^+} (\operatorname{tg} x)^{\frac{2}{\ln 2x}} = \lim\limits_{x\to 0^+} e^{\frac{2\ln \operatorname{tg} x}{\ln(2x)}}$

Seja $u = \dfrac{2 \ln \operatorname{tg} x}{\ln(2x)}$. Então $\lim\limits_{x\to 0^+} u = \lim\limits_{x\to 0^+} \dfrac{\overbrace{2\ln \operatorname{tg} x}^{\to -\infty}}{\underbrace{\ln(2x)}_{\to -\infty}} \overset{(\frac{\infty}{\infty})}{\underset{L'H}{=}} \lim\limits_{x\to 0^+} \dfrac{\frac{2\sec^2 x}{\operatorname{tg} x}}{\frac{1}{2x}\cdot 2}$

$= \lim\limits_{x\to 0^+} 2\left(\dfrac{x}{\operatorname{sen} x}\right)\cdot \dfrac{1}{\underbrace{\cos x}_{\to 1}} = 2$. Logo $\lim\limits_{x\to 0^+} e^{\frac{2\ln \operatorname{tg} x}{\ln 2x}} = \lim\limits_{u\to 2} e^{u} = e^{2}$.

(b) Seja $F(x)$ uma primitiva qualquer de e^{x^2}.

Então $\lim\limits_{x\to 0} \dfrac{\int_0^{\operatorname{sen} x} 3x\, e^{t^2}\, dt}{\int_{\cos x}^{1} e^{t^2}\, dt} = \lim\limits_{x\to 0} \dfrac{(F(\operatorname{sen} x) - F(0))\, 3x}{\underbrace{F(1) - F(\cos x)}_{\to 0}}$

$\overset{L'H}{\underset{(\frac{0}{0})}{=}} \lim\limits_{x\to 0} \dfrac{F'(\operatorname{sen} x)\cos x \cdot 3x + 3\,(F(\operatorname{sen} x) - F(0))}{-F'(\cos x)(-\operatorname{sen} x)}$

$= \lim\limits_{x\to 0} \dfrac{3x\, e^{\operatorname{sen}^2 x}\cos x + 3\,(F(\operatorname{sen} x) - F(0))}{e^{\cos^2 x}\operatorname{sen} x}$

$\overset{L'H}{\underset{(\frac{0}{0})}{=}} \lim\limits_{x\to 0} \dfrac{3e^{\operatorname{sen}^2 x}\cos x + 3x\left(e^{\operatorname{sen}^2 x}\cos x\right)' + 3e^{\operatorname{sen}^2 x}\cos x}{\underbrace{e^{\cos^2 x}\, 2\cos x\,(-\operatorname{sen} x)\operatorname{sen} x}_{\to 0} + \underbrace{e^{\cos^2 x}\cos x}_{\to e}}$

$= \dfrac{6}{e}$

(2,5) **Questão 2.** Calcule as seguintes integrais:

a) $\displaystyle\int \frac{x+1}{(x^2-2x+5)^2}\,dx$; b) $\displaystyle\int x\,\text{sen}^2 x\,dx$.

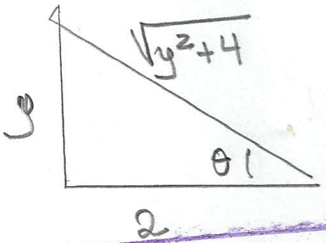
(a) $\displaystyle\int \frac{x+1}{(x^2-2x+5)^2}\,dx = \int \frac{(x+1)}{((x-1)^2+4)^2}\,dx \underset{\substack{y=x-1\\ dy=dx\\ x=y+1}}{=} \int \frac{(y+2)\,dy}{(y^2+4)^2}$

$\displaystyle = \underbrace{\int \frac{y}{(y^2+4)^2}\,dy}_{(1)} + 2\underbrace{\int \frac{dy}{(y^2+4)^2}}_{(2)}$

(1) $\displaystyle\int \frac{y}{(y^2+4)^2}\,dy \underset{\substack{u=y^2+4\\ du=2y\,dy}}{=} \frac{1}{2}\int \frac{du}{u^2} = -\frac{1}{2u} + C_1 = -\frac{1}{2(y^2+4)} + C_1$

(2) $\displaystyle\int \frac{dy}{(y^2+4)^2} \underset{\substack{y=2\,tg\,\theta\\ dy=2\sec^2\theta\,d\theta}}{=} \int \frac{2\sec^2\theta\,d\theta}{4^2(\sec^2\theta)^2}$

$\displaystyle = \frac{1}{2^3}\int \cos^2\theta\,d\theta = \frac{1}{2^3}\frac{1}{2}\left(\theta + \frac{\text{sen}\,2\theta}{2}\right) + C_2$



