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Demographic Modeling of the Geography of Migration and Population: A Multiregional Perspective

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This article focuses on the development and evolution of migration and population redistribution modeling within the spatial context of multiregional demography. It begins in 1965, when the state-of-the-art consisted largely of ideas and techniques imported from other disciplines (regression analysis, gravity models, Markov chains, and matrix cohort-survival population projection models) and then continues on to tell the story of multiregional demography, its evolution and emergence as a fully developed paradigm for studying the spatial dynamics of migration and population redistribution and, more recently, its approach for estimating the necessary migration input measures from inadequate data.

Introduction: a retrospective view

The literature on the topic of migration modeling has grown enormously over the past 40 years, and comprehensive reviews of the kind produced by Shaw (1975) and Clark (1982) are no longer feasible or appropriate. The field has split off into too many branches. The list of important publications has increased exponentially, well beyond the number that a single reviewer can tackle successfully. So instead I focus on a single strand of work in the field: the biography of an idea. *Multiregional demography* is the idea, and its development over the past some four decades is the biography. The organizing structure of this biography is my own involvement in its evolution and dissemination.

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Multiregional demography: a personal journey

I begin this article with a brief overview of the state-of-the-art in migration modeling and regional population projections in 1965, shortly after I had started to work on two reports for the California State Development Plan, as a member of the Center for Planning and Development Research at the University of California at Berkeley. I had never taken a course in demography, but I was at that time a postdoctoral student in operations research and had just completed an advanced course on stochastic processes, which included lectures on Markov Chains. I drew on those lectures in my efforts to introduce a *spatial* dimension to the demographer's nonspatial cohort-component population projection model—efforts which culminated in the publication in 1968 of my book: *Matrix Analysis of Interregional Population Growth and Distribution* (Rogers 1968).

Two years later, I moved to Northwestern University, and with the help of two superior doctoral students, Jacques Ledent and Frans Willekens, developed a formal demographic paradigm of what I called multiregional demography. Soon thereafter, in 1975, my second book, *Introduction to Multiregional Mathematical Demography*, was published, and Willekens and I moved to Austria to join the International Institute of Applied Systems Analysis (IIASA), an East–West think-tank housed in a Habsburg palace, located just outside of Vienna, in a little town called Laxenburg. Shortly after, we were joined by Ledent and another graduate student of mine, Luis Castro, as well as a multinational collection of scholars who joined us for varying periods of time at IIASA to contribute to the ongoing work on multiregional demography.

From IIASA I moved, in 1983, to the University of Colorado at Boulder where, with the help of my graduate students, I continued to carry out research on topics related to multiregional demography, focusing especially on various applications of the methodology.

Initial conditions: 1965

Four small streams of work characterized the state-of-the-art in the modeling of migration and population redistribution analysis in 1965. First, there were the various linear regression models, usually focused on net migration (Blanco 1964). Second, there was the gravity model (Carrothers 1956), which generally was estimated with a log-linear regression model (Lowry 1964). Third, there were simple Markov chain models (Tarver and Gurley 1965). Fourth, there were the matrix population models that projected a single regional population, expressed as a vector, forward through time by means of a matrix operator that multiplied the vector (Keyfitz 1964a, b). Over the past 40 years, the Markov chain models gradually disappeared, and the gravity models evolved into what now are known as spatial interaction models (Fotheringham and O'Kelley 1989). The uniregional matrix models evolved into multiregional formulations (Rogers 1968, 1975; Rees and Wilson 1977), and the regression models bred both macro- and microeconometric models, and also statistical survival models (Cox and Oakes 1984) that in sociology appear as ''event history'' models (Yamaguchi 1991) and in economics as ''limited dependent variable'' models (Greene 2000).

Early efforts

In the fall of 1964, I was asked to take on two research projects for California's State Development Plan: one on regional population projections for the state, the other on analysis of interregional migration within California. Thus began my four decades of work contributing to a branch of demography now called *multiregional demography*, which analyzes the *spatial dynamics* of a *system* of several *interdependent* populations linked by directional migration flows.

