ARTICLE IN PRESS

Computational Statistics and Data Analysis ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

The log-exponentiated Weibull regression model for interval-censored data

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ARTICLE INFO

Article history:
Received 17 December 2008
Received in revised form 1 October 2009
Accepted 2 October 2009
Available online xxxx

ABSTRACT

In interval-censored survival data, the event of interest is not observed exactly but is only known to occur within some time interval. Such data appear very frequently. In this paper, we are concerned only with parametric forms, and so a location-scale regression model based on the exponentiated Weibull distribution is proposed for modeling interval-censored data. We show that the proposed log-exponentiated Weibull regression model for interval-censored data represents a parametric family of models that include other regression models that are broadly used in lifetime data analysis. Assuming the use of interval-censored data, we employ a frequentist analysis, a jackknife estimator, a parametric bootstrap and a Bayesian analysis for the parameters of the proposed model. We derive the appropriate matrices for assessing local influences on the parameter estimates under different perturbation schemes and present some ways to assess global influences. Furthermore, for different parameter settings, sample sizes and censoring percentages, various simulations are performed; in addition, the empirical distribution of some modified residuals are displayed and compared with the standard normal distribution. These studies suggest that the residual analysis usually performed in normal linear regression models can be straightforwardly extended to a modified deviance residual in log-exponentiated Weibull regression models for interval-censored data.

1. Introduction

In several studies, the survival response can be interval-censored such that the event of interest is not observed exactly but is only known to occur within time intervals that may overlap and vary in length. The literature presents many applications of survival models for interval-censored data with respect to the Weibull family of distributions (Lawless, 2003). This family is very suitable for situations in which the failure rate function is constant or monotone. However, it is not suitable in situations in which the failure rate function presents a bathtub or unimodal shape. For example, according to Zimmer et al. (1998) and Silva et al. (2008), the failure rate function of the Burr XII distribution can be decreased or unimodal. To cope with these situations, several distributions derived from the Weibull distribution that exhibit bathtub-shaped or unimodal failure rate functions were developed, one of which is the exponentiated Weibull (EW) distribution proposed by Mudholkar et al. (1995).

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0167-9473/$ – see front matter
doi:10.1016/j.csda.2009.10.014

Please cite this article in press as: Hashimoto, E.M., et al., The log-exponentiated Weibull regression model for interval-censored data. Computational Statistics and Data Analysis (2009), doi:10.1016/j.csda.2009.10.014

In some situations, the times of the events of interest T may be only known to have occurred within an interval of time $[U, V]$, where $U \leq T \leq V$. This may occur in a clinical trial, for example, when patients are only assessed at pre-scheduled visits. If the event T has not been observed to have occurred at the time of one visit at time U but has occurred by the following visit at time V. Thus, it is only known that T lies at some point within the interval $[U, V]$. Thus, this is an example of interval-censored data. Note that exactly observed, right- and left-censored data are all special cases of interval-censored data. Note that $U = V$ for exactly observed data; $V = \infty$ for right-censored data; and $U = 0$ for left-censored data.

This paper examines aspects of statistical inference for modeling interval-censored data by using the log-exponentiated Weibull (LEW) regression model. The inferential component was carried out using the asymptotic distribution of the maximum likelihood estimators; this is useful, because in situations in which the sample size is small, normality is difficult to justify as an assumption. As an alternative to frequentist analysis, we explored the use of Markov chain Monte Carlo (MCMC) techniques to develop a Bayesian inference and jackknife estimator for the LEW regression model for interval-censored data. A punctual and interval estimation method based on bootstrap re-sampling is also proposed.

After modeling, it is important to check the assumptions of the model and conduct robustness checks to detect possible influential or extreme observations that may distort the results of the analysis. In this paper, we discuss the influence diagnostic based on case deletion (Cook, 1977) in which the influence of the ith observation on the parameter estimates is evaluated by removing cases from the analysis. We propose diagnostic measures based on case deletion for the LEW regression models for interval-censored data to determine which data points are influential in the analysis. This method has been applied in various statistical models. See, for instance, Christensen et al. (1992), Davison and Tsai (1992) and Xie and Wei (2007a).

Nevertheless, when case deletion is used, all information on a single data point is deleted at once, and therefore, it is difficult to determine whether that data point has some influence on a specific aspect of the model. A solution for the earlier problem (case deletion) can be found using the local influence approach in which one again investigates how the results of an analysis change under small perturbations in the model or data. Cook (1986) proposed a general framework to detect the influence of observations in order to evaluate how sensitive the analysis is to small perturbations that are provoked within the data or model. Some authors have investigated the evaluation of local influences in survival analysis models: for instance, Pettitt and Bin Daud (1989) investigated local influences in proportional hazard regression models; Escobar and Meeker (1992) adapted local influence methods to regression analysis with censoring; and Ortega et al. (2003) considered the problem of evaluating local influences in generalized log-gamma regression models with censored observations. More recently, Magnus and Vasnev (2007) analyzed sensitivity analysis using diagnostic testing, which resulted in applications in econometrics. Xie and Wei (2007b) developed an application of influence diagnostics in censored generalized Poisson regression models based on the case-deletion method and local influence analysis. In addition, Ortega et al. (2009) derived curvature calculations under various perturbation schemes in regression models with cure fraction, and Fachini et al. (2008) adapted local influence methods to polyhazard models in the context of explanatory variables. Carrasco et al. (2008) investigated influence diagnostics in log-modified Weibull regression models with censored data, and Silva et al. (2008) adapted global and local influence methods using log-Burr XII regression models with censored data. We propose a similar methodological approach to detect influential data points in LEW regression models for interval-censored data.

The paper is organized as follows. In Section 2, we discuss an LEW regression model for interval-censored data in addition to maximum likelihood estimators, Bayesian inference and the jackknife estimator. Score functions and observed information matrix are derived, while the process for estimating the regression coefficients and the remaining parameters is discussed. In Section 3, we use several diagnostic measures to consider case deletion, and the normal curvatures of local influences are derived under various perturbation schemes in the context of the proposed LEW regression model with interval-censored data. In Section 4, a deviance residual is proposed to assess departures from the LEW error assumption as well as outlying observations. In addition, we present and analyze results from various simulation studies, including graphic displays to further illustrate our findings. In Section 5, a real data set is analyzed; finally the last section presents some concluding remarks.

2. The log-exponentiated Weibull regression models for interval-censored data

We assume that the random variable T follows an EW distribution with parameters $(\alpha, \gamma, \lambda)^T$. The probability density function for the EW distribution is given by

$$f(t; \alpha, \gamma, \lambda) = \frac{\alpha\lambda}{\gamma}\left\{1 - \exp\left[-\left(\frac{t}{\gamma}\right)^{\alpha}\right]\right\}^{\lambda-1} \exp\left[-\left(\frac{t}{\gamma}\right)^{\alpha}\right]\left(\frac{t}{\gamma}\right)^{\alpha-1}. \tag{1}$$

The survival function reduces to

$$S(t; \alpha, \gamma, \lambda) = 1 - \left\{1 - \exp\left[-\left(\frac{t}{\gamma}\right)^{\alpha}\right]\right\}^{\lambda}. \tag{2}$$

Note that $t > 0$, $\alpha > 0$, $\lambda > 0$ are shape parameters, and $\gamma > 0$ is a scale parameter. The hazard function is given by $h(t; \alpha, \gamma, \lambda) = f(t; \alpha, \gamma, \lambda)/S(t; \alpha, \gamma, \lambda)$. The great flexibility of this model to fit lifetime data is due to the different forms

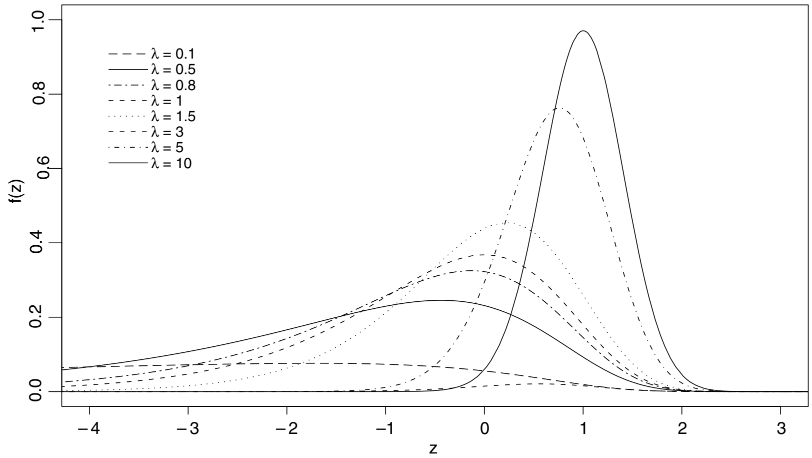

Fig. 1. Probability density function graphs of the Z variable for the indicated parameter.

that the hazard function can take. That is, (i) if $\alpha \geq 1$ and $\alpha\lambda \geq 1$, then the hazard function is monotonically increasing; (ii) if $\alpha \leq 1$ and $\alpha\lambda \leq 1$, then the hazard function is monotonically decreasing; (iii) if $\alpha > 1$ and $\alpha\lambda < 1$, then the hazard function is bathtub-shaped; and (iv) if $\alpha < 1$ and $\alpha\lambda > 1$, then the hazard function is unimodal.

Applications of the EW distribution in the context of reliability and survival studies have been investigated by Mudholkar et al. (1995). Cancho et al. (1999) developed a Bayesian analysis for the EW distribution, while Cancho and Bolfarine (2001) proposed an EW mixture model for the presence of a cure fraction in lifetime data. Some properties of this distribution have been studied in more detail by Mudholkar and Hutson (1996) and Nassar and Eissa (2003).

Let T be a random variable following the EW density function (1). The random variable $Y = \log(T)$ follows a LEW distribution with density function parametrized in terms of $\alpha = 1/\sigma$ and $\mu = \log(\gamma)$

$$f(y;\lambda,\sigma,\mu) = \frac{\lambda}{\sigma}\left\{1-\exp\left[-\exp\left(\frac{y-\mu}{\sigma}\right)\right]\right\}^{\lambda-1}\exp\left[\left(\frac{y-\mu}{\sigma}\right)-\exp\left(\frac{y-\mu}{\sigma}\right)\right]. \tag{3}$$

Note that $-\infty < y < \infty, \lambda > 0, \sigma > 0$ and $-\infty < \mu < \infty$. The corresponding survival function reduces to

$$S(y;\lambda,\sigma,\mu) = 1-\left\{1-\exp\left[-\exp\left(\frac{y-\mu}{\sigma}\right)\right]\right\}^{\lambda}.$$

The standardized random variable $Z = (Y-\mu)/\sigma$ has the density function

$$f(z) = \lambda\left\{1-\exp\left[-\exp(z)\right]\right\}^{\lambda-1}\exp\left[z-\exp(z)\right], \quad \infty < z < \infty. \tag{4}$$

The extreme-value standard distribution corresponds to the particular choice of $\lambda = 1$. Plots of the density function (4) for some parameter values are given in Fig. 1.

We hardly need to emphasize the necessity and importance of moments in any statistical analysis, especially in applied work. Some of the most important features and characteristics of a distribution can be studied through moments, including statistical tendency, dispersion, skewness and kurtosis. In particular, we draw on the following theorem.

Theorem 1. *For a random variable Z, the kth ordinary moment is*

$$\mu'_k = E(Z^k) = \lambda\Gamma(\lambda)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma(\lambda-j)j!}\times\left.\frac{\partial^k[(j+1)^{-a}\Gamma(a)]}{\partial a^k}\right|_{a=1}.$$

Note that $\Gamma(\cdot)$ is the gamma function; the proof given in Appendix B.

In many practical applications, lifetimes are affected by explanatory variables, such as cholesterol level or blood pressure. Let $\mathbf{x} = (x_1,\ldots,x_p)^T$ be the explanatory variable vector associated with the response variable y. Based on the LEW density, we construct a linear regression model linking the response variable y_i and the explanatory variable vector $\mathbf{x}_i$ as follows:

$$y_i = \mathbf{x}_i^T\boldsymbol{\beta} + \sigma z_i, \quad i = 1,\ldots,n. \tag{5}$$

Note that the random error z_i has the distribution (4), $\boldsymbol{\beta} = (\beta_1,\ldots,\beta_p)^T$, $\sigma > 0$ and $\lambda > 0$ are unknown parameters; and $\mathbf{x}_i^T = (x_{i1},\ldots,x_{ip})$ is the explanatory variable vector that models the location parameter μ_i. Hence, the location

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parameter vector $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_n)^T$ of the LEW regression model can be expressed as a linear model $\boldsymbol{\mu} = \mathbf{X}\boldsymbol{\beta}$, where $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_n)^T$ is a known model matrix.

