(1)

$$\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} = M^{-1} \begin{bmatrix} a & c \\ c & b \end{bmatrix} M$$

$$2u^2 + 4v^2 = 1$$

$$\frac{u^2}{\left(\frac{1}{\sqrt{2}}\right)^2} + \frac{v^2}{\left(\frac{1}{2}\right)^2} = 1$$

$\frac{1}{\sqrt{2}}$ $\quad \frac{\sqrt{2}}{2}$

$\frac{2}{\sqrt{2}}$, $\quad 1$

(2)

$$A = \begin{bmatrix} 5 & 3 \\ 3 & -3 \end{bmatrix}$$

$$P_A(x) = \det \begin{bmatrix} (5-x) & 3 \\ 3 & -(3+x) \end{bmatrix} = -(3+x)(5-x) - 9$$

$$= -[15 - 3x + 5x - x^2] - 9 = -(15 + 2x - x^2) - 9$$

$$= -15 - 2x + x^2 - 9 = x^2 - 2x - 24$$

$$x = \frac{2 \pm \sqrt{4 - 4(-24)}}{2} = \frac{2 \pm 10}{2} = \begin{cases} 6 \\ -4 \end{cases}$$

$\underline{\lambda = 6}$

$$\begin{bmatrix} -1 & 3 \\ 3 & -(9) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$+27(x\sqrt{9}$

$-x + 3y = 0 \quad v = \frac{1}{\sqrt{10}}(3, 1)$

$x = 3y$

(2) $\lambda = -4$

$$\begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$3x + y = 0$

$v_2 = \frac{1}{\sqrt{10}}(1, -3)$

$$M = \frac{1}{\sqrt{10}} \begin{bmatrix} 3 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = M \begin{bmatrix} x' \\ y' \end{bmatrix} = \frac{1}{\sqrt{10}} \begin{bmatrix} 3 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{10}}(3x' + y') \\ \frac{1}{\sqrt{10}}(x' - 3y') \end{bmatrix}$$

$$6(x')^2 - 4(y')^2 + \frac{1}{\sqrt{10}} \left[\frac{44}{\sqrt{10}}(3x' + y') - \frac{12}{\sqrt{10}}(x' - 3y') \right] = k$$

$$6(x')^2 - 4(y')^2 + \frac{1}{10}(120x' + 80y') = k$$

$$6(x')^2 + 12x' - 4(y')^2 + 8y' = k$$

$$6\left[(x')^2 + 2x' + 1\right] - 6 - 4\left[(y')^2 - 2y' + 1\right] + 4 = k$$

$$6(x' + 1)^2 - 4(y' - 1)^2 = k + 2$$

$k \neq -2$

(5)

$v_1 = (1,0,1,0) \qquad v_2 = (1,-1,-1,1)$

$\ker(T-4I) = [v_1, v_2]$

$v_3 = (0,1,0,1) \qquad Tv_3 = 3v_3$

So $\quad v_3 \in \ker(T-3I)$

$v_4 = (\alpha, \beta, \gamma, \delta)$

$\alpha + \gamma = 0 \Rightarrow \gamma = -\alpha$

$\alpha - \beta - \gamma + \delta = 0$

$\beta + \delta = 0 \Rightarrow \delta = -\beta$

$\alpha - \beta + \alpha - \beta = 0$

$2\alpha - 2\beta = 0$

$\alpha = \beta \qquad v_4 = (1,1,-1,-1)$

$Tv_4 = 3v_4$

$$(2,0,-2,0) = \alpha(1,0,1,0) + \beta(1,-1,-1,1) + \gamma(0,1,0,1) + \delta(1,1,-1,-1)$$

$$\begin{cases} \alpha + \beta + \delta = 2 \\ -\beta + \gamma + \delta = 0 \\ \alpha - \beta - \delta = -2 \\ \beta + \gamma - \delta = 0 \end{cases}$$

(2) $\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 & 0 \\ 1 & -1 & 0 & -1 & -2 \\ 0 & 1 & 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & -2 & 0 & -2 & -4 \\ 0 & 1 & 1 & -1 & 0 \end{array}\right]$

$\sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & +1 & 0 & +1 & +2 \\ 0 & 1 & 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 0 & 0 \end{array}\right]$

$\Rightarrow \alpha + \beta \quad + \delta = 2$

$\qquad -\beta + \gamma + \delta = 0 \Rightarrow \beta = \delta = 1$

$\qquad \gamma + 2\delta = 2 \Rightarrow \delta = 1$

$\qquad 2\gamma = 0 \Rightarrow \gamma = 0$

$\alpha + 1 + 1 = 2 \Rightarrow \alpha = 0$

$v_4 = \overset{v_2}{(1,-1,-1,1)} + \overset{v_4}{(1,1,-1,-1)}$

$Tv_4 = Tv_2 + Tv_4 = (4,-4,-4,4) + (3,3,-3,$

$= (7,-1,-7,1)$

5

Q4) $T: \mathbb{R}^3 \to \mathbb{R}^3$

$$[T]_{can} = \begin{bmatrix} 1 & 1 & b \\ 2 & 1 & 2 \\ 1 & a & 2 \end{bmatrix}$$

$W = \{(x,y,z) \in \mathbb{R}^3 \mid x+y+z=0\}$.