Before the emergence of the multiregional perspective, the standard procedure for analyzing and projecting multiple interactive populations focused on each of the regional populations *one at a time*, linking each population to the others by means of *net* (in minus out) migration. That form of linkage introduces a bias into the dynamics, because it confounds propensities to migrate with the relative distribution of population sizes. My particular interest was to generalize the standard paradigm of mathematical demography so that it would incorporate *directional* migration flows and deal with *the entire system of multiple interacting populations* simultaneously. My first efforts to do this appeared in December of 1965, in a pair of reports produced for the California State Development Plan, and in 1966 I published my first two articles on the subject: one in the journal *Demography* (Rogers 1966a), entitled ''The Multiregional Matrix Growth Operator and the Stable Interregional Age Structure,'' and another in the *Papers and Proceedings of the Regional Science Association*, entitled ''A Markovian Policy Model of Migration'' (Rogers 1966b).

Later, others joined me in my effort: notably, Phil Rees and Alan Wilson, geographers at the University of Leeds in England, who in their 1977 book *Spatial Population Analysis*, adopted a detailed accounting framework as their central paradigm for projecting multiregional populations (Rees and Wilson 1977).

Modeling the age and spatial dynamics of multiregional populations

The mathematical description of human populations, particularly their structure with regard to age and sex, and the components of change, such as births and deaths, which occur over time to alter that structure, lies in the domain of formal demography. In this branch of demography, analysts have focused their attention on population *stocks* and on population *events*. Formal *multiregional* demography extends that focus to include the *flows* that interconnect and weld several regional populations into a multiregional population system. The trifold focus of such descriptions is on the *stocks* of human population groups at different points in time and locations in space, the vital *events* that occur among these populations, and the *flows* of members of such populations across the spatial borders that delineate the constituent regions of the multiregional population system.

Multiregional mathematical demography

Two principal features distinguish the multi- from the uniregional perspective: the population being examined and the definition of rates of flow. The multiregional approach considers a national population as an interacting system of regional subpopulations; the uniregional approach examines each regional subpopulation one at a time. Moreover, the multiregional approach employs rates of flow that refer to the appropriate at-risk populations; the uniregional approach cannot do that because it considers only a single population at risk for both out- and immigration.

Two classes of models are commonly used to examine how the growth and structure of a national multiregional population evolves from particular regimes of fertility, mortality, and migration: the life table model and the projection model. Both allow one to separate out the impacts, on population growth and structure, of the demographic processes prevailing at a particular moment, and of the age composition and spatial distribution of the national multiregional population at that moment.

Multiregional life tables

The life table, a central concept in classical uniregional demography, expresses the facts of mortality in terms of survival probabilities and their combined impact on the lives of a cohort of people born at the same moment. In 1973 I published a multiregional generalization of the conventional uniregional life table (Rogers 1973a). This generalization posited a life table with multiple radices, one for each of the regional populations in the system and followed each birth cohort as it redistributed itself spatially and eventually left the system through death. Estimates of the required input probabilities normally are developed from observed data on rates and/or conditional proportions surviving (Pressat 1972; Rogers and Ledent 1976; Rogers and Willekens 1986, Ch. 9). And to ensure consistency, the data may be adjusted by basic accounting identities that are embedded in a set of demographic accounts (Rees 1979, 1986a, b).

Along with others, I have focused on variations in estimation procedures brought about by differences in how migration is observed and measured (Courgeau 1973; Rogers 1975; Ledent and Rees 1986). Counts of *moves* call for different estimation procedures than do counts of *movers* (Ledent 1980); therefore, migration data obtained from population registers require a different method for estimating transition probabilities than do migration data obtained from national censuses. (For a comparison of the results obtained using each of these two types of migration data, see Kupiszewski 1988).

Multiregional population projections

Population projections are numerical estimates of future demographic totals and are often based on rates that are extrapolations of past and current trends. The mechanics of multiregional population projection typically revolve around three basic steps. The first ascertains the starting age-region distributions and the age-specific regional schedules of fertility, mortality, and migration to which the multiregional population has been subject during a past period; the second adopts a set of assumptions regarding the future behavior of such schedules; and the third derives the consequences of applying these assumed schedules to the initial population stock.

