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From *Spatial Analysis* to Geospatial Science

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When Spatial Analysis *was published in 1968, it drew together the fruits of the first decade of geography's quantitative revolution. In the decades that have followed, quantitative geography has both diffused and concentrated, abandoned some themes, made major progress on others, and in the contemporary form of geospatial analysis has become an innovative multidisciplinary enterprise. In this article, we sketch the broad outlines of this history, lay out the main threads along which technical capabilities have developed, and describe what appear to us to be the leading questions at the research frontier. Even as many geographers disavow social science, geospatial science has emerged as a lusty arena marked by intellectual vigor, conceptual growth, and enhanced analytic abilities. What now is taking shape is a spatially integrated social–environmental science that is transcending older disciplinary attachments, boundaries, and constraints.*

Introduction

At the time that the forces of Marxism, deconstruction, and postmodernism have combined with romantic reattachment to exceptionalism to deflect geography from the cusp of scientific respectability, geospatial analysis has emerged as a vital new force supporting spatially integrated investigation that crosscuts the natural and human sciences. The roots of this force go back five decades. Berry and Marble's *Spatial Analysis* (1968) gathered key contributions that illustrate the themes that emerged in the first of these decades, the period of geography's quantitative revolution. Many of these contributions have proved to be prescient—the stirpes of contemporary geospatial research. It may thus be useful to examine their contemporary manifestations and then to ask what is being contributed today that may be a harbinger of the geospatial science that is yet to come.

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Prescient roots

Spatial Analysis began with the obligatory obeisance to history, set down in the editors' introduction and Burton's brief account of the quantitative revolution of the 1950s. This concern for history and for the legitimacy of scientific geography has continued. Burton's paper was followed by Curry (1967) who in turn was followed by Marchand (1974). After members of the discipline debated the merits and drawbacks of quantification for the next two decades, Getis (1995) renewed the discussion and Baker and Boots (2005) penned their 55-year retrospect.

Beyond legitimization, most other papers in *Spatial Analysis* were foundational in nature. The entries by Chorley and Lowry helped usher in new modes of thinking, a new concern for models in geography, highlighted by the contemporaneous publication of Chorley and Haggett's *Models in Geography* (1967) and followed by Haring and Lounsbury's (1971) *Introduction to Scientific Geographic Research*. The scientific approach promoted by these writings was bolstered by the applications furnished by Court, King, Sabbagh, and Bryson, Robinson et al., and Wong and Kendall. Meanwhile, Curry's focus on stochastic conceptualizations was a forerunner of thinking about superpopulations and contemporary model-based geographic inference. Although these papers have disappeared from most contemporary reference lists, their indirect effects thus live on.

In a similar vein, the cluster of papers by MacKay, Warntz, and Knos, while building on the older tradition of social physics, anticipated the revolutionary work on spatial interaction forged by Wilson (1970). Today, widely practiced gravity modeling of spatial interaction rests on a solid theoretical foundation. The recent Massey–Lane (Massey 1999; Lane 2001) exchange is indicative of continued interest in this type of underpinning for spatial analysis.

Closely related is the enormous amount of diffusion research motivated by Hägerstrand, which also stimulated the adoption of simulation experimentation as part of spatial analysis. Hägerstrand built the foundation for such masterpieces as Cliff et al.'s (1981) *Spatial Diffusion*, an instrumental predecessor of the present-day U.S. Centers for Disease Control's AIDS data animation project (Stephenson 1995) and its modeling of diffusion of the 2003 SARS epidemic (2004). The foundation will no doubt also play a pivotal role in analysis of the current avian flu crisis, as it is in continuing work on the spread of influenza (Viboud et al. 2006).

Among the variety of other beginnings, Nystuen's promotion of geographic concepts has exploded from his initial list. Choynowski's treatment of probability maps is a forerunner of auto-binomial modeling. Matui's and Dacey's papers pointed to modeling spatially correlated Poisson random variables. Berry's, Garrison's, and Nystuen and Daceys' papers laid the foundations for the eigenfunction analysis championed by Gould (1967) and Tinkler (1972), Tiefelsdorf and Boots (1995), and Griffith (2003). The entries by Chorley and Haggett, Harbaugh and Preston, and Haggett similarly foreshadowed the development of the eigenfunction-based spatial filtering model specification (Griffith 2003), whereas Berry and Baker's paper established the groundwork for the later spatial sampling designs developed by Stehman and Overton (1996) and refined by Griffith (2005). Griffith translated geographic context into the independent and identical distribution (i.i.d.) framework of classical statistics, which led him to the notion of effective sample size, or the equivalent i.i.d. size of a geographic sample once spatial autocorrelation effects have been taken into account. As geographic sample intensity increases, positive spatial autocorrelation in a sample tends to increase, resulting in increasingly redundant information and hence a continual decrease in the amount of new information being acquired with each new sampled location. This insight has enabled not only kriging but also spatial autoregressive and spatial filtering models to be related to imputation via the estimation-maximization (EM) algorithm of statistics, enabling the redundant information contained in spatial autocorrelation to be exploitable to produce estimates for locations for which values are missing (Griffith and Layne 1999).