Using the log-linear model in (5), the survival function of $Y_i|\mathbf{x}$ is given by

$$S(y_i|\mathbf{x}) = 1 - \left\{1 - \exp\left[-\exp\left(\frac{y_i - \mathbf{x}_i^T\boldsymbol{\beta}}{\sigma}\right)\right]\right\}^{\lambda}.$$

For interval-censored data, the observed data consist of an interval $(\log(u_i), \log(v_i))$ for each individual, and such intervals are known to include $y_i = \log(t_i)$ with probability one. That is, $P\left[\log(u_i) \leq y_i \leq \log(v_i)\right] = 1$; if $\log(v_i) = \infty$, then it is a right-censored time for y_i. This model is referred to as the LEW regression model for interval-censored data. It is an extension of an accelerated failure time model using the EW distribution for interval-censored data.

If $\lambda = 1$ in model (5), then the log-Weibull (LW) regression model for interval-censored data results. Moreover, if $\sigma = 1$ also holds, then model (5) reduces to the log-exponential (LE) regression model for interval-censored data. If $\sigma = 1/2$, then the log-generalized Rayleigh (LGR) regression for interval-censored data results. Finally, if $\sigma = 1$, then we obtain the log-exponentiated exponential (LEE) (see Gupta and Kundu, 1999, 2001) regression models for interval-censored data.

2.1. Estimation by maximum likelihood

Let $(\log(u_1), \log(v_1), \mathbf{x}_1), \ldots, (\log(u_n), \log(v_n), \mathbf{x}_n)$ be a set of n interval-censored observations and explanatory variables, where $(\log(u_i)$ and $\log(v_i))$ are the observed data; $\mathbf{x}_i$ is the explanatory variable vector; and the observed full log-likelihood function for the parameter vector $\boldsymbol{\theta} = (\lambda, \sigma, \boldsymbol{\beta}^T)^T$ is given by

$$l(\boldsymbol{\theta}) = \sum_{i \in F} l_1(\lambda, zu_i, zv_i) + \sum_{i \in C} l_2(\lambda, zu_i), \tag{6}$$

where

$$l_1(\lambda, zu_i, zv_i) = \log\left[\left\{1 - \exp\left[-\exp(zv_i)\right]\right\}^{\lambda} - \left\{1 - \exp\left[-\exp(zu_i)\right]\right\}^{\lambda}\right] \quad \text{and}$$

$$l_2(\lambda, zu_i) = \log\left[1 - \left\{1 - \exp\left[-\exp(zu_i)\right]\right\}^{\lambda}\right].$$

Note that F represents the set of individuals with interval censoring, that is, $y_i \in \left(\log(u_i), \log(v_i)\right]$. In addition, C represents the set of individuals with right censoring, that is, $y_i \in \left(\log(u_i), +\infty\right)$, $zu_i = \left[\log(u_i) - \mathbf{x}_i^T\boldsymbol{\beta}\right]/\sigma$ and $zv_i = \left[\log(v_i) - \mathbf{x}_i^T\boldsymbol{\beta}\right]/\sigma$. The maximum likelihood estimates (MLEs) of the parameter vector $\boldsymbol{\theta}$ can be obtained by maximizing the likelihood function. We use the software Ox (MAXBFGS subroutine) (Doornik, 2001) to compute the MLEs. The estimate of the covariance matrix of the MLEs $\widehat{\boldsymbol{\theta}}$ can also be obtained through the Hessian matrix. Confidence intervals and hypothesis testing can be conducted by employing a large sample distribution of the MLEs, which has a multivariate normal distribution with a covariance matrix given by the inverse of the information matrix since regularity conditions are satisfied. More specifically, the asymptotic covariance matrix of $\widehat{\boldsymbol{\theta}}$ is given by $\mathbf{I}^{-1}(\boldsymbol{\theta})$ with $\mathbf{I}(\boldsymbol{\theta}) = E[\ddot{\mathbf{L}}(\boldsymbol{\theta})]$ such that $\ddot{\mathbf{L}}(\boldsymbol{\theta}) = -\frac{\partial^2 l(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^T}$.

It is not possible to compute the Fisher information matrix $\mathbf{I}(\boldsymbol{\theta})$ due to censored observations, as censoring is random and non-informative. However, it is possible to use the negative of the matrix of the second derivatives of the log-likelihood, $-\ddot{\mathbf{L}}(\boldsymbol{\theta})$, evaluated at the MLE $\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}$, which is consistent. The asymptotic normal approximation for $\widehat{\boldsymbol{\theta}}$ may be expressed as $\widehat{\boldsymbol{\theta}} \sim N_{(p+2)}\{\boldsymbol{\theta}; -\ddot{\mathbf{L}}(\boldsymbol{\theta})^{-1}\}$, where $\ddot{\mathbf{L}}(\boldsymbol{\theta})$ is the $(p+2)(p+2)$ observed information matrix, expressed as

$$\ddot{\mathbf{L}}(\boldsymbol{\theta}) = \begin{pmatrix} \mathbf{L}_{\lambda\lambda} & \mathbf{L}_{\lambda\sigma} & \mathbf{L}_{\lambda\beta_j} \\ . & \mathbf{L}_{\sigma\sigma} & \mathbf{L}_{\sigma\beta_j} \\ . & . & \mathbf{L}_{\beta_j\beta_s} \end{pmatrix}$$

Note that $j, s = 1, \ldots, p$; in addition, the sub-matrices are defined in Appendix A.

To test the adequacy of the LW regression model for interval-censored data, i.e., $H_0 : \lambda = 1$, we consider the likelihood ratio (LR) statistic, given by

$$\Lambda_n = -2[\ell(\tilde{\boldsymbol{\theta}}) - \ell(\hat{\boldsymbol{\theta}})]. \tag{7}$$

Note that $\hat{\boldsymbol{\theta}}$ is the MLE that is derived from maximizing the log-likelihood in (6), and $\tilde{\boldsymbol{\theta}}$ is the restricted MLE computed assuming H_0, that is, with $\lambda = 1$. For large samples, Λ_n approximately has a chi-square distribution with one degree of freedom. For testing the adequacy of the LE regression model for interval-censored data, that is, $H_0 : (\sigma, \lambda)^T = (1, 1)^T$, the likelihood deviance Λ_n is as given in Eq. (7), but $\tilde{\boldsymbol{\theta}}$ is the restricted MLE computed under H_0, that is, with $\sigma = 1$ and $\lambda = 1$. In this case, for large samples, Λ_n approximately has chi-square distribution with two degrees of freedom.

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2.2. Jackknife estimator

Jackknifing involves transforming the problem of estimating a population parameter into the problem of estimating a population mean. According to this method, a mean value is first estimated, although the approach estimation is unusual. A framework for implementing the jackknife method is given by Lipsitz et al. (1990), who suggest an alternative robust estimator of the covariance matrix based on jackknifing in order to analyze data from repeated measures studies. In this paper, we use this method as an alternative to estimate the population parameters.

Suppose that $T_1, \ldots, T_n$ is a random sample of n values and that $\bar{T} = \sum_{i=1}^{n} \frac{T_i}{n}$ is the sample mean used to estimate the mean of the population.

The sample mean calculated with the lth observation missing is

$$\bar{T}_{-l} = \frac{\sum_{i=1}^{n} T_i - T_l}{n-1},$$

for which

$$T_l = n\hat{T} - (n-1)\bar{T}_{-l}. \tag{8}$$

Using a general example, let θ be a parameter estimated by $\hat{E}(T_1, \ldots, T_n)$. For ease of notation, we drop $(T_1, \ldots, T_n)$. Finally, $\hat{E}_{-l}$ is calculated, which is obtained with the T_l observation missing. It follows from Eq. (8) that pseudo-values can be calculated as follows:

$$\hat{E}_l^* = n\hat{E} - (n-1)\hat{E}_{-l}, \quad l = 1, \ldots, n.$$

The average of the pseudo-values is the jackknife estimate of θ given by

$$\hat{E}^* = \frac{\sum_{l=1}^{n} \hat{E}_l^*}{n}.$$

Manly (1997) suggested that an approximate $100(1-\alpha)\%$ confidence interval for θ is given by $\hat{E}^* \pm t_{\alpha/2,n-1} s/\sqrt{n}$, where s is the standard deviation of the pseudo-values; and $t_{\alpha/2,n-1}$ is the upper $(1-\alpha/2)$ point of the t-distribution with $(n-1)$ degrees of freedom, which has the effect of removing bias of order $1/n$.

The jackknife estimation calculations for the LEW regression model for interval-censored data are performed for λ, σ and $\beta_j (j = 1, \ldots, p)$, and confidence intervals are calculated separately for each parameter.

2.3. Bootstrap re-sampling methods

The bootstrap re-sampling method was proposed by Efron (1979). The method treats the observed sample as if it represented the population. From the information obtained from such a sample, B bootstrap samples of similar size to that of the observed sample are generated, from which it is possible to estimate various characteristics of the population, such as mean, variance, percentiles and so on.

According to the literature, the re-sampling method may be non-parametric or parametric. In this study, the parametric bootstrap method is addressed, according to which the distribution function F can be estimated by $\hat{F}_{\boldsymbol{\theta}}$ from B bootstrap samples generated by a parametric model.

Let $(T_1, \ldots, T_n)$ be an observed random sample with which an estimator $\hat{\boldsymbol{\theta}} = s(\hat{F})$ is calculated from a parameter of interest $\boldsymbol{\theta} = s(F)$. Given this supposition, $(T_1^*, \ldots, T_B^*)$ samples are randomly generated through parametric bootstrap sampling. For the B bootstrap samples generated, $T_1^*, \ldots, T_B^*$, the bootstrap replication of the parameter of interest for the bth sample is given by:

$$\hat{\boldsymbol{\theta}}_b^* = s(T_b^*).$$

That is, this is the value of $\hat{\boldsymbol{\theta}}$ for sample $T_b^*, b = 1, \ldots, B$.

The bootstrap estimator of the standard error (Efron and Tibshirani, 1993) is the standard deviation of these bootstrap samples; it is denoted by $\hat{EP}_B$ and obtained by the following expression

$$\hat{EP}_B = \left[\frac{1}{(B-1)} \sum_{b=1}^{B} \left(\hat{\theta}_b^* - \bar{\theta}_B\right)^2\right]^{1/2},$$

in which $\bar{\theta}_B = \frac{1}{B}\sum_{b=1}^{B} \hat{\theta}_b^*$. Note that B is the number of bootstrap samples generated. According to Efron and Tibshirani (1993), assuming $B \geq 200$, it is generally sufficient to present good results to determine the bootstrap estimations. However, to achieve greater accuracy, a reasonably high B value must be considered. We describe the bias corrected and accelerated (BCa) method for constructing approximated confidence intervals based on the bootstrap re-sampling method. For further details on bootstrap intervals, see, Efron and Tibshirani (1993), DiCiccio and Efron (1996) and Davison and Hinkley (1997).

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BCa bootstrap interval

The bootstrap interval based on the BCa method assumes that the percentiles used in delimitating the bootstrap confidence intervals depend on the corrections to tendency $\hat{a}$ and acceleration $\hat{z}_0$.

The bias correction value $\hat{z}_0$ is generated based on the proportion of estimations of bootstrap samples that are smaller than the original estimation $\hat{\boldsymbol{\theta}}$. The expression of $\hat{z}_0$ is given by

$$\hat{z}_0 = \Phi^{-1}\left(\frac{\sharp(\hat{\boldsymbol{\theta}}_b^* < \hat{\boldsymbol{\theta}})}{B}\right), \quad b = 1, \ldots, B.$$

Note that $\Phi^{-1}(\cdot)$ is the inverse of the accumulated standard normal distribution; B is the number of generated bootstrap samples; $\hat{\boldsymbol{\theta}}$ is the MLE of the observed sample; and $\hat{\boldsymbol{\theta}}_b^*$ is the MLE of the bth bootstrap sample.

Let $\hat{\boldsymbol{\theta}}_{(i)}$ be the MLE of the sample without the ith observation. Then $\hat{a}$ is given by

$$\hat{a} = \frac{\sum_{i=1}^{n}\left[\hat{\boldsymbol{\theta}}_{(\cdot)} - \hat{\boldsymbol{\theta}}_{(i)}\right]^3}{6\left\{\sum_{i=1}^{n}\left[\hat{\boldsymbol{\theta}}_{(\cdot)} - \hat{\boldsymbol{\theta}}_{(i)}\right]^2\right\}^{3/2}}.$$

Note that $\hat{\boldsymbol{\theta}}_{(\cdot)} = \sum_{i=1}^{n} \hat{\boldsymbol{\theta}}_{(i)}/n$ and n is the sample size.