A useful discrete model of multiregional demographic growth expresses the population projection process by means of a matrix operation in which a multiregional population, set out as a vector, is multiplied by a growth matrix that survives that population forward over time (Rogers 1973b). Such a projection calculates the regional and age-specific survivors of a multiregional population of a given sex and adds to this total the new births that survive to the end of the unit time interval. As in the uniregional model, the survival of individuals from one moment in time to another, say 5 years later, is calculated by diminishing each regional population to take into account the decrement due to mortality. In the multiregional model, however, we also need to include the decrement resulting from outmigration and the increment contributed by inmigration. In models ''open'' to international migration, the decrement from emigration and the increment from immigration also need to be incorporated. Surviving children born during the 5-year interval, migrate with their parents or are born after their parents have migrated but before the time interval has elapsed.

A multinational comparative study of migration and settlement patterns

In 1975, I left Northwestern University to begin a sabbatical that ended up being an 8-year stay at IIASA, where I set out to disseminate the new paradigm defined in my 1975 book. The most important vehicle for conveying this was a multinational comparative study that combined the efforts of some 40 scholars from IIASA's member nations.

IIASA's study of migration and settlement began with two basic components: a set of computer programs for multiregional demographic analysis and a network of collaborating investigators from nations of the Institute's then 17 member organizations. The principal goal was a case study of each country to be conducted by a scholar from that country. Each case study was to use a common methodology and to follow a common outline of substantive topics. Much of the data analysis was to be carried out at IIASA using a standard package of computer programs, and most of the scholars involved had to be trained in the methodology by those at IIASA familiar with the mathematical theory. The success of this training led the Institute to offer short courses on multiregional demography in Austria, Mexico, and Bulgaria.

The Migration and Settlement Study was concluded in 1982, 7 years after its initiation. An important outcome of the study was the set of 17 country reports authored by 27 scholars. Each report presents a national overview of recent regional patterns of fertility, mortality, and internal migration, illustrates the application of multiregional demographic techniques and the additional insights into population redistribution that can be gained from it, and concludes with a very brief review of population distribution issues and policies.

The theoretical work of the Migration and Settlement Study has received wide dissemination. It served as the focus of two sessions on mathematical demography at the annual meetings of the Population Association of America (in 1979 and in 1981). Two special issues of the journal *Environment and Planning A* were devoted to an exposition of its principal findings (the May 1978 and the May 1980 issues), and a substantial part of a National Science Foundation (NSF)-sponsored conference on multidimensional mathematical demography focused on its theoretical advances (Land and Rogers 1982).

Collaborators recruited in each of the 17 member nations assembled the necessary demographic data for their country reports, sent them to us at IIASA for data processing and then, with our help, authored their national case study reports. Their 17 national multiregional demographic analyses and data appendices were published and assembled, along with reprints of our journal articles and the manual for the software used by everyone (Willekens and Rogers 1978), into the three-volume boxed set (Willekens 1982a, b). This collection of 23 reports was soon followed by a book that summarized the principal results of the study for a wider audience (Rogers and Willekens 1986).

Most of what was accomplished and learned in the course of the Migration and Settlement Study was methodological and descriptive in character. Because the study's principal aim was to disseminate and expand a methodological tool, this is not surprising. The dissemination led to the accumulation of a large data bank, assembled by those adopting the new tool, and this in turn led to a comparative analysis of the data (Willekens 1986). New problems and ideas arising out of the implementation of the tool in different national settings led to the development of new theoretical findings and methods, most of which contribute to three principal themes: spatial population dynamics, migration measurement and analysis, and finally, multistate demography.

The comparative study encountered a number of issues and problems related to the available data on migration. Irregularities in the age patterns of the observed migration rate schedules led to the development of graduation models, called model migration schedules, which were used to smooth out the irregularities (Rogers, Raquillet, and Castro 1978). Missing data led to the development of statistical methods for estimating their probable values (Willekens, Por, and Raquillet 1981). National differences in data collection procedures (e.g., registration versus census enumeration) led to the development of methods for dealing both with the counts of migrations and the counts of migrants (Ledent 1980; Rees and Willekens 1986).