Spatial Analysis also included cautionary notes. The papers by Thomas and Anderson and by Goodman on the use of aggregate geographic data anticipated the publication of subsequent papers such as Openshaw and Taylor's (1979) classic about the modifiable areal unit problem and the myriad of more recent work about the ecological fallacy authored by Amrhein (1995), Steel and Holt (1996), King (1997), and Richardson and Montfort (2000). The ecological fallacy problem remains unsolved, however, and as such continues to be the topic of continuing research.

Fundamental harbingers

The most important of the beginnings in *Spatial Analysis* have been left for last—those that presaged the emergence of geographic information systems (GIS) and geospatial science. Let us look briefly at the first and with some more detail at the second.

GIS

With respect to GIS, Tobler's entry signified the unfolding of new approaches to map projection. This was followed by Snyder's (1983) tome and its update by Bugayevskiy and Snyder (1995). Map projections have become of sufficient general interest that Robinson's obituary in *The New York Times* (Wilford 2004) commemorates them. Beyond map projection, the matrix organization of geographic phenomena spelled out by Berry has become the GIS attribute table (Sui 1995), while Robinson's concern for isarithmic interpolation may be viewed as a predecessor of present-day kriging.

From these beginnings, and the crude computer-generated maps that accompanied them, the expansion of GIS has been truly phenomenal, driven by rapid expansions in computer power—far beyond those anticipated by Kao in *Spatial Analysis*—satellite technology and global positioning (themselves creatures of military and intelligence demands), and pushing much further into computer science (Goodchild 1992). To cite just one example, there is growing concern for the collection, management, processing, analysis, and delivery of real-time geospatial data using distributed geosensor networks. The sensors can be static or mobile and can be used to passively collect information about the environment or—and this is clearly becoming critical—to actively influence it. Supportive research addresses such issues as data stream processing, temporal–spatial queries over geosensor networks, and sensor data integration and mining. Whether this makes GIS a toolkit or a science is a matter of continuing debate (Wright et al. 1997).

Geospatial science

There is no question with respect to emergent geospatial science. The important harbingers were Geary's article on spatial autocorrelation, Dacey's paper about two- and K-color maps, and that of Bachi on geographic series. Their focus on geographic dependence was quickly followed by Cliff and Ord's (1973) pioneering treatment of spatial autocorrelation that heralded the work on spatially autoregressive and geostatistical analysis that now is central to geospatial inference.

Three spatial regression model specifications have emerged and seem to be forming the backbone of the science. The autoregressive (AR) response model is written as $Y = \rho WY + X\beta + \varepsilon$, where W is the row-standardized version of a binary geographic neighbors matrix, C. The simultaneous autoregressive (SAR) model is $Y = \rho WY + (I - \rho W)X\beta + \varepsilon$. Third, the conditional autoregressive (CAR) model is $DY = DX\beta + \varepsilon$, where $D^{T}D = (I - \rho C)M$, and M is a diagonal matrix, often set equal to I. The CAR model describes spatial autocorrelation in terms of a first-order inverse covariance structure; the other two describe it in terms of a second-order structure.

All three models are connected. The SAR and CAR models are both members of the family of Gaussian spatial processes. They differ only in their specification of the covariance matrix but originate in the same spatial link matrix. A third member in this family is the Gaussian moving average spatial process. The AR model, also termed in spatial econometrics the spatial lagged model, is a special case of the SAR model in which the lag component in the exogenous variables is restricted to zero. Geostatistics shies away from this formulation because its underlying probability distribution cannot be given in closed form.