Hence, the BCa bootstrap interval of coverage $100(1-2\alpha)\%$ is given by

$$\left[\hat{\boldsymbol{\theta}}^*_{(B\alpha_1)}, \hat{\boldsymbol{\theta}}^*_{(B\alpha_2)}\right],$$

in which

$$\alpha_1 = \Phi\left\{\hat{z}_0 + \frac{\hat{z}_0 + \Phi^{-1}(\alpha)}{1 - \hat{a}\left[\hat{z}_0 + \Phi^{-1}(\alpha)\right]}\right\} \quad \text{and} \quad \alpha_2 = \Phi\left\{\hat{z}_0 + \frac{\hat{z}_0 + \Phi^{-1}(1-\alpha)}{1 - \hat{a}\left[\hat{z}_0 + \Phi^{-1}(1-\alpha)\right]}\right\}.$$

Note that α_1 and α_2 are corrections to the bootstrap percentiles; $\Phi(\cdot)$ is an accumulated distribution function of the standard normal distribution; and $\Phi^{-1}(\cdot)$ is the inverse of the accumulated distribution function of the standard normal distribution. The percentile bootstrap interval is considered a particular case of the BCa bootstrap interval (Efron and Tibshirani, 1993).

2.4. *A Bayesian analysis of the model*

The use of an alternative Bayesian method allows the incorporation of previous knowledge of the parameters through informative priori densities. When this information is not available, one may consider non-informative priori densities. In the Bayesian approach, the information that refers to model parameters is obtained through the posterior marginal distribution. In this way, two difficulties arise. The first involves attaining the marginal posterior distribution, while the second relates to the calculation of moments. In both cases, integral calculations that many times do not present analytical solutions are necessary. We use MCMC method, such as the Gibbs sampler and Metropolis–Hasting algorithm, to overcome these difficulties.

The LEW regression model (5) for interval-censored data and the likelihood function (6) for $\beta_0, \beta_1, \ldots, \beta_p$ and σ are considered here. For a Bayesian analysis, we assume the following priori densities for β_j, σ and λ;

- $\beta_j \sim N(\mu_{0j}, \sigma_{0j}^2)$, μ_{0j} and σ_{0j}^2 known, $j = 1, \ldots, p$;
- $\sigma \sim IG(a, b)$, a and b known;
- $\lambda \sim G(a_1, b_1)$ a_1 and b_1 known.

Note that $IG(a, b)$ denotes an inverse gamma distribution with a density function given by

$$f(\omega; a, b) = \frac{b^a \omega^{-(a_i+1)} \exp(-b/\omega)}{\Gamma(a)}. \tag{9}$$

In addition, $G(a_1, b_1)$ denotes a gamma distribution with mean a_1/b_1 and variance a_1/b_1^2.

From the likelihood function $L(\boldsymbol{\beta}, \lambda, \sigma) = \exp\left[\ell(\boldsymbol{\beta}, \lambda, \sigma)\right]$ and assuming independence among parameters β_j, $j = 0, \ldots, p$ and σ, we can show that the joint posterior distribution for $\boldsymbol{\beta} = (\beta_0, \ldots, \beta_1)$, σ and λ is given by

$$\pi(\boldsymbol{\beta}, \lambda, \sigma | D) \propto \frac{\lambda^{a_1-1}}{\sigma^{a+1}} \exp\left\{-\frac{1}{2}\sum_{j=1}^{p}\frac{(\beta_j - \mu_{0j})^2}{\sigma_{0j}} - \frac{b}{\sigma} + b_1\lambda\right\} L(\boldsymbol{\beta}, \lambda, \sigma), \tag{10}$$

where

$$L(\boldsymbol{\beta}, \lambda, \sigma) = \prod_{i\in F}\left[\left\{1-\exp\left[-\exp(zv_i)\right]\right\}^{\lambda} - \left\{1-\exp\left[-\exp(zu_i)\right]\right\}^{\lambda}\right]\prod_{i\in C}\left[1-\left\{1-\exp\left[-\exp(zu_i)\right]\right\}^{\lambda}\right],$$

is the likelihood function, $zu_i = \left[\log(u_i) - \mathbf{x}_i^T\,\boldsymbol{\beta}\right]/\sigma$ and $zv_i = \left[\log(v_i) - \mathbf{x}_i^T\,\boldsymbol{\beta}\right]/\sigma$.

The joint posterior density of $(\boldsymbol{\beta}, \lambda, \sigma)$ in (10) is analytically problematic because the integration of the joint posterior density is not easy to perform. An alternative is the Gibbs sampler algorithm, which is discussed next. In this regard, we first obtain the conditional distributions of the parameters

$$\pi(\boldsymbol{\beta}|\lambda, \sigma, \mathbf{D}) \propto \exp\left\{-\frac{1}{2}\sum_{j=1}^{p}\frac{(\beta_j-\mu_{0j})^2}{\sigma_{0j}} + \log[L(\boldsymbol{\beta}, \lambda, \sigma)]\right\},$$

$$\pi(\sigma|\boldsymbol{\beta}, \lambda, \mathbf{D}) \propto \frac{1}{\sigma^{a+1}}\exp\left\{-\frac{b}{\sigma} + \log\left[L(\boldsymbol{\beta}, \lambda, \sigma)\right]\right\} \quad \text{and}$$

$$\pi(\lambda|\boldsymbol{\beta}, \sigma, \mathbf{D}) \propto \lambda^{a_1-1}\exp\left\{-b_1\lambda + \log\left[L(\boldsymbol{\beta}, \lambda, \sigma)\right]\right\}.$$

Note that we must use the Metropolis–Hasting algorithm to generate variables $\boldsymbol{\beta}$, λ and σ from the conditional posterior densities.

3. Sensitivity analysis

3.1. Global influence

As previously stated, the first step involved in performing sensitivity analysis is to focus on global influences according to case deletion (Cook, 1977). Case deletion is a common approach to study the effect of dropping the ith case from the data set. Case deletion for model (5) is given by

$$Y_l = \mathbf{x}_l^T\,\boldsymbol{\beta} + \sigma z_l, \quad l = 1, 2, \ldots, n,\ l \neq i. \tag{11}$$

In the following section, a quantity with subscript "(i)" indicates that the original data set with the ith observation deleted. For model (11), the log-likelihood function is denoted by $l_{(i)}(\boldsymbol{\theta})$.

Let $\hat{\boldsymbol{\theta}}_{(i)} = (\hat{\sigma}_{(i)}, \hat{\boldsymbol{\beta}}_{(i)}^T)^T$ be the MLE based on $\boldsymbol{\theta}$ obtained from maximizing $l_{(i)}(\boldsymbol{\theta})$. To assess the influence of the ith observation on the MLE $\hat{\boldsymbol{\theta}} = (\hat{\sigma}, \hat{\boldsymbol{\beta}}^T)^T$, we compare the difference between $\hat{\boldsymbol{\theta}}_{(i)}$ and $\hat{\boldsymbol{\theta}}$. If the deletion of an observation seriously influences an estimates, more attention should be paid to that observation. Hence, if $\hat{\boldsymbol{\theta}}_{(i)}$ is far from $\hat{\boldsymbol{\theta}}$, then this case is regarded as an influential observation. An initial measure of global influence is defined as the standardized norm of $\hat{\boldsymbol{\theta}}_{(i)} - \hat{\boldsymbol{\theta}}$ (i.e., generalized Cook distance):

$$GD_i(\boldsymbol{\theta}) = (\hat{\boldsymbol{\theta}}_{(i)} - \hat{\boldsymbol{\theta}})^T\left\{-\ddot{\mathbf{L}}(\boldsymbol{\theta})\right\}(\hat{\boldsymbol{\theta}}_{(i)} - \hat{\boldsymbol{\theta}}).$$

Another alternative is to assess the values $GD_i(\boldsymbol{\beta})$ and $GD_i(\sigma)$, which reveal the impact of the ith observation on the estimates of $\boldsymbol{\beta}$ and σ, respectively. Another popular measure of the difference between $\hat{\boldsymbol{\theta}}_{(i)}$ and $\hat{\boldsymbol{\theta}}$ is the following likelihood displacement:

$$LD_i(\boldsymbol{\theta}) = 2\left\{l(\hat{\boldsymbol{\theta}}) - l(\hat{\boldsymbol{\theta}}_{(i)})\right\}.$$

Furthermore, we can also compute $\beta_j - \beta_{j(i)} (j = 1, 2, \ldots, p)$ to calculate the difference between $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\beta}}_{(i)}$. Other global influence measures are also possible. One might think of the behavior of a test statistic under a case-deletion scheme; such statistics may include the Wald test for explanatory variables or censoring effects.

To avoid employing direct model estimation for all observations, we can use the following one-step approximation to reduce the burden of calculation:

$$\hat{\boldsymbol{\theta}}_{(i)}^1 = \hat{\boldsymbol{\theta}} + \ddot{\mathbf{L}}(\hat{\boldsymbol{\theta}})^{-1}\dot{\mathbf{l}}_{(i)}(\hat{\boldsymbol{\theta}}).$$

Note that $\dot{\mathbf{l}}_{(i)}(\hat{\boldsymbol{\theta}}) = \frac{\partial l_{(i)}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$ is evaluated at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$ (Cook and Weisberg, 1982).

3.2. Local influence

Another approach suggested by Cook (1986) is to weight observations instead of removing then. The calculation of local influences can be carried out for model (5). If the likelihood displacement $LD(\boldsymbol{\omega}) = 2\{l(\hat{\boldsymbol{\theta}}) - l(\hat{\boldsymbol{\theta}}_{\boldsymbol{\omega}})\}$ is used, where $\hat{\boldsymbol{\theta}}_{\boldsymbol{\omega}}$ denotes MLE under the perturbed model, then the normal curvature for $\boldsymbol{\theta}$ at direction $\mathbf{d}$, $\|\mathbf{d}\| = 1$, is given by $C_{\mathbf{d}}(\boldsymbol{\theta}) = 2|\mathbf{d}^T\boldsymbol{\Delta}^T\left[\ddot{\mathbf{L}}(\boldsymbol{\theta})\right]^{-1}\boldsymbol{\Delta}\mathbf{d}|$, where $\boldsymbol{\Delta}$ is a $(p+2)\times n$ matrix that depends on the perturbation scheme. The elements of this scheme are given by $\Delta_{vi} = \partial^2 l(\boldsymbol{\theta}|\boldsymbol{\omega})/\partial\theta_v\partial\omega_i, i = 1, 2, \ldots, n$ and $v = 1, 2, \ldots, p+2$ evaluated at $\hat{\boldsymbol{\theta}}$ and $\boldsymbol{\omega}_0$; $\boldsymbol{\omega}_0$ is the no-perturbation vector. For the LEW regression model, the elements of $\ddot{\mathbf{L}}(\boldsymbol{\theta})$ are given in Appendix A. We can also calculate the normal curvatures $C_{\mathbf{d}}(\lambda)$, $C_{\mathbf{d}}(\sigma)$ and $C_{\mathbf{d}}(\boldsymbol{\beta})$ to perform various index plots, including, for instance, the index plot of $\mathbf{d}_{max}$, the eigenvector corresponding to $C_{\mathbf{d}_{max}}$, the largest eigenvalue of the matrix $\mathbf{B} = -\boldsymbol{\Delta}^T\left[\ddot{\mathbf{L}}(\boldsymbol{\theta})\right]^{-1}\boldsymbol{\Delta}$, and the index plots of $C_{\mathbf{d}_i}(\lambda)$, $C_{\mathbf{d}_i}(\sigma)$ and $C_{\mathbf{d}_i}(\boldsymbol{\beta})$, which are together denoted as the total local influence. See, for example, Lesaffre and Verbeke (1998) in which $\mathbf{d}_i$ denotes an $n \times 1$ vector of zeros with one at the ith position. Thus, the curvature at direction $\mathbf{d}_i$ assumes the form $C_i = 2|\boldsymbol{\Delta}_i^T\left[\ddot{\mathbf{L}}(\boldsymbol{\theta})\right]^{-1}\boldsymbol{\Delta}_i|$, where $\boldsymbol{\Delta}_i^T$ denotes the ith row of $\boldsymbol{\Delta}$. It is commonplace to point out cases in which $C_i \geq 2\bar{C}$, where $\bar{C} = \frac{1}{n}\sum_{i=1}^{n} C_i$.

Next, for five perturbation schemes, we calculate the following matrix:

$$\boldsymbol{\Delta} = (\boldsymbol{\Delta}_{vi})_{(p+2)\times n} = \left(\frac{\partial^2 l(\boldsymbol{\theta}|\boldsymbol{\omega})}{\partial\theta_i\partial\boldsymbol{\omega}_v}\right)_{(p+2)\times n}, \quad v = 1, \ldots, p+2 \text{ and } i = 1, \ldots, n.$$

We consider the model (5) and its log-likelihood function given by (6). Consider the vector of weights $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_n)^T$.