Finally, we recognized early on that most demographic processes can be viewed as transitions that are experienced during a life course. Individuals are born, age with the passage of time, enroll in school, enter the labor force, get married, reproduce, move from one region to another, retire, and ultimately die. Moreover, the arithmetic for tracing people's transitions from one region of residence to another (*multiregional* demography) is the same as that for handling their transitions into and out of various status categories, such as marriage and employment, for example (*multistate* demography). Transitions are at the core of all such applications, and the tools of formal demography may be applied to determine what happens if these transitions are chained over successive periods of age and time (Keyfitz 1980; Land and Rogers 1982). Multistate life tables were used during this period (1975–1982) to illuminate the study of, for example: marital status changes (Schoen and Nelson 1974; Willekens, Shah, and Ramachandran 1982), labor force behavior (Hoem and Fong 1976; Willekens 1980b), and pensions and annuities (Keyfitz and Rogers 1982).

Applications and extensions

The wide distribution of the results of IIASA's multinational comparative study of migration and settlement created a growing network of scholars interested in the topic. Linked by a quarterly newsletter, POPNET, scholars contributed further insights into national patterns, data problem resolutions, and extensions of the methodology.

In 1983, I left IIASA to assume the directorship of the Population Program at the University of Colorado at Boulder, where I continued my work in multiregional demography. Others joined me and several of my graduate students in our common efforts to advance the state-of-the-art.

Elderly migration and settlement

Locally, we began by emulating the IIASA migration and settlement comparative study, this time focusing on *elderly* migration and settlement patterns (Rogers and Watkins 1987). Elderly migration and settlement patterns may be characterized by two attributes. First, the elderly constitute a relatively small segment of the total population and, second, they comprise an even smaller portion of the total migrants. However, as their fraction of the total population has increased over time, and as more and more older persons have the desire, resources, and health to move, elderly migration has become a topic of growing interest to scholars and policymakers alike. Much of the recent attention in the United States has been focused on the analysis of various kinds of moves, their destination patterns, the socioeconomic characteristics of the movers, and on the consequences of such movements for the origin and destination regions.

In 1984, the National Institute of Aging (NIA) awarded us a grant to conduct a project that focused on elderly migration (with the late Professor William Serow of Florida State University as my co-Principal Investigator). A multinational collaborative network was formed, and several conferences in Colorado were convened (again with the financial support of the NIA) to bring the collaborating scholars together. One of the conferences led to a commercially published book on elderly migration (Rogers 1992), which included important chapters from prominent scholars, such as Anthony Warnes and Charles Longino.

Population forecasting: a parameterized time series approach

Despite the importance of forecasts in demography, systematic development of forecasting methodology has not been a major preoccupation of demographers, and forecasters have generally relied on simple extrapolative methods and demographic accounting procedures.

> Ask anyone outside the profession what he thinks demographers do and he will give a much bigger place to forecasting than the editors of our journals give it space on their pages. Our most distinguished demographers do not put their major efforts into forecasting . . . we assign the highest professional standing to those who derive relations among variables in application to past data, in short who can most convincingly explain the past (Keyfitz 1985, p. 60).

The practical importance of population forecasts call for basic research and innovation in the methods of demographic forecasting, a strategy that involves the integration of two—and ultimately three—traditionally independent approaches to demographic forecasting: time series methods, demographic accounting, and explanatory models.

In 1984, the NSF awarded us the first of two consecutive grants to apply multiregional demographic models in the production of multiregional population projections and forecasts. This project led to a series of articles on the use of parameterized model schedules (Rogers 1986) and modern time series methods in generating such projections and forecasts (McNown and Rogers 1989; McNown, Rogers, and Little 1995).

Our approach to forecasting combined parameterized model schedules and time series methods in generating forecasts of mortality and fertility. It demonstrated the feasibility and accuracy of the forecasting methodology and indicated how one could extend this approach to forecasts of interregional migration were adequate time series data available. A major component of this descriptive analysis was the fitting of parameterized model schedules to historical data, permitting a relatively concise representation of such data by age and sex, and enabling one to make structural comparisons at different points in time.

The estimation of the parameterized schedules for each year yields a set of observations on each parameter over time. Time series methods can then be applied to the data on each parameter to develop forecasting models of sex-specific schedules and hence of probabilities by age and sex. Improvements in accuracy arise from the formal acknowledgment in the forecasting model of the pervasive age-specific regularities in such patterns.

A special issue of the international journal *Mathematical Population Studies* (Rogers 1995b) was devoted entirely to a series of papers on the topic written by Ronald Lee, Warren Sanderson, and others. Additional research efforts by Wilson and Rees (2005) and Booth (2006) have helped build a solid foundation for future efforts.