What is interesting is that there appear to be significant differences in user preferences. The AR model is preferred by spatial econometricians who see the term ρWY controlling for spatial autocorrelation in a manner that is analogous to procedures for controlling serial autocorrelation in time series models. The approach was pioneered by Paelinck and Klaasen (1979) and Anselin (1998), who built off the treatment in Cliff and Ord's (1973) work to introduce spatial effects into the econometrics literature. The AR model has facilitated numerous tests of interdependent behavior that are revealed through spatial dependence in cross–sectional data. Behavior is often interdependent because of an externality, that is, a situation where the behavior of one agent spills over and directly influences the behavior of another. When the spillover is spatial, there are spatial externalities—the behavior of agents at one location is influenced by behavior of agents at another location. In such cases the normal assumptions of regression analysis fail. Of the numerous instances of spatial econometric models, perhaps the most noteworthy involve strategic interaction between governments on a variety of issues including provision of public goods, tax competition, and environmental quality. Examples of other applications involve criminal behavior, regional industrial clusters, and valuing real estate. Anselin, Florax, and Rey (2004) contains a compendium of recent applications and presents the current specification and estimation issues.

Whereas the rapid growth of AR applications is in spatial econometrics, the SAR model pioneered by Whittle (1954) is favored by spatial statisticians and the CAR model by remote-sensing researchers, although it also is used in hierarchical Bayesian modeling by spatial statisticians, and hence by epidemiologists who engage in disease mapping. The reasons for these different preferences are complex (Paelinck and Griffith 2004). The SAR model casts spatial autocorrelation as a function of a model's error term, and hence renders unbiased ordinary least-squares estimators in the presence of spatial autocorrelation. The CAR model involves a first-order covariance structure (i.e., only immediate neighbors have a direct effect of a given location's value), which relates to a more natural Markov-type dependency structure and which always can capture positive spatial autocorrelation when used to specify hierarchical models.

Spatial structure emerges in residuals when covariates are missing from a mean response specification, when a nonlinear relationship is mistakenly specified as a linear one, when heterogeneity arises because of the geographic aggregation of nonhomogeneous units, or when spatial interaction among locations is present (i.e., spatial processes are at work). The SAR model accounts for redundant information in georeferenced data that is due simply to geographic nearness (i.e., a spillover effect) with a second-order covariance structure (i.e., both first- and second-order neighbors have a direct effect on a given location's value). The CAR model is often preferred in image analysis because it achieves the maximum entropy among the set of all stationary models with a given finite set of variances and covariances, because it yields minimum variance predictions, and because it is directly extendable to space–time model specifications. Both the SAR and the CAR specifications have strong conceptual linkages with geostatistical semivariogram modeling. Early applications of these models were to data obtained during agricultural field experiments. Cliff and Ord (1973) popularized them for a much wider variety of situations, many of which involve socio-economic and demographic variables. Griffith and Layne (1999) furnish a set of example applications illustrating this scope.

Key issues

The nature of the spatial relationship

Almost all analyses in spatial statistics, spatial econometrics, geostatistics, or spatial epidemiology rely on simple topological relationships among the spatial objects. These binary indicator relationships are based either on dissimilarity metrics such as functions of interobject distances or on similarity metrics such as neighborhood characteristics. These different metrics are closely linked to the configuration of the spatial objects. The Young and Householder theorem can be used to retrieve a configuration of the spatial objects which is approximately congruent with their original planar layout (Tiefelsdorf 2000, section 3.2.3). This implies that statistical methods of spatial autocorrelation analysis that utilize either distance or adjacency relationships will produce highly correlated results because they ultimately are based on the same spatial structure, an idea probed by Berry in *Spatial Analysis*.

Alternative specifications of spatial relationships are far less commonly adopted, despite Haggett's (1976) initial success. He identified the most likely diffusion pathways of a measles epidemic at different phases of the diffusion process, explicitly specified hypothetical pathways of spatial interaction among the disease carriers, and then checked the degree of correspondence of the endogenously observed spatial patterns to the exogenously specified spatial diffusion link matrices. Exogenous spatial link matrices may reflect functional relationships among the spatial objects rather than simple distance decay or adjacency. For instance, a pattern embodying a spatial hierarchy cannot be captured at higher levels of the hierarchy by simple neighborhood relationships because observations are spatially disjoint (see Berry 1972).

Spatial mixing in observational data

The values of variables observed at particular locations may result from spatial mixing, in which case spatial modeling which controls for spillovers in the presence of local causal relationships will be inadequate. For example, a high incidence of a particular disease in a particular location may not be a matter of local causation but arise because immigrants bring the condition with them, simultaneously affecting the disease rate in their areas of origin. In such cases a spatial mixing or exchange mechanism needs to be embodied in the analysis.