- **Case-weight perturbation**
  In this case, the log-likelihood function takes the form $l(\boldsymbol{\theta}|\boldsymbol{\omega}) = \sum_{i\in F}\omega_i l_1(\lambda, zu_i, zv_i) + \sum_{i\in C}\omega_i l_2(\lambda, zu_i)$, where $0 \leq \omega_i \leq 1$ and $\boldsymbol{\omega}_0 = (1, \ldots, 1)^T$; $l_1(\cdot)$ and $l_2(\cdot)$ is defined in (6). The matrix $\boldsymbol{\Delta} = (\boldsymbol{\Delta}_\lambda, \boldsymbol{\Delta}_\sigma, \boldsymbol{\Delta}_{\boldsymbol{\beta}})^T$ is given in Appendix C.
- **Response perturbation** ($\log(u_i)$)
  We here consider that each u_i is perturbed as $u_{iw} = u_i + \omega_i S_u$, where S_u is a scale factor that may be equal to the estimated standard deviation of U, $\omega_i \in \mathbf{R}$. Here, the perturbed log-likelihood function becomes expressed as $l(\boldsymbol{\theta}|\boldsymbol{\omega}) = \sum_{i\in F} l_1(\lambda, zu_i^*, zv_i) + \sum_{i\in C} l_2(\lambda, zu_i^*)$. Note that $(zu)_i^* = [\log(u_i^*) - \mathbf{x}_i^T\hat{\boldsymbol{\beta}}]/\hat{\sigma}$, $u_i^* = [\log(u_i) + \omega_i S_u]$ and $\boldsymbol{\omega}_0 = (0, \ldots, 0)^T$; $l_1(\cdot)$ and $l_2(\cdot)$ is defined in (6). The matrix $\boldsymbol{\Delta} = (\boldsymbol{\Delta}_\lambda, \boldsymbol{\Delta}_\sigma, \boldsymbol{\Delta}_{\boldsymbol{\beta}})^T$ is given in Appendix D.
- **Response perturbation** ($\log(v_i)$)
  We now consider that each v_i is perturbed as $v_{iw} = v_i + \omega_i S_v$, where Sv is a scale factor that may be equal to the estimated standard deviation of V, $\omega_i \in \mathbf{R}$.
  Here, the perturbed log-likelihood function can be expressed as $l(\boldsymbol{\theta}|\boldsymbol{\omega}) = \sum_{i\in F} l_1(\lambda, zu_i, zv_i^*) + \sum_{i\in C} l_2(\lambda, zu_i)$, where $zv_i^* = [\log(v_i^*) - \mathbf{x}_i^T\hat{\boldsymbol{\beta}}]/\hat{\sigma}$, $v_i^* = [\log(v_i) + \omega_i S_v]$ and $\boldsymbol{\omega}_0 = (0, \ldots, 0)^T$; $l_1(\cdot)$ and $l_2(\cdot)$ is defined in (6). The matrix $\boldsymbol{\Delta} = (\boldsymbol{\Delta}_\lambda, \boldsymbol{\Delta}_\sigma, \boldsymbol{\Delta}_{\boldsymbol{\beta}})^T$ is provided in Appendix E.
- **Simultaneous response perturbation** ($\log(u_i), \log(v_i)$)
  We now assume that each u_i, v_i is perturbed as $u_{iw} = u_i + \omega_i S_u$, $v_{iw} = v_i + \omega_i S_v$, respectively, where S_u and S_v are scale factors that may be equal to the estimated standard deviations of U and V, $\omega_i \in \mathbf{R}$.
  Here, the perturbed log-likelihood function can be expressed as $l(\boldsymbol{\theta}|\boldsymbol{\omega}) = \sum_{i\in F} l_1(\lambda, zu_i^*, zv_i^*) + \sum_{i\in C} l_2(\lambda, zu_i^*)$, where $zu_i^* = [\log(u_i^*) - \mathbf{x}_i^T\hat{\boldsymbol{\beta}}]/\hat{\sigma}$, $\log(u_i^*) = (\log(u_i) + \omega_i S_u)$, $zv_i^* = [\log(v_i^*) - \mathbf{x}_i^T\hat{\boldsymbol{\beta}}]/\hat{\sigma}$, $\log(v_i^*) = [\log(v_i) + \omega_i S_v]$ and $\boldsymbol{\omega}_0 = (0, \ldots, 0)^T$; $l_1(\cdot)$ and $l_2(\cdot)$ are defined in (6). The matrix $\boldsymbol{\Delta} = (\boldsymbol{\Delta}_\lambda, \boldsymbol{\Delta}_\sigma, \boldsymbol{\Delta}_{\boldsymbol{\beta}})^T$ is given in Appendix F.
- **Explanatory variable perturbation**
  Consider an additive perturbation on a particular continuous explanatory variable, namely, X_t, by allowing $x_{it\omega} = x_{it} + \omega_i S_x$, where S_x is a scale factor, $\omega_i \in \mathbf{R}$. This perturbation scheme leads to the following expression of the log-likelihood function: $l(\boldsymbol{\theta}|\boldsymbol{\omega}) = \sum_{i\in F} l_1(\lambda, zu_i^{**}, zv_i^{**}) + \sum_{i\in C} l_2(\lambda, zu_i^{**})$, where $zu_i^{**} = [\log(u_i) - \mathbf{x}_i^{*T}\boldsymbol{\beta}]/\sigma$, $zv_i^{**} = [v_i - \mathbf{x}_i^{*T}\boldsymbol{\beta}]/\sigma$, $\mathbf{x}_i^{*T}\boldsymbol{\beta} = \beta_1 + \beta_2 x_{i2} + \cdots + \beta_t(x_{it} + \omega_i S_x) + \cdots + \beta_p x_{ip}$ and $\boldsymbol{\omega}_0 = (0, \ldots, 0)^T$; $l_1(\cdot)$ and $l_2(\cdot)$ are defined in (6). The matrix $\boldsymbol{\Delta} = (\boldsymbol{\Delta}_\lambda, \boldsymbol{\Delta}_\sigma, \boldsymbol{\Delta}_{\boldsymbol{\beta}})^T$ is given in Appendix G.

4. Residual analysis

An analysis of residuals may be carried out for a number of purposes after a statistical model is fitted to the data. These purposes include checking the assumptions of the model, checking the validity of the data, and examining the data to reveal useful information. To study any departures from the error assumption as well as the presence of outliers, there are various residual analysis proposed in the literature; see, for example, Collett (2003) and Ortega et al. (2008). Defining residuals for interval-censored data is complicated by the fact that the observations are incomplete. Farrington (2000) developed an analysis of residuals for proportional hazard models using interval-censored survival data. In this study, we extend the residuals proposed by Farrington (2000) to the LEW regression model for interval-censored data.

4.1. Adjusted Cox–Snell residual

This residual for LEW regression models for interval-censored data takes the form

$$r_{C_i} = \begin{cases} \dfrac{(1-\hat{a}_i)^{\hat{\lambda}}\left[1 - \hat{\lambda}\log\left(1-\hat{a}_i\right)\right] - \left(1-\hat{b}_i\right)^{\hat{\lambda}}\left[1 - \hat{\lambda}\log\left(1-\hat{b}_i\right)\right]}{\left(1-\hat{b}_i\right)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} & \text{if } i \in F, \\ 1 - \hat{\lambda}\log\left(1-\hat{a}_i\right) & \text{if } i \in C, \end{cases}$$

where

$$\hat{a}_i = \exp[-\exp(\hat{zu}_i)], \qquad \hat{b}_i = \exp[-\exp(\hat{zv}_i)],$$
$$\hat{zu}_i = [\log(u_i) - \mathbf{x}_i^T \widehat{\boldsymbol{\beta}}]/\hat{\sigma} \quad \text{and} \quad \hat{zv}_i = [\log(v_i) - \mathbf{x}_i^T \widehat{\boldsymbol{\beta}}]/\hat{\sigma}.$$

4.2. Martingale residual

The martingale residual was introduced by Lagakos (1980); it is also referred to as the Lagakos residual. Thus, the martingale residual for the LEW regression models for interval-censored data takes the form

$$r_{M_i} = \begin{cases} \dfrac{\hat{\lambda}\left[(1-\hat{a}_i)^{\hat{\lambda}}\log(1-\hat{a}_i) - (1-\hat{b}_i)^{\hat{\lambda}}\log(1-\hat{b}_i)\right]}{\left(1-\hat{b}_i\right)^{\hat{\lambda}} - \left(1-\hat{a}_i\right)^{\hat{\lambda}}} & \text{if } i \in F, \\ \hat{\lambda}\log\left(1-\hat{a}_i\right) & \text{if } i \in C. \end{cases}$$

4.3. Modified deviance residual

The martingale residual is transformed in order to exhibit a new residual symmetrically distributed around zero. The modified deviance applied to the LEW regression models for interval-censored data can be expressed as

$$r_{D_i} = \begin{cases} \operatorname{sgn}(\hat{r}_{M_i})\left\{-2\left[\dfrac{\hat{\lambda}\left[(\hat{a}_i)_\lambda - (\hat{b}_i)_\lambda\right]}{\left(1-\hat{b}_i\right)^{\hat{\lambda}} - \left(1-\hat{a}_i\right)^{\hat{\lambda}}} + \log\left(1 - \left\{\dfrac{\hat{\lambda}\left[(\hat{a}_i)_\lambda - (\hat{b}_i)_\lambda\right]}{\left(1-\hat{b}_i\right)^{\hat{\lambda}} - \left(1-\hat{a}_i\right)^{\hat{\lambda}}}\right\}\right)\right]\right\}^{1/2} & \text{if } i \in F, \\ \operatorname{sgn}(\hat{r}_{M_i})\left[-2\hat{\lambda}\log(1-\hat{a}_i)\right]^{1/2} & \text{if } i \in C. \end{cases}$$

Note that

$$(\hat{a}_i)_\lambda = (1-\hat{a}_i)^{\hat{\lambda}}\log(1-\hat{a}_i), \qquad (\hat{b}_i)_\lambda = (1-\hat{b}_i)^{\hat{\lambda}}\log(1-\hat{b}_i), \qquad \hat{a}_i = \exp[-\exp(\hat{zu}_i)],$$
$$\hat{b}_i = \exp[-\exp(\hat{zv}_i)], \qquad \hat{zu}_i = [\log(u_i) - \mathbf{x}_i^T \widehat{\boldsymbol{\beta}}]/\hat{\sigma} \quad \text{and} \quad \hat{zv}_i = [\log(v_i) - \mathbf{x}_i^T \widehat{\boldsymbol{\beta}}]/\hat{\sigma}.$$

4.4. Simulation studies

To investigate the empirical distributions of the residuals r_i, r_{M_i} and r_{D_i} for the values $n = 50$, $n = 100$ and $n = 200$, $\lambda = 0.3$ and 1.00, $\sigma = 0.5$, and censoring percentages 0, 0.10 and 0.30, we performed a small simulation study described below. The log-lifetimes denoted by $\log(T_1), \ldots, \log(T_n)$ were generated from the LEW distribution given in (4) by considering the re-parametrization $\alpha = \sigma^{-1}$ and $\gamma = \exp(\mu)$ and by assuming $\mu_i = \beta_0 + \beta_1 x_i$, with x_i generated from a uniform distribution in the range [0, 1]. Note that β_0 and β_1 were fixed. The censoring times denoted by $C_1, \ldots, C_n$ were generated from a uniform distribution $[0, \tau]$, where τ was adjusted until censoring percentages of 0, 0.10 or 0.30, were reached. The lifetimes considered in each fit were calculated as $y_i = \min\{\log(C_i), \log(T_i)\}$, and $\log(u_i)$ and $\log(v_i)$ were generated for the times with interval censoring such that $y_i \in (\log(u_i), \log(v_i))$ with probability one (Zhao, 2004).

For each configuration of n, λ, σ and censoring percentage, 1000 samples were generated, and each one was fitted under the LEW regression model (5) for interval-censored data. For each fit, the residuals r_i, r_{M_i} and r_{D_i} were calculated and stored. We then performed normal probability plots between the mean quartiles of the residuals and the expected quartiles of the standard normal distribution.

From Figs. 2 and 3, we note the following conclusions:

- We clearly observe that the empirical distribution of the modified deviance residual follows the standard normal distribution.
- The empirical distributions of the adjusted Cox–Snell residuals and the martingale residuals exhibit similar behaviors.
- The empirical distributions of the adjusted Cox–Snell and the martingale residuals in general present accentuated asymmetry.
- As censoring decreases, the empirical distribution of the modified deviance residual seems to approach the standard normal distribution faster than the other two residuals considered in the analysis.
- As the sample size increases, the empirical distribution of the modified deviance residual seems to exhibit the best alignment with the standard normal distribution.