Active life expectancies: a multistate perspective

A number of studies of longevity and health among the elderly published in the 1980s compared changes in total life expectancy with corresponding changes in disability-free life expectancy and concluded that the positive trends in the prolongation of life have not been matched by similar trends in the extension of healthy life (see Wilkins and Adams 1983; Bebbington 1988; Crimmins, Saito, and Ingegneri 1989; McKinlay, McKinlay, and Beaglehole 1989). Typical of their assessments was the study of Crimmins, Saito, and Ingegneri (1989), who, after examining data from the National Health Interview Survey for the United States, concluded that Americans were not living longer healthy lives. They argued that additions to life expectancy between 1970 and 1980 were concentrated in the disabled years—primarily years of long-term disability.

We questioned such findings, and in 1988 obtained grant support from the NIA to support our exploratory effort to apply the arithmetic of multiregional demography to the subject of disability dynamics and patterns among the elderly, examining fundamental conceptual issues related to the measurement and modeling of the mortality–morbidity process, and arguing that the models used to measure the health of a population over time have been defined in a manner that introduces bias in favor of a pessimistic conclusion. Our argument put forward two principal points. First, the life tables used to measure changes in health status were inappropriately specified and estimated. Second, the almost exclusive focus on prospects for delaying the onset of disability or dependency (referred to as ''compression of morbidity'') diverted attention from another form of transition that also can extend disability-free life expectancies—namely, recovery. Our project showed that the widespread use of *prevalence* measures of disability often guaranteed a pessimistic finding, one that was reversed with the use of the *incidence* measures used by multiregional (multistate) demographic models (Rogers, Rogers, and Belanger 1990). Since then, a number of studies have adopted a similar perspective (e.g., Crimmins, Hayward, and Saito 1994).

Immigration and the foreign-born population

Imagine the geography of the foreign-born population in the United States at the middle of the 20th century, and consider its changes since then. How did the demographic processes of immigration, emigration, internal migration, and mortality shape the changing geography? How did the fertility patterns of native and foreign-borns combine with the migration and mortality patterns of the latter to shape the geography of the native-born population and the consequent foreign-born shares of regional populations? How have the internal migration patterns of the foreign-borns differed from those of the native-born population? Assisted by several graduate and undergraduate students and a grant in 1986 from the National Institute for Child Health and Human Development, James Raymer and I embarked on a 4-year effort to apply multiregional demographic models to track and project the spatial dynamics of the native and foreign-born regional populations in the United States

(Rogers, Little, and Raymer 1999; Rogers and Raymer 2001). This effort required an extension of the basic multiregional model to include origin-dependence in the model's survivorship probabilities and region-specific *international* migration flows, features missing in my 1975 book, but detailed in my subsequent 1995 book (Rogers 1995a).

Although the principal focus of the multiyear study of the foreign-born population was interregional migration and spatial redistribution in the United States, the absence of adequate data on territorial mobility—for example, on emigration—necessitated some of the estimates of some migration streams to be made indirectly. In the process, new methodological approaches had to be explored and tested for the first time, drawing on the emerging literature on the statistical analysis of data with missing values. This led us to seek grant support for efforts to develop improved methods for estimating migration, our next phase of research in multiregional demography.

Modeling and estimating the age and spatial structures of migration flows

The estimation of migration from aggregate and incomplete data generally has been carried out with a focus on *net* migration and approximated by the population change that cannot be attributed to natural increase. Such methods are reviewed in, for example, United Nations (1967) and Bogue, Hinze, and White (1982).

Methods for inferring *gross* (directional) migration streams have a much more limited history (Rogers 1975; Rogers and Willekens 1986). Recently, indirect estimations of migration have relied on the use of models and on the theory of statistical inference to infer the parameters from available data. Some models describe age patterns of migration, while others describe spatial interaction patterns. Both categories of models are considered in a special issue of *Mathematical Population Studies* (Rogers 1999).