$\displaystyle = \frac{1}{2^4}\left(\text{arctg}\left(\frac{y}{2}\right) + \frac{2y}{y^2+4}\right) + C_2$

Logo $\displaystyle\boxed{\int \frac{x+1}{(x^2-2x+5)}\,dx = -\frac{1}{2(x^2-2x+5)} + \frac{2}{2^4}\left[\text{arc tg}\left(\frac{x-1}{2}\right) + \frac{2(x-1)}{x^2-2x+5}\right] + C_4}$

(b) $\int x \operatorname{sen}^2 x \, dx$

Usando integração por partes com:

$u = x \Rightarrow du = dx$

$dv = \operatorname{sen}^2 x \, dx \Rightarrow v = \int \operatorname{sen}^2 x \, dx = \int \frac{1}{2}(1 - \cos 2x) \, dx$

$$= \frac{1}{2}\left(x - \frac{\operatorname{sen} 2x}{2}\right) + k$$

$$\int x \operatorname{sen}^2 x \, dx = x \cdot \frac{1}{2}\left(x - \frac{\operatorname{sen} 2x}{2}\right) - \int \frac{1}{2}\left(x - \frac{\operatorname{sen} 2x}{2}\right) dx$$

$$= \frac{x^2}{2} - \frac{x \operatorname{sen} 2x}{4} - \frac{1}{4} x^2 - \frac{1}{8} \cos 2x + C$$

$$\boxed{= \frac{x^2}{4} - \frac{x \operatorname{sen} 2x}{4} - \frac{1}{8} \cos 2x + C}$$

(2,0) **Questão 3.** Esboce o gráfico de $f(x) = x^3 e^{-x}$, determinando o seu domínio, os intervalos de crescimento e de decrescimento, concavidades e assíntotas (caso existam).

$D_f = \mathbb{R}$

$$\lim_{x\to+\infty} \frac{x^3}{e^x} \underset{\left(\frac{\infty}{\infty}\right)}{\overset{L'H}{=}} \lim_{x\to+\infty} \frac{3x^2}{e^x} \overset{L'H}{=} \lim_{x\to+\infty} \frac{6x}{e^x} \overset{L'H}{=} \lim_{x\to+\infty} \frac{6}{x} = 0$$

$y = 0$ é assíntota horizontal

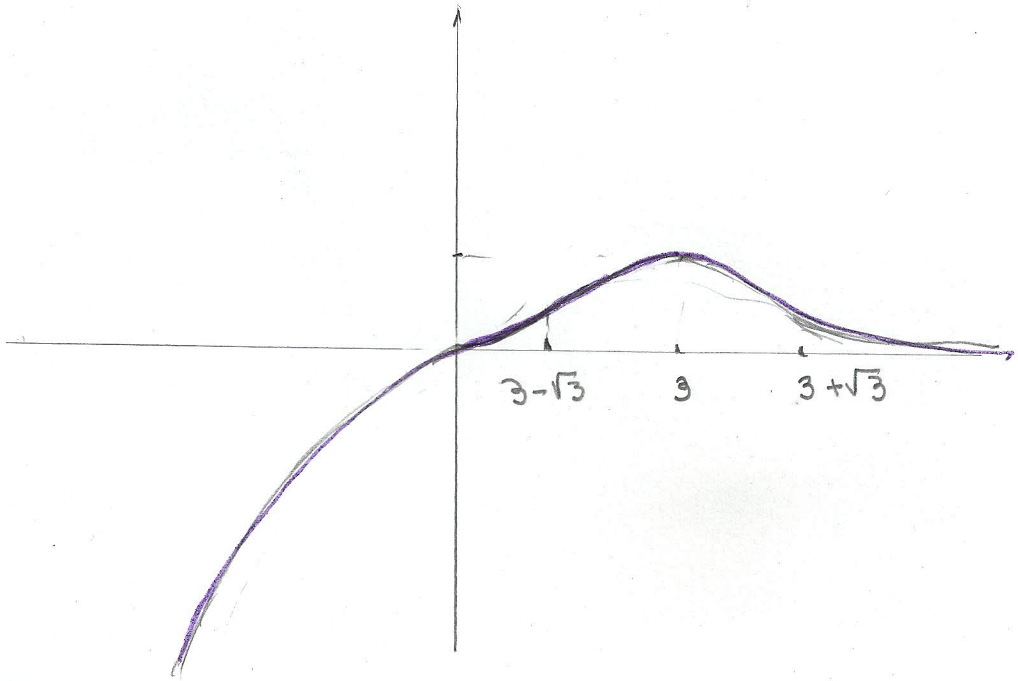
$$\lim_{x\to-\infty} \underbrace{x^3}_{\to -\infty} \cdot \overbrace{\frac{1}{e^x}}^{\to +\infty} = 0$$

$$f'(x) = 3x^2 e^{-x} - x^3 e^{-x} = \underbrace{x^2 e^{-x}}_{\geq 0}(3-x)$$

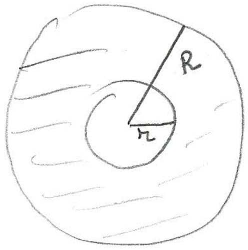
	0		3	
f'	$+\ +$	$+\ +\ +$		$-\ -\ -$
f	↗	↗		↘

$$f''(x) = e^{-x}(6x - 3x^2) + (3x^2 - x^3)e^{-x} \cdot (-1)$$
$$= e^{-x}(x^3 - 6x^2 + 6x) = x e^{-x}(x^2 - 6x + 6)$$