Thus, we recommend the use of normal probability plots for r_{D_i} with a simulated envelope, as suggested by Atkinson (1985).

Please cite this article in press as: Hashimoto, E.M., et al., The log-exponentiated Weibull regression model for interval-censored data. Computational Statistics and Data Analysis (2009), doi:10.1016/j.csda.2009.10.014

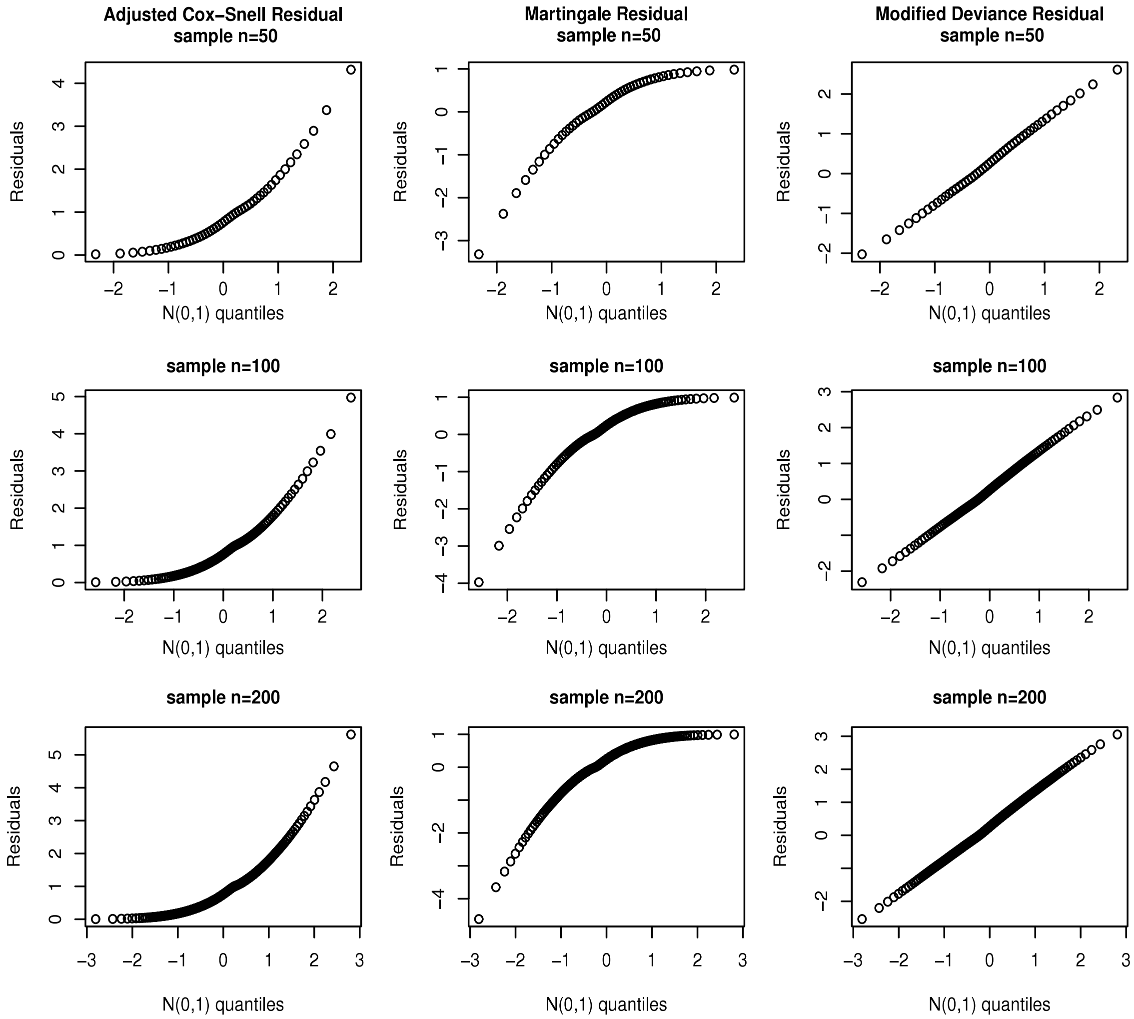

Fig. 2. Normal probability plots for adjusted Cox–Snell residuals (r_i), martingale residual (r_{M_i}) and modified deviance residual (r_{D_i}). Sample sizes $n = 50$, $n = 100$ and $n = 200$, percentage of right-censored = 30 and $\lambda = 1$.

5. Application

We provide an application of the results derived in the previous sections using real data. The required numerical evaluations were implemented using the software program Ox (Doornik, 2001). We illustrate the proposed model using the cancer data on $n = 94$ breast cancer patients, 48 of whom were treated with radiation therapy and adjuvant chemotherapy ($x_{i1} = 1$) and 46 of whom were treated with radiation therapy alone ($x_{i1} = 0$). The response variable (y_i) is the log-time to cosmetic deterioration, for all $i = 1, 2, \ldots, 94$.

5.1. Estimation

5.1.1. Maximum likelihood and jackknife estimation

To obtain the MLEs of the parameters in the LEW regression model for interval-censored data, we used the subroutine MAXBFGS in Ox; the results are given in Table 1. Additionally, in Table 1, we report the jackknife estimates. We can observe that the explanatory variable x_1 is significant at the level of 5%. From Table 1, we can observe that the variable x_1 is significant (at 5%) for the log-survival time. Note that estimates from the two methods seem to be very similar.

5.1.2. Bootstrap re-sampling method

We considered $B = 5000$ bootstrap samples of the LEW regression model with interval-censored data. By using the bootstrap method described in Section 2.3, we found the estimated bootstrap and the BCa confidence intervals, which are presented in Table 2.

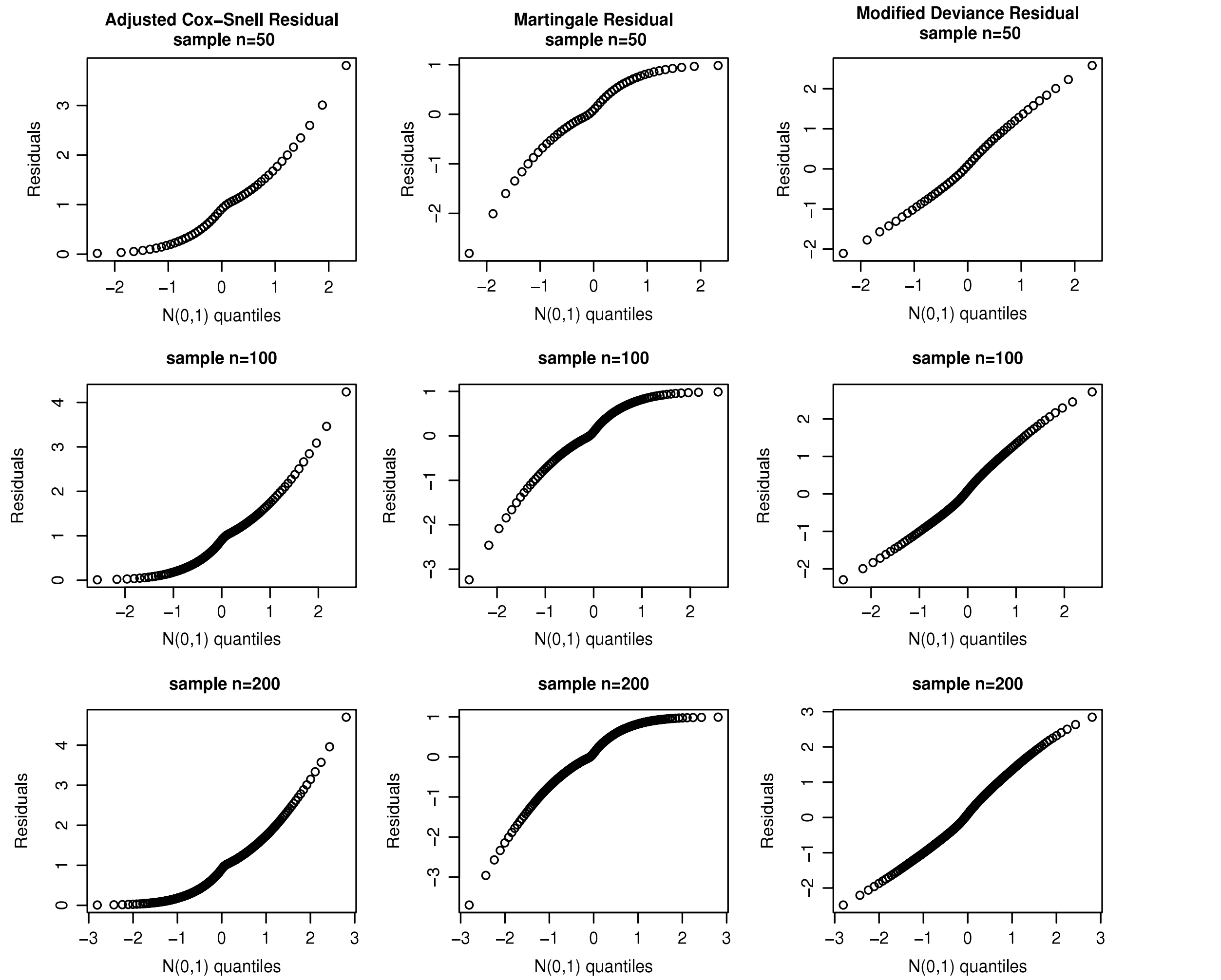

Fig. 3. Normal probability plots for adjusted Cox–Snell residuals (r_i), martingale residual (r_{M_i}) and modified deviance residual (r_{D_i}). Sample sizes $n = 50$, $n = 100$ and $n = 200$, percentage of right-censored = 30 and $\lambda = 0.5$.

Table 1
Maximum likelihood and jackknife estimates for the parameters of the LEW regression model for interval-censored data fitted to cancer data.

$\boldsymbol{\theta}$	MLEs			Jackknife estimates		
	Estimate	S.E.	p-value	Estimate	S.E.	95% C.I.
λ	0.451	0.349	–	0.383	0.355	(0.000; 1.080)
σ	0.341	0.216	–	0.341	0.220	(0.000; 0.773)
β_0	4.187	0.233	<0.001	4.245	0.226	(3.802; 4.688)
β_1	−0.569	0.169	<0.001	−0.577	0.160	(−0.891; −0.263)

Table 2
Parametric bootstrap estimate and confidence intervals based on the parametric bootstrap re-sampling method using cancer data.

$\boldsymbol{\theta}$	Estimate	S.E.	95% C.I. (BCA)
λ	0.705	1.536	(0.000; 1.548)
σ	0.455	0.272	(0.153; 0.826)
β_0	4.079	0.616	(2.113; 4.555)
β_1	−0.592	0.040	(−0.613; −0.485)

5.1.3. Bayesian analysis

We considered the LEW regression model (5) for interval-censored data by considering the prior densities $\beta_j \sim N(0, 1000)\lambda \sim G(1, 0.01)$ and $\sigma \sim IG(1, 0.01)$. We generated two parallel independent runs of the Gibbs sampler chain with size 50,000, discarding the first 10,000 iterations for each parameter. To eliminate the effect of initial values and to

Table 3
Posterior summaries of the LEW regression model for interval-censored data fitted to cancer data.

Parameters	Mean	S.D	95% Credible interval	$\hat{R}$
λ	0.422	0.384	(0.083; 1.601)	1.008
σ	0.301	0.208	(0.083; 0.861)	1.007
β_0	4.230	0.254	(3.608; 4.683)	1.015
β_1	−0.557	0.177	(−0.946; −0.249)	1.001

Table 4
Posterior summaries of the LEW regression model with ridge shrinkage priors for interval-censored data fitted to cancer data.

Parameters	Mean	S.D	95% Credible interval	$\hat{R}$
λ	0.735	0.392	(0.094; 1.731)	1.000
σ	0.443	0.221	(0.086; 0.901)	1.000
β_0	3.946	0.212	(3.511; 4.482)	1.001
β_1	−0.352	0.164	(−0.740; −0.239)	1.001

Table 5
Results of Monte Carlo simulation based on 500 simulated samples of model in (12). MC mean, MC SD and RMSE are the posterior mean average, standard deviation average and the square root of the mean square error of the estimates, respectively.

Parameter	Vague prior			Ridge shrinkage prior		
	Mean	SD	REQM	Mean	SD	REQM
λ	1.489	0.620	0.823	1.357	0.577	0.678
σ	0.382	0.119	0.424	0.358	0.110	0.180
β_0	4.058	0.207	0.502	4.190	0.166	0.252
β_1	0.897	0.332	0.438	0.882	0.191	0.192

avoid correlation problems, we considered a spacing of size 10, thereby obtaining a sample of size 8,000 from each chain. We monitored the convergence of the Gibbs samples using the Gelman and Rubin (1992) method, which uses an analysis of variance technique if further iterations are needed. In Table 2, we report the posterior summaries for the parameters. Also, in Table 3, we present the estimated potential scale reduction $\hat{R}$ (Gelman and Rubin, 1992), which is an index to check the convergence of the algorithm. Since $\hat{R} < 1.1$ for all parameters, it seems that the chains converge. To evaluate the robustness of the models with regard to the choice of the hyper-parameters of the prior distributions, a small sensitivity analysis was undertaken with larger standard deviations for the prior distributions. The posterior summaries of the parameters do not present remarkable differences and do not influence the results in Table 3.