Model migration schedules and spatial interaction models

A number of studies of regularities in age patterns of migration, over the past quarter of a century (e.g., Rogers and Castro 1981, Rogers and Watkins 1987, Rogers and Little 1994) have discovered that the mathematical expression called the *multiexponential function* provides a remarkably good fit to a wide variety of empirical interregional migration schedules. That goodness-of-fit has led a number of demographers and geographers to adopt it in various studies of migration all over the world (Bates and Bracken 1982, 1987; Liaw and Nagnur 1985; Hofmeyr 1988; Potrykowska 1988; Kawabe 1990; United Nations 1992; George 1994; Pimienta 1999). More recently, Congdon (2005) has applied a Bayesian perspective to the modeling of model migration schedules, and proposes two alternatives:

> The baseline model is in Rogers and Castro (1981). The first alternative retains the Rogers-Castro parametric approach model schedule but with random

> effect options . . . The second involves a completely nonparametric random effects approach
>
> (Congdon 2005, p. 1).

He concludes that models which introduce random effects into a fully parametric multiexponential model are competitive with purely nonparametric approaches.

The problem of estimating gravity models, or, more accurately, spatial interaction models, has been approached from different perspectives over the past four decades. First formulated in analogy to Newton's law of gravitation (Stewart 1948; Olsson 1965), the resulting purely mechanical approach was revised by Alan Wilson some 30 years ago in terms of entropy maximization theory (Wilson 1970). This was followed by a behavioral microtheoretical approach called random utility modeling (McFadden 1978). And some 25 years ago geographers recognized that models developed in the field of discrete multivariate analysis could fruitfully be applied to express spatial interaction patterns. Foremost among these models has been the log-linear model (Willekens 1980a, b, 1982a, b, 1983).

The indirect estimation of migration

Demographic estimation typically is based on data collected by censuses and vital registration systems. In countries with inadequate or inaccurate data reporting systems, analysts often must rely on methods that are ''indirect.'' The estimation of the probability of dying before age 2 by using the proportion of children dead, among those borne by women 20–24 years of age, is an example of indirect estimation. Such estimation techniques often rely on model schedules, collections of age-specific rates that are based on patterns observed in various populations other than the one being studied, selecting one of them on the basis of some data describing the observed population.

Fertility and mortality processes involve single populations. Migration links two populations: the population of the origin region and that of the destination region. This greatly complicates its estimation by indirect methods, because what this means in practical terms is that a focus on *age patterns* (e.g., with model migration schedules) is not enough—one also must focus on *spatial patterns* (e.g., with log-linear models). The imposition of observed regularities in both the age and spatial patterns of interregional migration on inadequate data on territorial mobility holds great promise as a means for developing detailed age- and destination-specific migration flow data from inaccurate, partial, and even nonexistent information on this most fundamental process underlying population redistribution.

Over the past several years, a number of us have been developing a formal model-based approach to the estimation of migration, when the needed data are inadequate, inaccurate, or incomplete (Rogers, Willekens, and Raymer 2003; Raymer and Rogers 2007). Our formal approach indicates that rough estimates of interregional age-specific migration streams can be developed by indirect estimation methods applied to two age-region-specific population counts, disaggregated by region of births, and some auxiliary information obtained from historical data. For example, especially robust estimates have been obtained using infant migration data of a current period and regression relationships prevailing during an earlier period (Rogers and Jordan 2004). Because children who have been born in region *i*, and who are, say, 0–4 years old at the time of the census and living in region *j*, must have migrated during the immediately preceding 5-year interval, we can obtain a ''proxy'' infant migration rate by ''backcasting'' them to their region of birth and then calculating their prospective propensity to migrate. Given their young age, and the fact that they were on average born 2.5 years ago, it is unlikely that they experienced more than one migration. Regression equations and model migration schedules can be used to expand these child-migration levels and spatial patterns into the corresponding levels and patterns for every age.

Much of the research on the indirect estimation of migration has taken on added significance, with the U.S. Census Bureau's transition from the so-called ''long form'' questionnaire used in earlier censuses, to the new American Community Survey, a smaller and ongoing survey of migration. Raymer and Rogers (2007) illustrate the problem and offer a possible solution.

Conclusion: a prospective view

An indicator of the usefulness of a new idea is the degree to which it becomes widely disseminated and applied. Assessed in such terms, the models and computer programs of the Migration and Settlement Study can be said to have attained some measure of success. Expositional articles that deal with applications of multiregional/multistate demography and use IIASA's computer programs continue to appear in different languages in various international scholarly journals. Governmental agencies, such as the Quebec Bureau of Statistics (1981), Statistics Canada (George 1994), and the U.S. Bureau of Labor Statistics (Smith 1982), have adopted this work, and the *International Encyclopedia of Population* (Ross 1982) refers to it as a fundamental new departure in life table application.