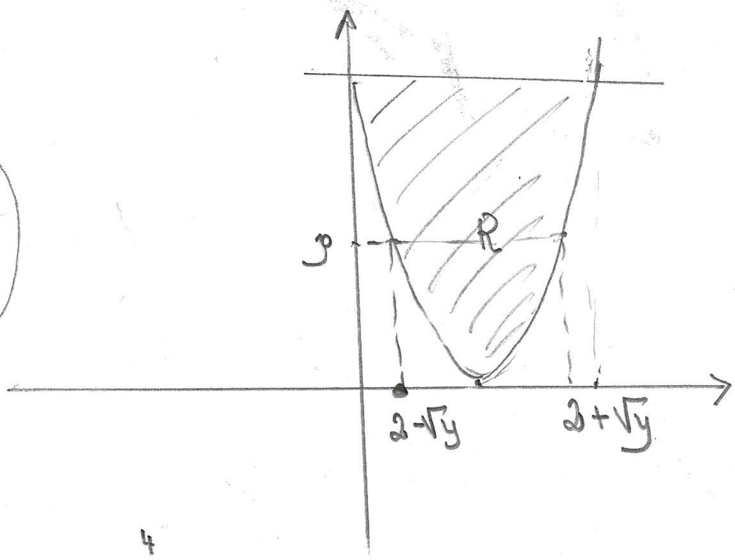
		0	$3-\sqrt{3}$	$3+\sqrt{3}$
x	$-\ -$	$+$	$+\ +\ +\ +$	$+\ +\ +\ +$
x^2-6x+6	$+\ +\ +$	$+$	$-\ -\ -$	$+\ +\ +$
f''	$-$	$+$	$-$	$+$
f	∩	∪	∩	∪



(2,0) **Questão 4.** Seja $R = \{(x, y) \in R^2 : (x-2)^2 \leq y \leq 4\}$. Determine o volume do sólido obtido pela rotação de R em torno do eixo Oy.



$R = 2 + \sqrt{y}$
$r = 2 - \sqrt{y}$



$y \in [0, 4]$

$y = (x-2)^2$

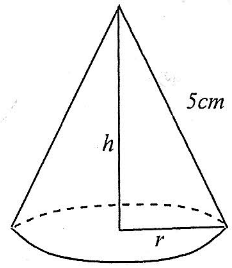
$\Rightarrow x - 2 = \pm\sqrt{y}$

$x = 2 \pm \sqrt{y}$

$$V = \pi \int_0^4 \left[(2+\sqrt{y})^2 - (2-\sqrt{y})^2\right] dy = \pi \int_0^4 8\sqrt{y}\, dy$$

$$= 8\pi \cdot \frac{2}{3}\, y^{3/2} \Big|_0^4 = \frac{16\pi}{3} \cdot 8 = \boxed{\frac{128\pi}{3}}$$

(1,5) **Questão 5.** Determine as dimensões de um cone circular reto de maior volume cuja geratriz tenha $5cm$ de comprimento.



$$V_{cone} = \frac{1}{3}\pi r^2 h$$

$$r^2 = 25 - h^2 \qquad V(h) = \frac{1}{3}\pi (25 - h^2) h$$

$$h \in]0, 5[$$

$$V'(h) = \frac{1}{3\pi}\left[25 - 3h^2\right]$$

$$V'(h) = 0 \text{ e } h > 0 \implies h = \frac{5}{\sqrt{3}} \text{ cm}$$

$$\begin{array}{ccccc} + \; + \; + & & - \; - \; - & \\ 0 & \nearrow & \frac{5}{\sqrt{3}} & \searrow & 5 \end{array}$$

$h = 5/\sqrt{3}$ é ponto de máximo de $V(h)$.

$$r^2 = 25 - \frac{25}{3} = \frac{50}{3} \quad \therefore \quad r = \frac{\sqrt{50}}{\sqrt{3}} = \frac{5\sqrt{2}}{\sqrt{3}}$$

Dimensões: $h = \frac{5}{\sqrt{3}}$ cm e $r = \frac{5\sqrt{2}}{\sqrt{3}}$ cm