For comparison, we implemented a Bayesian analysis with $\beta_j \sim N(0, \tau_j^2)$ where $\tau_j \sim IG(1, 0.001), j = 0, 1$. This prior distribution induces a ridge shrinkage prior (Park and Casella, 2008). MCMC computations were done similar to those as described above. The posterior summaries in Table 4 show that the posterior mean is different to Table 3 with less variability for β_0 and β_1, but for θ and λ, standard deviations are very close to those in Table 3.

Simulation

A Monte Carlo simulation was carried out in order to compare the Bayesian estimates with vague prior and ridge shrinkage prior. We took the following regression model:

$$\log T_i = \beta_0 + \beta_1 x_i + \sigma z_i, \quad i = 1, \ldots, 100. \tag{12}$$

Note that the random error z_i has the distribution (4), and $\beta_0 = 4, \beta = 1, \sigma = 0.5$ and $\lambda = 1$, with x_i are generated from a uniform distribution on the range [0, 1]. The censoring times denoted by $C_1, \ldots, C_n$ were generated from a uniform distribution $[0, \tau]$, where τ was adjusted until the censoring percentage 10%, was reached. The log of lifetimes considered in the fit was calculated as $y_i = \min\{\log(C_i), \log(T_i)\}$, and $\log(u_i)$ and $\log(v_i)$ were generated for the times with interval censoring such that $y_i \in (\log(u_i), \log(v_i))$ with probability one. In each simulation, 500 Monte Carlo samples were generated from the Section 4.4 and were fitted via the strategies outlined in the previous section and considering the same prior distribution of our application. Table 5 shows the results of this analysis; we observe from the table that the posterior mean under a vague prior is close to the true value, but the variability is not high when compared with those obtained with a ridge shrinkage prior.

The estimates from the four methods seem to be very similar. The MLEs appear more conservative with large standard errors. Therefore, since normality for the jackknife estimator is expected for this sample size ($n = 94$), one may also expect some symmetric distribution with heavy tails for the MLEs. We continue this analysis by using the MLEs and considering LEW regression models.

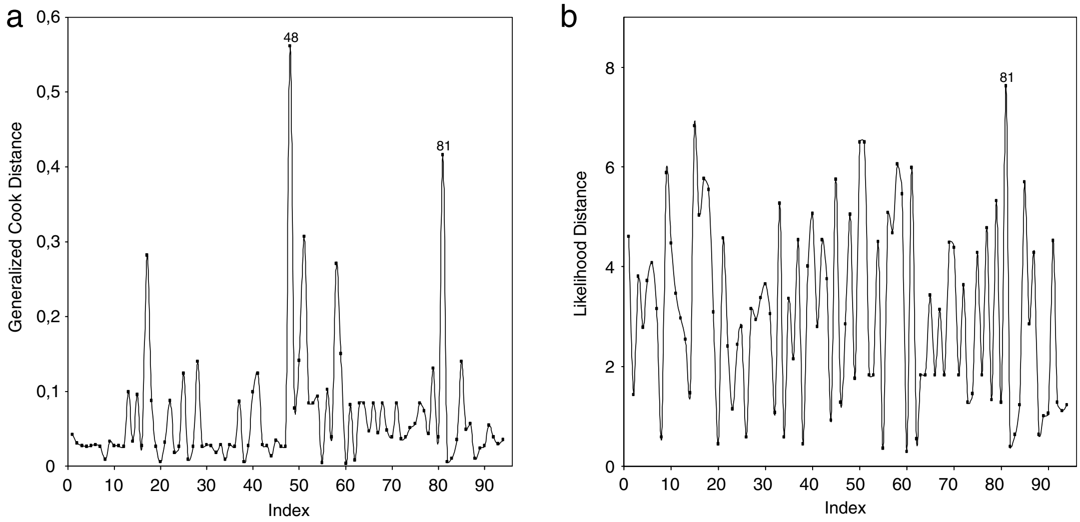

Fig. 4. Index plot of $GD_i(\boldsymbol{\theta})$ (Generalized Cook's distance) (Fig. 2(a)). Index plot of $LD_i(\boldsymbol{\theta})$ (Likelihood distance) (Fig. 2(b)).

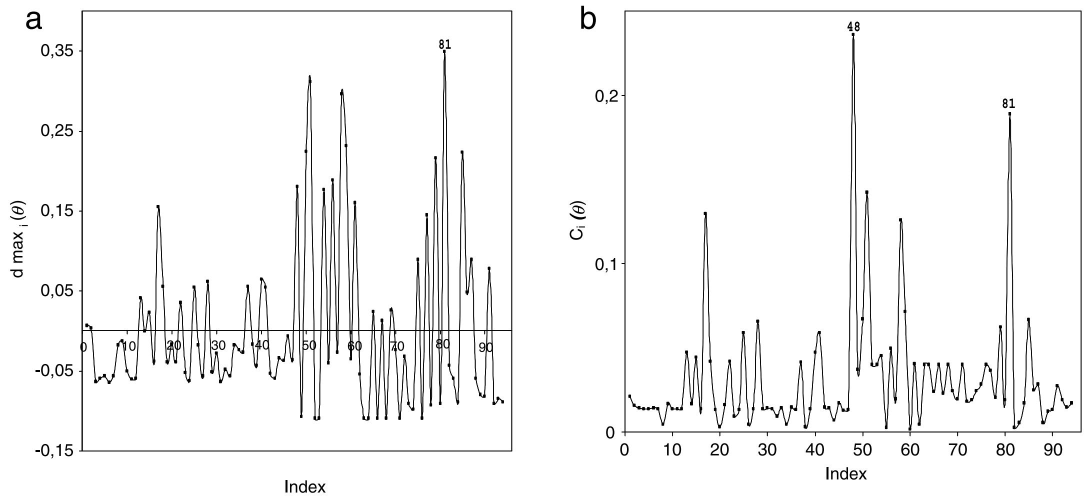

Fig. 5. (a) Index plot of $\mathbf{d}_{max}$ for $\boldsymbol{\theta}$ (case-weight perturbation) and (b) total local influence for $\boldsymbol{\theta}$ (case-weight perturbation) based on the fit of the model to the cancer data.

5.2. Global influence analysis

In this section, we use the Ox software to compute the case-deletion measures $GD_i(\boldsymbol{\theta})$ and $LD_i(\boldsymbol{\theta})$ presented in Section 3.1. The results of such influence measures index plots are displayed in Fig. 4. From this figure, we note that cases ♯48 and ♯81 are possible influential observations.

5.3. Local and total influence analysis

In this section, we analyze local influences with respect to the cancer data set using the LEW regression model for interval-censored data.

Case-weight perturbation

By applying the local influence framework developed in Section 3.2 in which case-weight perturbation is used, the value $C_{\mathbf{d}_{max}} = 1.17$ was obtained as a maximum curvature. In Fig. 5(a), the plot of the eigenvector corresponding to $\mathbf{d}_{max}$ is presented, and the total influence C_i is shown in Fig. 5(b). Observations ♯48 and ♯81 are very distinguished in relation to the others.

Response variable perturbation

Next, the influence of perturbations on the observed survival times is analyzed (i.e., simultaneous response perturbation $(\log(U_i), \log(V_i))$). The value for the maximum curvature calculated was $C_{\mathbf{d}_{max}} = 137.51$. Fig. 6(a) provides the plot for $\mathbf{d}_{max}$

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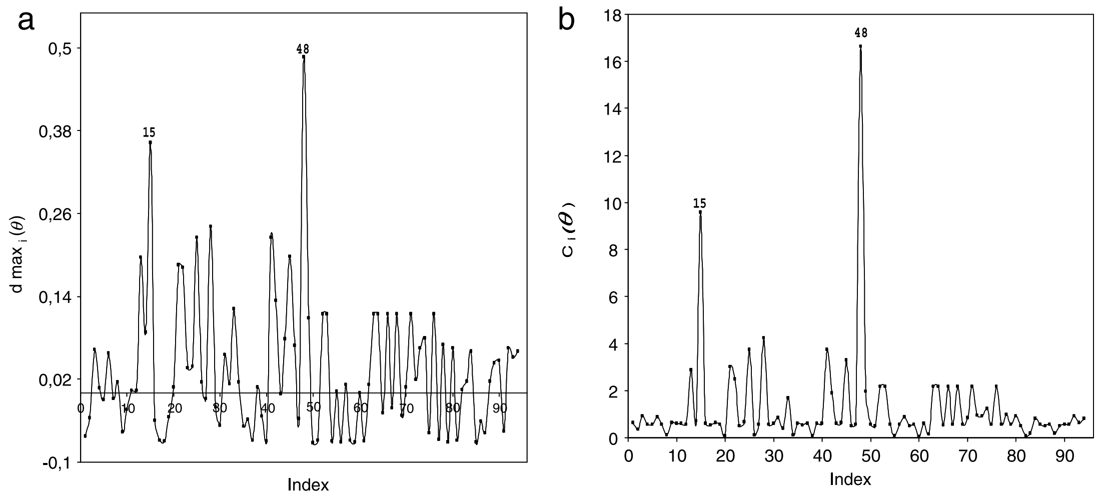

Fig. 6. (a) Index plot of $\mathbf{d}_{max}$ for $\boldsymbol{\theta}$ (simultaneous response perturbation) and (b) total local influence for $\boldsymbol{\theta}$ (simultaneous response perturbation) based on the model fitted to the cancer data.

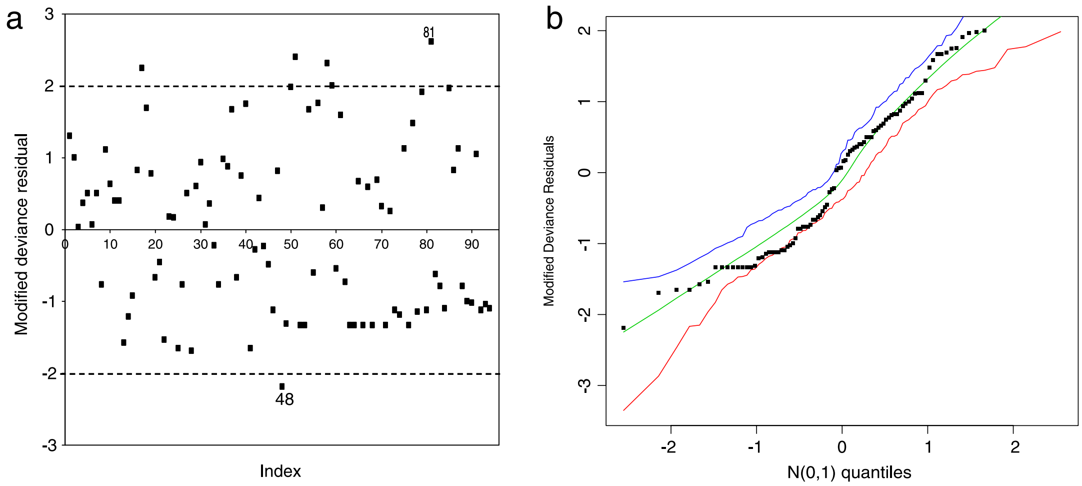

Fig. 7. (a) Index plot of the deviance residual r_{D_i} and (b) normal probability plot for the deviance residual r_{D_i} with envelopes.

versus the observation index that shows observations ♯15 and ♯48 to be more salient in relation to others. Fig. 6(b) presents plots for the total local influence (C_i), according to which observations ♯15 and ♯48 again stand out.

5.4. Residual analysis

To detect possible outlying observations in fitting the LEW regression models for interval-censored data, Fig. 7(a) provides the index plot of r_{D_i}.

By analyzing the modified deviance residual plot, few observations appear as possible outliers (♯48 and ♯81), indicating that the model is well-fitted.

5.5. Impact of the detected influential observations

Hence, the diagnostic analysis (including an analysis of global influence and local influence as well as residual analysis) detected two observations ♯48 and ♯81 as potentially influential. Observation ♯48 represents a censored observation to the right and presents the interval of time $[48, +\infty)$. Observation ♯81, represents an interval-censored whose length is the smallest interval. In order to reveal the impact of these two observations on the parameter estimates, we refitted the model under some situations. First, we individually eliminated each one of these two cases. Next, we removed the totality of potentially influential observations from the set "A", that is, the original data set.