After four decades of development and publications on the subject, multiregional demography has become an established subfield within demography. For example, the text by Halli and Rao (1992), entitled *Advanced Techniques of Population Analysis*, devotes an entire chapter to multiregional demographic models; a British text on spatial demography also identifies multiregional demography as an important branch of demography (Rees 1986a, p. 124); a review of a monograph by Jozwiak (1992) on mathematical models of population concludes:

> . . . multiregional models of population reproduction in continuous and discrete time are described . . . Much of the material is now firmly established in the literature . . .
>
> (Pollard 1993, p. 369).

Further evidence of that establishment in the literature is the illustration of a *multiregional* Lexis diagram (borrowed from one of my multiregional demography texts: Rogers 1975) on the cover of the professional journal of U.S. demographers—*Demography*—during the entire year of 1996.

The fundamental ideas of multiregional/multistate demography have become widespread and are receiving continued attention from scholars in a number of countries. In conclusion, I will now touch on three important topics that have not been covered in this already too long review, and that no doubt will receive increased attention in the future: (1) the linking of demographic and economic variables in *demoeconomic* models, (2) the marriage of the multiregional/multistate demographic *macromodels* of mathematical demography with the individual biographic micromodels of statistical demography, and (3) the further development of multiregional/multistate *probabilistic* population projection models. Studies such as these suggest that the active period of methodological development in multiregional demography of the past decades will continue their evolution.

Demoeconomic models

In the introduction to the book: *Population Change and the Economy*, the editor, Andrew Isserman, reminded us that population change both affects and is affected by economic conditions and that, therefore, the variables of both should be linked to form demoeconomic models (Isserman 1986, p. xiii). In their contribution to the edited volume, Rogers and Williams (1986, Ch. 8) addressed this topic and set out a framework for a multistate demoeconomic projection model that links a demographic model with an economic model. Other chapters offered alternative frameworks, including a team effort by five scholars led by Paul Beaumont presenting the details of a particularly large interregional demoeconomic model of the United States called ECESIS (Ch. 9) that has

> 7,400 endogenous variables and 884 exogenous variables. In addition, 25,500 state-to-state migration flows are modeled . . . .ECESIS was designed so that further demographic and economic disaggregation is feasible
>
> (Beaumont et al. 1986, pp. 203–4).

Large-scale models, such as the ECSIS model are no longer in vogue. They have been criticized for being overly complicated, opaque, atheoretical, multipurposeful, structurally inflexible, beyond validation, hypercomprehensive, data hungry, mechanical, expensive, and often exceeding the modeler's span of control (Lee 1973; Arthur and McNicoll, 1975). Several authors have pointed particularly to the lack of needed theory underlying the models, arguing that the amount of theory is nowhere near sufficient to support such efforts, thereby limiting the uses to which the models can be put. Perhaps Alonso (1968, p. 252) put it best when he observed:

> I am questioning whether we have arrived at the design of skyscrapers but we have only lumber for construction material. If we do, we had better build low to the ground while we improve upon our materials.

None of the above authors, however, urged the abandonment of demoeconomic modeling activities; rather, each argued for ''simpler'' models. And so although their further development has slowed, the broader topic continues to merit further research efforts, albeit directed more at smaller models.

Micro–macro models: statistical and mathematical demography combined

In the field of economics, a division is generally made between the areas of *mathematical economics* and *econometrics*. The former deals principally with abstract mathematical descriptions of dynamics and growth; the latter treats statistically estimated relationships between basic variables. In a similar vein, mathematical demography may be distinguished from the statistical branch of demography that, couched in the perspective of causal analysis, emphasizes the effects of population heterogeneity on rates with stochastic process models. It focuses, for example, on the impacts of observed and unobserved heterogeneity, the effects of duration in a state on rates of exit from the state, the reasonableness of assumptions population homogeneity over time, and the influence of previous experiences on current and prospective patterns of behavior.