There are few such studies. Examples are Tiefelsdorf (1998), who argues that migration is spatially mixing of observed disease rates for degenerative diseases with a long latency, and Kim, Elliott, and Wang (2003), who use intercounty commuting flows to capture the economic integration of counties in a voting pattern analysis. Tiefelsdorf demonstrates that in spatial epidemiology, spatial interaction relationships are more relevant from a substantive point of view than simple spatial spillovers, pointing to the need for more informed specification of spatial relationships, as opposed to mechanical use of first- or second-order adjacency.

Spatial heterogeneity

Global models, which assume a common functional structure, may not be sufficient to fully characterize underlying data-generating spatial processes. Systematic local departures from the underlying global relationships lead to spatial heterogeneity. The common approach to understanding such heterogeneity, presaged by Thomas in *Spatial Analysis*—and still good practice—is to identify influential observations, outliers, and patterns in global model residuals. This information may point to local mechanisms operating on particular subsets of objects.

The evolution of local spatial analysis beyond such information-gathering efforts has three sources. First, spatial interaction modelers noted spatial heterogeneity in local distance-decay parameters. Fotheringham (1991) put forward a behavioral theory of competing destinations to address these variations and modeled them using a local accessibility proxy to the potential destinations. Second, Anselin (1995) and Ord and Getis (1995) simultaneously introduced local indicators of spatial association to the methodological repertoire of spatial autocorrelation analysis. They realized that the tendency of spatial data either to cluster or to manifest as spatial outliers may vary locally throughout a map. Third, Fotheringham, Brunsdon, and Charlton (2002) extended the standard regression model to its geographically weighted form, arguing that the relationship between an endogenous variable and a set of exogenous variables may vary across a study area. Marked spatial heterogeneities in local regression parameters imply the possibility of locally varying data-generating mechanisms and thus locally varying theories.

Common to all systematic attempts to codify local variations in model parameters is a spatial filter that ties a local estimate at a pivot to its surrounding neighborhood. These local neighborhoods can be specified either by an adjacency metric or by a distance-decay relationship. In making such estimates, the available data are reused several times because they fall simultaneously into the domain of various pivots sliding slowly over the study region, however. While this provides smooth map patterns of the local estimates, it comes at the price of high intercorrelations among these estimates. The effect of these intercorrelations has been investigated by Tiefelsdorf (2000, 2003), who showed that local Moran's I's are correlated even if their pivots are more than two spatial lags apart and that, for spatial interaction models, a spatial pattern in the local distance-decay parameters may very well be an outcome of a misspecified distance-decay function. Local distance-decay parameters also have the tendency to correlate highly with other estimated parameters of the model. This multicollinearity hampers substantive interpretation of the estimated parameters because their individual effects can no longer be identified. There will also be strong correlations among the spatial patterns of different sets of local regression parameters (Wheeler and Tiefelsdorf 2005). These also induce artifacts that confound substantive meaning and highlight the necessity for more research on the properties of local statistical methods and interpretation of their results.

Local statistical methods are a youthful addition to the repertoire of spatial analysis and initial setbacks should not surprise us. A research agenda on these methods needs to focus on three issues: first, the local statistics must link to the associated global parameter because local parameters involve variation around the global model structure (Anselin 1995). Second, the correlation among the local statistics must either be controlled or explicitly incorporated in the local model interpretations—a clear identification of the unique local effect is required (Ord and

Getis 1995). And third, a joint statistical framework for testing local parameters must be adopted to enable the significance of local patterns and the statistical power of the local estimates to be assessed. Of course, these three issues converge into a unified model structure that allows simultaneous estimation and testing of all local and global effects.

Conclusion

There are, of course, a myriad of other technical problems to be solved. Not the least of these are codifying the impacts of spatial autocorrelation on the full battery of descriptive statistics, understanding negative spatial autocorrelation, developing the distribution theory for testing for spatial autocorrelation in residuals from generalized linear and autoregressive models, and developing generalized linear mixed models that account for spatial autocorrelation. All of which is to say that, 40 years after *Spatial Analysis*, Geospatial Science has emerged as a powerful new locus with a technically proficient and socially significant research agenda—a multidisciplinary enterprise that involves not only statisticians, geographers, and economists, but also epidemiologists, demographers, and many others. Each brings particular spatial problems and solution strategies to the table. The obvious question is whether the result will be a new discipline or an integrative interdisciplinary pursuit that fosters the broader development of spatially integrated human-environmental inquiry.

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