Table 6
Relative changes [-**RC**-in %], estimates and the corresponding p-values in parentheses for the regression coefficients to explain the log-survival time.

Set{I}	λ	σ	β_0	β_1
A	– 0.451 (–)	– 0.341 (–)	– 4.187 (0.000)	– −0.569 (0.001)
A-{♯48}	[47] 0.241 (–)	[44] 0.191 (–)	[3] 4.257 (0.000)	[−2] −0.588 (0.001)
A-{♯81}	[30] 0.584 (–)	[17] 0.400 (–)	[2] 4.122 (0.000)	[−2] −0.582 (0.001)
A-{♯48 and ♯81 }	[32] 0.307 (–)	[34] 0.226 (–)	[−1] 4.228 (0.000)	[6] −0.601 (0.001)

In Table 6, we present the relative changes (in percentage) of each parameter estimate defined by $\mathbf{RC}_{\theta_j} = \left[(\hat{\theta}_j - \hat{\theta}_{j(I)})/\hat{\theta}_j\right] \times 100$, parameter estimates and the corresponding p-values, where $\hat{\alpha}_{j(I)}$ denotes the MLE of θ_j after the set "I" of observations was removed. From Table 6, we note that the MLEs from the LEW regression model for interval-censored data are highly robust under the deletion of outstanding observations. In general, the significance of the parameter estimates does not change at the level of 1% after removing set I. Therefore, we do not have inferential changes after removing the observations identified in the diagnostic plots.

5.6. Goodness-of-fit

To detect possible departures from the assumptions of distribution errors made for model (5) as well as outlying observations, we present in Fig. 7(b) the normal probability plot for the modified deviance residual with the generated envelope, as suggested by Atkinson (1985). As we can see, the plot in Fig. 7(b) indicates that the LEW regression model for interval-censored data does not seem unsuitable to fit the data. Also, no observation appears as a possible outlier.

6. Concluding remarks

In this paper, an LEW regression model for interval-censored data is proposed. We used the Quasi-Newton algorithm to obtain the maximum likelihood estimates, and asymptotic tests were performed for the parameters based on the asymptotic distribution of the MLEs. However, as an alternative analysis, the paper discusses the use of Bayesian inference, the jackknife estimator and parametric bootstrapping for the LEW regression model for interval-censored data. In addition, various simulation studies developed in this work indicate that the distribution of a modified deviance residual presents high agreement with the standard normal distribution. The necessary matrices for application of these techniques were obtained by taking into account some usual perturbation in the model and data. By applying the procedures to a medical data set, we were able to assess the sensitivity aspects of the MLEs under the perturbation schemes as well as check the goodness-of-fit of the postulated model. Although the diagnostic plots detected some possible influential observations, their deletion did not cause substantial changes in the results.

Acknowledgements

This work was partially supported by grants from CNPq and was developed when the first author was a master student at the Departamento de Ciências Exatas, Escola Superior de Agricultura "Luiz de Queiroz", Universidade de São Paulo, Piracicaba. The authors are grateful to two anonymous referees and the Editor for very useful comments and suggestions.

Appendix A. Matrix of second derivatives $\ddot{\mathbf{L}}(\boldsymbol{\theta})$

Here we derive the necessary formulas to obtain the second-order partial derivatives of the log-likelihood function. After some algebraic manipulations, we obtain

$$\begin{aligned}\mathbf{L}_{\lambda\lambda} &= \sum_{i\in F}\left\{\frac{(b_i)_\lambda \log(1-b_i) - (a_i)_\lambda \log(1-a_i)}{(1-b_i)^\lambda - (1-a_i)^\lambda} - \left[\frac{(b_i)_\lambda - (a_i)_\lambda}{(1-b_i)^\lambda - (1-a_i)^\lambda}\right]^2\right\} \\ &\quad - \sum_{i\in C}\left\{\frac{(a_i)_\lambda \log(1-a_i)}{1-(1-a_i)^\lambda} + \left[\frac{(a_i)_\lambda}{1-(1-a_i)^\lambda}\right]^2\right\},\end{aligned}$$

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$$\begin{aligned}
\mathbf{L}_{\lambda\sigma} &= \sum_{i\in F}\left\{\frac{\sigma^{-1}\left(zu_i g_i p_i - zv_i h_i q_i\right)}{(1-b_i)^{\lambda}-(1-a_i)^{\lambda}} - \left[\frac{\left[\lambda\sigma^{-1}((a_i)_{\lambda}-(b_i)_{\lambda})(zu_i g_i - zv_i h_i)\right]}{[(1-b_i)^{\lambda}-(1-a_i)^{\lambda}]^2}\right]\right\}\\
&\quad + \sum_{i\in C}\left\{\frac{\sigma^{-1}zu_i g_i}{1-(1-a_i)^{\lambda}}\left[p_i + \frac{\lambda(1-a_i)^{\lambda-1}\log(1-a_i)}{1-(1-a_i)_{\lambda}}\right]\right\},\\
\mathbf{L}_{\lambda\beta_j} &= \sum_{i\in F}\left\{\frac{\sigma^{-1}x_{ij}\left[g_i p_i - h_i q_i\right]}{(1-b_i)^{\lambda}-(1-a_i)^{\lambda}} - \left[\frac{\lambda\sigma^{-1}x_{ij}((a_i)_{\lambda}-(b_i)_{\lambda})(g_i-h_i)}{[(1-b_i)^{\lambda}-(1-a_i)^{\lambda}]^2}\right]\right\}\\
&\quad + \sum_{i\in C}\left\{\frac{\sigma^{-1}x_{ij}g_i}{1-(1-a_i)^{\lambda}}\left[p_i + \frac{\lambda(1-a_i)^{\lambda-1}\log(1-a_i)}{1-(1-a_i)^{\lambda}}\right]\right\},\\
\mathbf{L}_{\sigma\sigma} &= \sum_{i\in F}\left\{\frac{\lambda\sigma^{-2}\left[zv_i h_i (b_i)_{\sigma} - zu_i g_i (a_i)_{\sigma}\right]}{(1-b_i)^{\lambda}-(1-a_i)^{\lambda}} - \left[\frac{\lambda\sigma^{-1}(zu_i g_i - zv_i h_i)}{(1-b_i)^{\lambda}-(1-a_i)^{\lambda}}\right]^2\right\}\\
&\quad - \sum_{i\in C}\left\{\frac{\lambda\sigma^{-2}zu_i g_i}{1-(1-a_i)^{\lambda}}\left[(a_i)_{\sigma} + \frac{\lambda zu_i g_i}{1-(1-a_i)^{\lambda}}\right]\right\},\\
\mathbf{L}_{\sigma\beta_j} &= \sum_{i\in F}\left\{\frac{\lambda\sigma^{-2}x_{ij}\left[h_i(b_i)_{\sigma\beta} - g_i(a_i)_{\sigma\beta}\right]}{(1-b_i)^{\lambda}-(1-a_i)^{\lambda}} - \left[\frac{\lambda^2\sigma^{-2}x_{ij}(zu_i g_i - zv_i h_i)(g_i-h_i)}{[(1-b_i)^{\lambda}-(1-a_i)^{\lambda}]^2}\right]\right\}\\
&\quad - \sum_{i\in C}\left\{\frac{\lambda\sigma^{-2}x_{ij}g_i}{1-(1-a_i)^{\lambda}}\left[(a_i)_{\sigma\beta} + \frac{\lambda zu_i g_i}{1-(1-a_i)^{\lambda}}\right]\right\}\\
\mathbf{L}_{\beta_j\beta_s} &= \sum_{i\in F}\left\{\frac{\lambda\sigma^{-2}x_{ij}x_{is}\left[h_i(b_i)_{\beta} - g_i(a_i)_{\beta}\right]}{(1-b_i)^{\lambda}-(1-a_i)^{\lambda}} - \left[x_{ij}x_{is}\left(\frac{\lambda\sigma^{-1}(g_i-h_i)}{(1-b_i)^{\lambda}-(1-a_i)^{\lambda}}\right)^2\right]\right\}\\
&\quad - \sum_{i\in C}\left\{\frac{\lambda\sigma^{-2}x_{ij}x_{is}g_i}{1-(1-a_i)^{\lambda}}\left[(a_i)_{\beta} + \frac{\lambda g_i}{1-(1-a_i)^{\lambda}}\right]\right\}.
\end{aligned}$$

Note that

$$\begin{aligned}
&zu_i = [\log(u_i) - \mathbf{x}_i^T\,\boldsymbol{\beta}]/\sigma, \qquad zv_i = [\log(v_i) - \mathbf{x}_i^T\,\boldsymbol{\beta}]/\sigma, \qquad a_i = \exp[-\exp(zu_i)],\\
&b_i = \exp[-\exp(zv_i)], \qquad (a_i)_{\lambda} = (1-a_i)^{\lambda}\log(1-a_i), \qquad (b_i)_{\lambda} = (1-b_i)^{\lambda}\log(1-b_i),\\
&g_i = (1-a_i)^{\lambda-1}a_i\exp(zu_i), \qquad h_i = (1-b_i)^{\lambda-1}b_i\exp(zv_i), \qquad p_i = \lambda\log(1-a_i)+1,\\
&q_i = \lambda\log(1-b_i)+1, \qquad (a_i)_{\sigma} = 2 + (1-a_i)^{-1}zu_i a_i\exp(zu_i)[\lambda-1] + zu_i[1-\exp(zu_i)],\\
&(b_i)_{\sigma} = 2 + (1-b_i)^{-1}zv_i b_i\exp(zv_i)[\lambda-1] + zv_i[1-\exp(zv_i)],\\
&(a_i)_{\sigma\beta} = 1 + (1-a_i)^{-1}zu_i a_i\exp(zu_i)[\lambda-1] + zu_i[1-\exp(zu_i)],\\
&(b_i)_{\sigma\beta} = 1 + (1-b_i)^{-1}zv_i b_i\exp(zv_i)[\lambda-1] + zv_i[1-\exp(zv_i)],\\
&(a_i)_{\beta} = 1 + (1-a_i)^{-1}a_i\exp(zu_i)[\lambda-1] - \exp(zu_i),\\
&(b_i)_{\beta} = 1 + (1-b_i)^{-1}b_i\exp(zv_i)[\lambda-1] - \exp(zv_i).
\end{aligned}$$

Appendix B. Proof of Theorem 1

We now derive an infinite sum representation for the kth ordinary moment of the distribution $f(z)$ given in (4), that is, $\mu'_k = E(Z^k)$. We have

$$\mu'_k = \int_{-\infty}^{\infty} z^k\lambda\left\{1-\exp\big[-\exp(z)\big]\right\}^{\lambda-1}\exp\big[z-\exp(z)\big]\mathrm{d}z. \tag{B.1}$$

We use the series representation

$$(1+x)^a = \sum_{j=0}^{\infty}\frac{\Gamma(a+1)x^j}{\Gamma(a-j+1)j!}. \tag{B.2}$$

Substituting $u = \exp(z)$ and using the representation (B.2), (B.1) can be further expanded as

$$\mu'_k = \lambda\Gamma(\lambda)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma(\lambda-j)j!}\int_0^{\infty}\big[\log(u)\big]^k\exp\big[-u(j+1)\big]\mathrm{d}u. \tag{B.3}$$

The integral in (B.3) follows from Prudnikov et al. (1986) and Nadarajah (2006); it can be calculated as

$$\int_0^{\infty}\left[\log(u)\right]^k \exp\left[-u(j+1)\right]\mathrm{d}u = \left.\frac{\partial^k[(j+1)^{-a}\Gamma(a)]}{\partial a^k}\right|_{a=1}. \tag{B.4}$$

Finally, inserting (B.4) in (B.3), the kth moment of Z can be expressed as

$$\mu_k' = \lambda\Gamma(\lambda)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma(\lambda-j)j!}\left.\frac{\partial^k[(j+1)^{-a}\Gamma(a)]}{\partial a^k}\right|_{a=1}. \tag{B.5}$$

Appendix C. The case-weight perturbation scheme

Here, we provide the elements necessary to consider the case-weight perturbation scheme. The elements of the matrix $\boldsymbol{\Delta} = (\boldsymbol{\Delta}_\lambda, \boldsymbol{\Delta}_\sigma, \boldsymbol{\Delta}_{\boldsymbol{\beta}})^T$ are expressed as

$$\Delta_i = \begin{cases} \dfrac{(\hat{b}_i)_\lambda - (\hat{a}_i)_\lambda}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} & \text{if } i \in F, \\[2ex] \dfrac{(\hat{a}_i)_\lambda}{1-(1-\hat{a}_i)^{\hat{\lambda}}} & \text{if } i \in C. \end{cases}$$

$$\Delta_i = \begin{cases} \dfrac{\hat{\lambda}\hat{\sigma}^{-1}(\hat{zu}_i\hat{g}_i - \hat{zv}_i\hat{h}_i)}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} & \text{if } i \in F, \\[2ex] \dfrac{\hat{\lambda}\hat{\sigma}^{-1}\hat{zu}_i\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}} & \text{if } i \in C. \end{cases}$$