Recent research has brought the mathematical tradition much closer to the statistical/causal one, and a successful marriage between the two perspectives is in its early phases, with statistical microdemographic models increasingly devoted to the formal causal analysis of the behavior of decision-making units, such as the individual or the family, and mathematical macrodemography continuing to examine the behavior of aggregates, for example, the relationships between various population subgroups and different measures of regional economic performance and well-being. An important consequence of such a merger is a further development of micro and macro branches of formal demography.

Willekens et al. (2005) illustrates such a marriage of the two multistate perspectives. As director of the Mic-Mac project, funded by the European Commission, he had been leading an effort to bridge the divide with a modeling methodology that complements the usual cohort-based demographic projections with projections of the ways people live their lives.

> This paper extends the multistate life table and generates biographies of individual cohort members . . . The individual biographies contain useful information not provided by the cohort biographies . . . the results provide insights in the diversity of individual life courses and the path dependence of the occurrence and timing of demographic events
>
> (Willekens et al. 2005, Abstract).

Probabilistic population projections

Many official forecasted multiregional populations are the projected consequences of particular combinations of expected low, medium, and high ''variants'' of fertility, mortality, and migration. Usually, the selection of a ''best'' or ''most likely'' projection is part of the output. But this is an imprecise formulation, and has led researchers such as Lutz, Sanderson, and Scherbov (2004), and Keilman, Pham, and Hetland (2002), to argue in favor of a formal recognition of the various uncertainties by simulating a large number of projected scenarios which arise from random drawings from the specified probability distributions of fertility, mortality, and migration, in addition to whatever other components of change are introduced in the analysis.

Probabilistic population projections translate uncertainties in the components of demographic growth and change into the corresponding probabilistic outputs that describe expected population stocks and their associated prediction intervals. Such descriptions generally are phrased as percentage distributions around a central number, leading to conclusions such as, for example, that the global population total in 2050 is 90% likely to lie between 9 and 10 billion people. The prediction intervals widen rapidly when extended into the future. For example, recent probabilistic projections for Norway state that:

> The odds are two against one that the Norwegian population, now 4.5 million, will number between 3.9 and 6 million in 2050. Compared to the median forecast of 4.8 million in 2050, this two-thirds prediction interval is 43 per cent wide. Odds of 19 to one (95 per cent probability) are attached to an interval between 3.2 and 7.2 million in 2050. This interval is twice as wide as the 67 per cent interval: 88 per cent, compared to the median forecast
>
> (Keilman, Pham, and Hetland 2002, p. 431).

A number of alternative frameworks have been proposed for generating probabilistic population forecasts. The earliest formulations often adopted time series frameworks (McNown, Rogers, and Little 1995), whereas more recent efforts have emphasized simulations based on random drawings from distributions of the fundamental components of change—distributions which, as in the Lutz, Sanderson, and Scherbov (2004) projections, were shaped by ''expert'' opinions. Booth (2006) offers an extensive review of probabilistic population projection models, as part of her authoritative overview of demographic forecasting models in general.

A final word: lessons learned

The above three categories of models suggest that further methodological developments in multiregional/multistate demography will continue to enrich this particular subfield of demography. Finally, I would like to conclude this article by mentioning just two of the ''lessons learned'' in the course of developing and promoting this particular methodology.

First, there is the surprisingly robust resistance to new ideas that one often encounters in proposing a different methodology for a particular application. For example, convincing academics and professionals that net migration rates and various forms of prevalence rates misspecify the underlying dynamics being modeled has been surprisingly difficult. My early efforts in this regard were not very successful. Indeed, my first paper on multiregional life tables and population projections (Rogers 1973a) was rejected by two journals. But I stubbornly persisted and ultimately succeeded in getting it published. The lesson here clearly is not to give up.

A second lesson concerns the usefulness of ''ransacking cognate fields for applicable ideas . . .''—a question raised by a second referee. And the answer, of course, is yes. Some of the instances that led to successful borrowings in my case, included the transfer of the basic ideas of the aggregation problem in the multisectoral economic input–output literature to multiregional demographic projection models (Rogers 1969); the heavy reliance on matrix formulations of transition processes—a reliance that ultimately led to the multiregional generalizations of the formulas of uniregional demography (Rogers 1995a); the impacts of unobserved heterogeneity (Heckman and Singer 1982), and the use of biproportional methods to ''update'' a matrix of flows—be they, for example, interzonal traffic flows, intersectoral economic flows, or interregional flows of people (Bacharach 1970).

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