Reminder: $f: S \to S$; $M_f = (S \times \mathbb{R})/\sim \; = S \times [0,1]/\sim$ with $(x,t) \sim (f(x), t+1)$ $\boxed{10/11/2015}$

f pseudoAnosov $\Leftrightarrow$ M_f has a hyperbolic structure
$\Updownarrow$
M_f homeo. to $\mathbb{H}^3/\Gamma_f$, $\Gamma_f \subset \mathrm{Aut}\,\mathbb{H}^3$

This Γ_f: $0 \to \pi_1(S,s) \subset \Gamma_f \to \mathbb{Z} \to 0$.

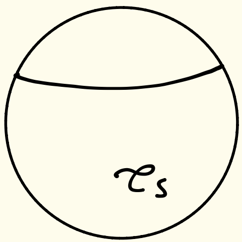
So we are looking for this group, in $\mathrm{Hom}(\pi_1(S,s), PSL_2\mathbb{C})$. $\longrightarrow G_S = \langle a_1, \dots, a_g, b_1, \dots, b_g ; \prod [a_i, b_i] = 1 \rangle$

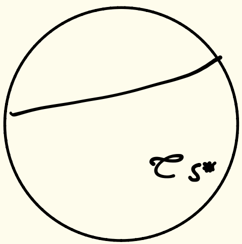
Looking for "the" representation that can be enriched appropriately.

There is a "big" subset of $\mathrm{Hom}(G_S, PSL_2\mathbb{C})$, consisting of the quasi-fuchsian groups;

the set of conj. classes of qf-rep. is isomorphic to $\mathcal{T}_S \times \mathcal{T}_{S^*}$.

Appropriately means you have a fixed pt of the action of f on $\mathrm{Hom}(\pi_1(S,s), PSL_2\mathbb{C})$.





We want to give a meaning to:

either a sequence of discrete faithful representations converge to a discrete injective representation or $\mathbb{H}^3$ degenerates into an $\mathbb{R}$-tree.

Chiswell's theorem:

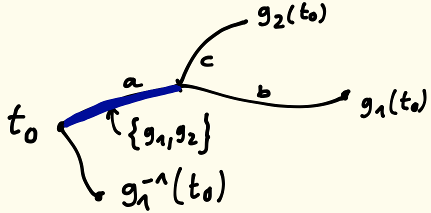
Gromov product: $\{g_1, g_2\} := \frac{1}{2}\left(|g_1| + |g_2| - |g_1^{-1} g_2|\right)$ given $|\cdot| : \Gamma \longrightarrow \mathbb{R}$.

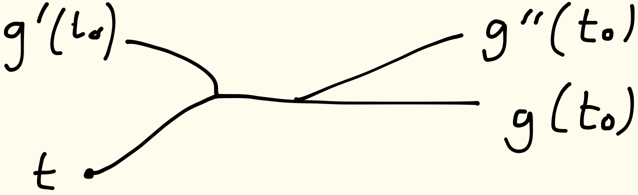
If $|\cdot| : G \to \mathbb{R}$ is such that : $\begin{cases} |id| = 0 \\ |g| = |g^{-1}| \text{ for all } g \\ \text{if } \{g, g'\} < \{g, g''\} \text{ then } \{g, g'\} = \{g', g''\} \end{cases}$

then there exists an $\mathbb{R}$-tree T, with base point t_0 and an action of G on T by isometries such that $|g| = d(t_0, g(t_0))$.

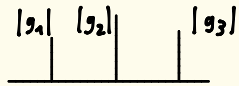
$d(t_0, g_1^{-1} g_2(t_0)) = d(g_1(t_0), g_2(t_0))$

$\{g_1, g_2\} = \frac{1}{2}\left((a+b) + (a+c) - (b+c)\right) = a$





How to build $\mathbb{R}$-tree: take segments & glue them:



if $t \leq \{g_1, g_2\}$ glue (g_1, t) to (g_2, t).

$$\coprod_{g \in G} [0, |g|] / \sim$$

And g acts: $g \cdot (h, t) = (gh, t)$.

$$\Psi_S : QF(G_S) \times \mathbb{H}^3 \longrightarrow \mathbb{P}(\mathbb{R}^{G_S})$$
$$(\rho, x) \longmapsto (g \mid d(x, \rho(g)(x)).$$

the image is cpct, & pts that are not in image are Chiswell's fc.