For $j = 1, \ldots, p+2$.

$$\Delta_{ji} = \begin{cases} \dfrac{\hat{\lambda}\hat{\sigma}^{-1}x_{ij}(\hat{g}_i - \hat{h}_i)}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} & \text{if } i \in F, \\[2ex] \dfrac{\hat{\lambda}\hat{\sigma}^{-1}x_{ij}\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}} & \text{if } i \in C. \end{cases}$$

Appendix D. Response perturbation $(\log(u_i))$

Here we provide the elements $\boldsymbol{\Delta}_{ji}$ considering the response variable perturbation scheme. The elements of matrix $\boldsymbol{\Delta} = (\boldsymbol{\Delta}_\lambda, \boldsymbol{\Delta}_\sigma, \boldsymbol{\Delta}_{\boldsymbol{\beta}})^T$ are expressed as

$$\Delta_i = \begin{cases} \dfrac{-S_u\hat{\sigma}^{-1}\hat{g}_i\hat{p}_i}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} + \dfrac{[(\hat{b}_i)_\lambda - (\hat{a}_i)_\lambda](\hat{\lambda}\hat{\sigma}^{-1}S_u\hat{g}_i)}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\[2ex] \dfrac{-\hat{\sigma}^{-1}S_u\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\left[\hat{p}_i + \dfrac{\hat{\lambda}(\hat{a}_i)_\lambda}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\right] & \text{if } i \in C. \end{cases}$$

$$\Delta_i = \begin{cases} \dfrac{\hat{\lambda}\hat{\sigma}^{-2}S_u\hat{g}_i(\hat{a}_i)_{\sigma\beta}}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} + \dfrac{\hat{\lambda}^2\hat{\sigma}^{-2}S_u\hat{g}_i(\hat{zu}_i\hat{g}_i - \hat{zv}_i\hat{h}_i)}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\[2ex] \dfrac{\hat{\lambda}\hat{\sigma}^{-2}S_u\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\left[(\hat{a}_i)_{\sigma\beta} + \dfrac{\hat{\lambda}\hat{zu}_i\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\right] & \text{if } i \in C. \end{cases}$$

For $j = 1, \ldots, p+2$.

$$\Delta_{ji} = \begin{cases} \dfrac{\hat{\lambda}\hat{\sigma}^{-2}x_{ij}S_u\hat{g}_i(\hat{a}_i)_\beta}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} + \dfrac{\hat{\lambda}^2\hat{\sigma}^{-2}x_{ij}S_u\hat{g}_i(\hat{g}_i - \hat{h}_i)}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\[2ex] \dfrac{\hat{\lambda}\hat{\sigma}^{-2}x_{ij}S_u\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\left[(\hat{a}_i)_\beta + \dfrac{\hat{\lambda}\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\right] & \text{if } i \in C. \end{cases}$$

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Appendix E. Response perturbation ($\log(v_i)$)

Here we provide the elements $\mathbf{\Delta}_{ji}$ necessary to consider the response variable perturbation scheme. The elements of the matrix $\mathbf{\Delta} = (\mathbf{\Delta}_\lambda, \mathbf{\Delta}_\sigma, \mathbf{\Delta}_{\boldsymbol{\beta}})^T$ are expressed as

$$\Delta_i = \begin{cases} \dfrac{S_v\hat{\sigma}^{-1}\hat{h}_i\hat{q}_i}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} - \dfrac{\hat{\lambda}\hat{\sigma}^{-1}S_v\hat{h}_i[(\hat{b}_i)_\lambda - (\hat{a}_i)_\lambda]}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\ 0 & \text{if } i \in C. \end{cases}$$

$$\Delta_i = \begin{cases} \dfrac{-\hat{\lambda}\hat{\sigma}^{-2}S_v\hat{h}_i(\hat{b}_i)_{\sigma\beta}}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} - \dfrac{\hat{\lambda}^2\hat{\sigma}^{-2}S_v\hat{h}_i(z\hat{u}_i\hat{g}_i - z\hat{v}_i\hat{h}_i)}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\ 0 & \text{if } i \in C. \end{cases}$$

For $j = 1, \ldots, p+2$.

$$\Delta_{ji} = \begin{cases} \dfrac{-\hat{\lambda}\hat{\sigma}^{-2}x_{ij}S_v\hat{h}_i(\hat{b}_i)_\beta}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} - \dfrac{\hat{\lambda}^2\hat{\sigma}^{-2}x_{ij}S_v\hat{h}_i(\hat{g}_i - \hat{h}_i)}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\ 0 & \text{if } i \in C. \end{cases}$$

Appendix F. Response perturbation with simultaneous ($\log(u_i)$, $\log(v_i)$)

Here we provide the elements $\mathbf{\Delta}_{ji}$ necessary to consider the response variable perturbation scheme. The elements of the matrix $\mathbf{\Delta} = (\mathbf{\Delta}_\lambda, \mathbf{\Delta}_\sigma, \mathbf{\Delta}_{\boldsymbol{\beta}})^T$ are expressed as

$$\Delta_i = \begin{cases} \dfrac{\hat{\sigma}^{-1}(S_v\hat{h}_i\hat{q}_i - S_u\hat{g}_i\hat{p}_i)}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} - \dfrac{\hat{\lambda}\hat{\sigma}^{-1}[(\hat{b}_i)_\lambda - (\hat{a}_i)_\lambda](S_v\hat{h}_i - S_u\hat{g}_i)}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\ \dfrac{-\hat{\sigma}^{-1}S_u\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\left[\hat{p}_i + \dfrac{\hat{\lambda}(\hat{a}_i)_\lambda}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\right] & \text{if } i \in C. \end{cases}$$

$$\Delta_i = \begin{cases} \dfrac{\hat{\lambda}\hat{\sigma}^{-2}[S_u\hat{g}_i(\hat{a}_i)_{\sigma\beta} - S_v\hat{h}_i(\hat{b}_i)_{\sigma\beta}]}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} - \dfrac{\hat{\lambda}^2\hat{\sigma}^{-2}(z\hat{u}_i\hat{g}_i - z\hat{v}_i\hat{h}_i)(S_v\hat{h}_i - S_u\hat{g}_i)}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\ \dfrac{\hat{\lambda}\hat{\sigma}^{-2}S_u\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\left[(\hat{a}_i)_{\sigma\beta} + \dfrac{\hat{\lambda}z\hat{u}_i\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\right] & \text{if } i \in C. \end{cases}$$

For $j = 1, \ldots, p+2$.

$$\Delta_{ji} = \begin{cases} \dfrac{\hat{\lambda}\hat{\sigma}^{-2}x_{ij}[S_u\hat{g}_i(\hat{a}_i)_\beta - S_v\hat{h}_i(\hat{b}_i)_\beta]}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} - \dfrac{\hat{\lambda}^2\hat{\sigma}^{-2}x_{ij}(\hat{g}_i - \hat{h}_i)(S_v\hat{h}_i - S_u\hat{g}_i)}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\ \dfrac{\hat{\lambda}\hat{\sigma}^{-2}x_{ij}S_u\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\left[(\hat{a}_i)_\beta + \dfrac{\hat{\lambda}\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\right] & \text{if } i \in C. \end{cases}$$

Appendix G. Explanatory variable perturbation

Here we provide the elements $\mathbf{\Delta}_{ji}$ necessary to consider the explanatory variable perturbation scheme. The elements of the matrix $\mathbf{\Delta} = (\mathbf{\Delta}_\lambda, \mathbf{\Delta}_\sigma, \mathbf{\Delta}_{\boldsymbol{\beta}})^T$ are expressed as

$$\Delta_i = \begin{cases} \dfrac{\hat{\sigma}^{-1}\hat{\beta}_tS_x[\hat{g}_i\hat{p}_i - \hat{h}_i\hat{q}_i]}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} - \dfrac{\hat{\lambda}\hat{\sigma}^{-1}\hat{\beta}_tS_x[(\hat{b}_i)_\lambda - (\hat{a}_i)_\lambda](\hat{g}_i - \hat{h}_i)}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\ \dfrac{\hat{\sigma}^{-1}\hat{\beta}_tS_x\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\left[\hat{p}_i + \dfrac{\hat{\lambda}(\hat{a}_i)_\lambda}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\right] & \text{if } i \in C. \end{cases}$$

$$\Delta_i = \begin{cases} \dfrac{\hat{\lambda}\hat{\sigma}^{-2}\hat{\beta}_tS_x[(\hat{h})_i(\hat{b}_i)_{\sigma\beta} - \hat{g}_i(\hat{a}_i)_{\sigma\beta}]}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} - \dfrac{\hat{\lambda}^2\hat{\sigma}^{-2}\hat{\beta}_tS_x(z\hat{u}_i\hat{g}_i - z\hat{v}_i\hat{h}_i)(\hat{g}_i - \hat{h}_i)}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\ \dfrac{-\hat{\lambda}\hat{\sigma}^{-2}\hat{\beta}_tS_x\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\left[(\hat{a}_i)_{\sigma\beta} - \dfrac{z\hat{u}_i\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\right] & \text{if } i \in C. \end{cases}$$

For $t \neq j$ and $j = 1, \ldots, p+2$.

$$\Delta_{ji} = \begin{cases} \dfrac{\hat{\lambda}\hat{\sigma}^{-2}x_{ij}S_x\hat{\beta}_t[\hat{h}_i(\hat{b}_i)_\beta - \hat{g}_i(\hat{a}_i)_\beta]}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} - \dfrac{\hat{\lambda}^2\hat{\sigma}^{-2}x_{ij}\hat{\beta}_t S_x(\hat{g}_i - \hat{h}_i)^2}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\ \dfrac{-\hat{\lambda}\hat{\sigma}^{-2}x_{ij}\hat{\beta}_t S_x \hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\left[(\hat{a}_i)_\beta + \dfrac{\hat{\lambda}\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\right] & \text{if } i \in C. \end{cases}$$

For $t = j$

$$\Delta_{ti} = \begin{cases} \dfrac{\hat{\lambda}\hat{\sigma}^{-1}S_x[\hat{h}_i(\hat{b}_i)_\omega - \hat{g}_i(\hat{a}_i)_\omega]}{(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}} - \dfrac{\hat{\lambda}^2\hat{\sigma}^{-2}x_{it}\hat{\beta}_t S_x(\hat{g}_i - \hat{h}_i)^2}{[(1-\hat{b}_i)^{\hat{\lambda}} - (1-\hat{a}_i)^{\hat{\lambda}}]^2} & \text{if } i \in F, \\ \dfrac{-\hat{\lambda}\hat{\sigma}^{-1}\hat{g}_i S_x}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\left[(\hat{a}_i)_\omega + \dfrac{\hat{\lambda}\hat{\sigma}^{-1}x_{it}\hat{\beta}_t\hat{g}_i}{1-(1-\hat{a}_i)^{\hat{\lambda}}}\right] & \text{if } i \in C, \end{cases}$$

where

$$(\hat{a}_i)_\omega = (1-\hat{a}_i)^{-1}\hat{\sigma}^{-1}x_{it}\exp(\hat{zu}_i)\hat{a}_i\hat{\beta}_t(\hat{\lambda}-1) + \hat{\sigma}^{-1}x_{it}\hat{\beta}_t[1-\exp(\hat{zu}_i)],$$
$$(\hat{b}_i)_\omega = (1-\hat{b}_i)^{-1}\hat{\sigma}^{-1}x_{it}\exp(\hat{zv}_i)\hat{b}_i\hat{\beta}_t(\hat{\lambda}-1) + \hat{\sigma}^{-1}x_{it}\hat{\beta}_t[1-\exp(\hat{zv}_i)].$$

Expressions, $\hat{zu}_i$, $\hat{zv}_i$, $\hat{a}_i$, $\hat{b}_i$, $(\hat{a}_i)_{\hat{\lambda}}$, $(\hat{b}_i)_\lambda$, $\hat{g}_i$, $\hat{h}_i$, $\hat{p}_i$, $\hat{q}_i$, $(\hat{a}_i)_\sigma$, $(\hat{b}_i)_\sigma$, $(\hat{a}_i)_{\sigma\beta}$, $(\hat{b}_i)_{\sigma\beta}$, $(\hat{a}_i)_\beta$, $(b_i)_\beta$ are defined in Appendix A.

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Please cite this article in press as: Hashimoto, E.M., et al., The log-exponentiated Weibull regression model for interval-censored data. Computational Statistics and Data Analysis (2009), doi:10.1016/j.csda.2009.10.014