W é invariável por T $\Leftrightarrow$

Sol:

$z = -x-y$

$w = (x,y,z) = (x,y,-x-y) = x(1,0,-1) + y(0,1,-1)$

$$\begin{bmatrix} 1 & 1 & b \\ 2 & 1 & 2 \\ 1 & a & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \alpha\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \beta\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{cases} 1-b = \alpha \\ 2-2 = \beta \Rightarrow \beta = 0 \\ 1-2 = -\alpha-\beta \Rightarrow -1 = -\alpha \Rightarrow \alpha = 1 \end{cases}$$

$1-b = 1 \Rightarrow \boxed{b=0}$

$$\begin{bmatrix} 1 & 1 & b \\ 2 & 1 & 2 \\ 1 & a & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \alpha\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \beta\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ -\alpha-\beta \end{bmatrix}$$

$$\begin{cases} 1-b = \alpha \Rightarrow \alpha = 1 \\ 1-2 = \beta \Rightarrow \beta = -1 \\ a-2 = -\alpha-\beta \\ \quad a-2 = -1+1 \Rightarrow \boxed{a=2} \end{cases}$$

(5)

Q.5)

Seja $z = \frac{1}{\sqrt{2}}(-1 + i) = \cos\left(\pi - \frac{\pi}{4}\right) + i\,\mathrm{sen}\left(\pi - \frac{\pi}{4}\right)$

$\mathcal{B} = \{1, i\}$ é uma base de $\mathbb{C} / \mathbb{R}$

Se $T: \mathbb{C} \to \mathbb{C}$ a transformação $\mathbb{R}$-linear dada por

$$T(w) = zw$$

Nestas condições

$$[T]_{\mathcal{B}} = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

e $$[T]_{\mathcal{B}}^{200} = \frac{1}{2^{100}}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{2^{100}} I$$

onde $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, pois

$$z^{200} = \cos 200\left(\pi - \frac{\pi}{4}\right) + i\,\mathrm{sen}\, 200\left(\pi - \frac{\pi}{4}\right) = 1.$$

(7)

Logo

$$\frac{A^{200}}{2^{199}} = \begin{bmatrix} \frac{1}{2^{199}} & 0 \\ 0 & 2 \end{bmatrix}$$

$$A^{200} = 2^{199} \begin{bmatrix} 2^{-199} & 0 \\ 0 & 2 \end{bmatrix}$$

$$\operatorname{tr} A^{200} = 2^{199}\left(2 + 2^{-199}\right) = 1 + 2^{200}$$

Q.8)

$$t^2 + t + 1 = 0$$

$$t = \frac{-1 \pm \sqrt{1 - 4(1)}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

Seja $\lambda = \frac{-1 + \sqrt{3}i}{2}$

Sabemos que $\ker(T - 2I) = [(1,0,1)]$

e $\ker[T - \lambda I] = [(i,1,0)]$

Logo $\ker[T - \bar{\lambda} I] = [(-i,1,0)]$

As soluções complexas do sistema são

$$x(t) = c_1 e^{2t}(1,0,1) + c_2 e^{\left(\frac{-1+\sqrt{3}i}{2}\right)t}(i,1,0)$$

$$+ c_3 e^{\left(\frac{-1-\sqrt{3}i}{2}\right)t}(-i,1,0)$$

(2)

mas

$$e^{(-1+\sqrt{3}i)\frac{t}{2}}(i,1,0)$$

$$= e^{-t/2}\, e^{\frac{\sqrt{3}ti}{2}}(i,1,0) =$$

$$= e^{-t/2}\left[\cos\left(\frac{\sqrt{3}}{2}t\right) + i\,\mathrm{sen}\left(\frac{\sqrt{3}}{2}t\right)\right](i,1,0)$$

$$e^{-t/2}\left(\cos\left(\frac{\sqrt{3}}{2}t\right)i - \mathrm{sen}\left(\frac{\sqrt{3}}{2}t\right), \cos\left(\frac{\sqrt{3}}{2}t\right) + i\,\mathrm{sen}\left(\frac{\sqrt{3}}{2}t\right), 0\right)$$

$$= e^{-t/2}\left[\left(-\mathrm{sen}\left(\frac{\sqrt{3}}{2}t\right), \cos\left(\frac{\sqrt{3}}{2}t\right), 0\right) + i\left(\cos\left(\frac{\sqrt{3}}{2}t\right), \mathrm{sen}\left(\frac{\sqrt{3}}{2}t\right), 0\right)\right]$$

Logo

$$e^{2t}(1,0,1), \quad e^{-t/2}\left(-\mathrm{sen}\left(\frac{\sqrt{3}}{2}t\right), \cos\left(\frac{\sqrt{3}}{2}t\right), 0\right),$$

$$e^{-t/2}\left(\cos\left(\frac{\sqrt{3}}{2}t\right), \mathrm{sen}\left(\frac{\sqrt{3}}{2}t\right), 0\right) \quad \text{são}$$

as funções cujas combinações lineares dão todas as soluções do sistema