<u>Thm:</u> (Morgan, Shalen, Bestvina, Paulin, Otal)

1) the image of Ψ_S is compact

2) <u>either</u> a conv. seq. lifts to a cv. sequence in $\mathbb{R}^{G_S}$ & in that case if $h_n(x_n) = x_0$

$h_n \circ \rho_n \circ h_n^{-1}$ cv. to a discrete inj. representation.

$\uparrow$ aut. of $\mathbb{H}^3$

OR $\Psi_S(\rho_n, x_n)$ converges to a similarity class of Chiswell Function.

Proof: → compactness is easy: take generators, scale so that generators moving the most move by 1. We are now in $\prod_{g\in G} [0, w^\ell(g)]$.

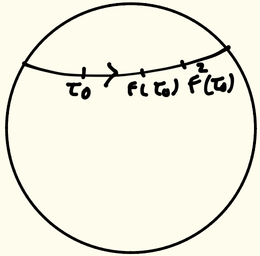
Rk: space of discrete injective rep. of non-elementary group is closed

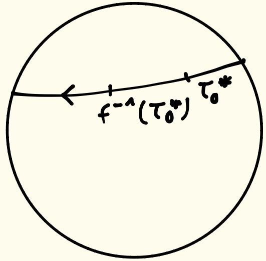
→ the first alternative of 2 is also easy.

→ second alt.: choose conj. class so that we can use Skora's thm: several applications of Margulis lemma.

Given diverging ρ_n, choose carefully the x_n. Hard part is minimality.

After rescaling, space becomes δ-hyperbolic, with $\delta \to 0$, but 0-hyperbolic space is another definition of an $\mathbb{R}$-tree.





$f^n(\tau_0) \not\perp f^{-n}(\tau_0^*)$ We can choose x_n s.t T_n belongs to 1^{st} altern. of 2.

Indeed: if not, then we would contradict Bers inequality: $\ell(\gamma) \leq 2 \inf(\ell^-(\gamma), \ell^+(\gamma))$.

If "or": $\exists \gamma_n$ s.t $\inf(\ell^+(\rho_n(\gamma_n)), \ell^-(\rho_n(\gamma_n)))$ bounded, but $\ell(\rho_n(\gamma_n)) \to \infty$ contradicting Bers.

Using Hatcher-Skora, the tree is the tree of some measured foliation $\mathcal{F}$ on S.

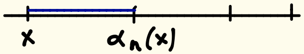
On, S, we already have $\mathcal{F}^+, \mathcal{F}^-$ invariant under f.

$\mathcal{F}$ can be made transversal to $\mathcal{F}^+$, or $\mathcal{F}^-$, say $\mathcal{F}^+$.

We will look at any curve α on S, and $f^{*n}(\alpha)$: claim $\ell^+(f_n(\alpha_n))$ is constant.

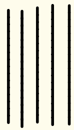
Same Riem. surface, different marking.

It remains to look at: $\ell(\rho_n(\gamma_n)) \to \infty$.



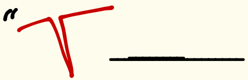
$\alpha_n \dashrightarrow \mathcal{F}^+$

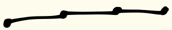
Tree looks like:

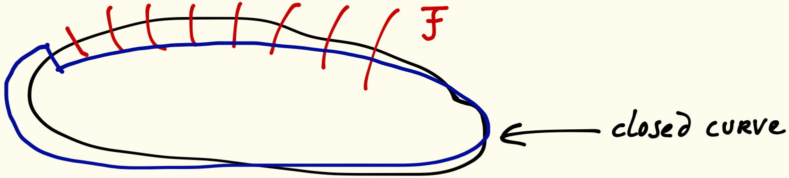


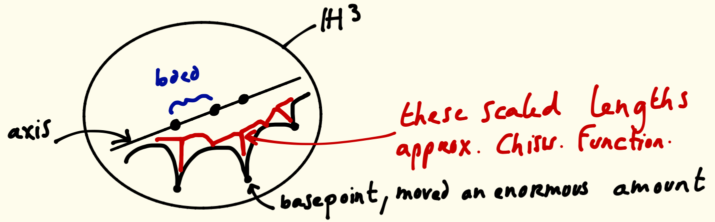
in $\mathbb{H}^3$

Part that goes to ∞:
the "leashes"



hence  can't be bdd.





One can visualize $\{\cdot,\cdot\}$ here.