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## Palavras do Editor

Este número do boletim é especial por diversas razões. No último mês de março, realizou-se o XI EBEB, quando foi eleita a diretoria da ISBrA para o biênio 2012-2014. Trazemos neste número um relato do ótimo encontro e marcamos a transição da gestão com as cartas dos presidentes: da chapa eleita, Adriano Polpo (UFSCar), e da última diretoria, Julio Stern (IME-USP). Como se esses textos não bastassem, trazemos ainda artigos notáveis escritos especificamente para o boletim.

Como lembramos nas últimas edições, 2012 marca o bicentenário da primeira edição do *Théorie Analytique des Probabilités*, de Pierre Simon Laplace. Para escrever sobre esta pedra angular da inferência estatística, convidamos dois pesquisadores que são autoridades sobre o assunto. O primeiro é o professor Christian Robert, da *Université Paris-Dauphine*, autor dos livros *The Bayesian Choice*, *Monte Carlo Statistical Methods* e *Introducing Monte Carlo Methods with R*, os dois últimos com George Casella. Embora tristemente, aproveito a oportunidade para lembrar o falecimento do professor Casella no último dia 17 de junho em Gainesville, Florida.

O outro pesquisador que convidamos para escrever sobre a obra de Laplace é o professor Richard Pulskamp, da *Xavier University*. Ele traduziu, praticamente na íntegra, o livro de Laplace para o inglês e mantém um ótimo *website* onde disponibiliza, além dos capítulos do *Théorie*, diversos artigos de Laplace traduzidos para o inglês[1]. Aos professores Robert e Pulskamp, agradecemos imensamente pela disposição e esforço que, sem dúvida, enriqueceram muito nosso boletim.

A última seção, como já é tradição, traz anúncios de eventos que ocorrerão nos próximos meses, no Brasil e no mundo.

Neste número, eu também me despeço. A partir do próximo boletim, Victor Fossaluza (UFSCar), que assina o relato do XI EBEB, será o responsável pela edição.

Gostaria de agradecer a algumas pessoas. Em primeiro lugar, aos componentes da diretoria 2010-2012: Julio Stern, Marcelo Lauretto e Adriano Polpo, por terem me confiado a tarefa de editar este boletim. Em especial ao Adriano, que me indicou para isso, mas também me ajudou muito quando pedi ou precisei. Também agradeço, mais uma vez, ao professor (e amigo) Carlinhos. Sua ajuda não se reduziu a boas idéias para o boletim, pois ele também pôs a mão na massa. Entre outras tarefas, Carlinhos juntou-se ao time de entrevistadores do boletim: os professores Francisco Louzada Neto e Jorge Achcar, que colheram ótimos depoimentos, respectivamente, dos professores Carlos A. B. Dantas, Basílio B. Pereira e Josemar Rodrigues. A toda essa turma de alto quilate, meu muito obrigado. Agradeço também à minha paciente esposa, Mirian, pela ajuda para minimizar os erros de português dessas linhas.

Dedico este último boletim ao professor José Galvão Leite. Minha probabilidade para o evento "ele vai apreciar muito as resenhas sobre o livro de Laplace" é próxima de um.

Boa leitura!

## Índice



[1]Para os pouco versados em francês, como eu, o endereço é fornecido no artigo do Professor Pulskamp.

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# Cartas da Presidência

Adriano Polpo - presidente eleito para o biênio 2012-2014
*(UFSCar)*

Apresento a nova diretoria que assume a gestão do biênio 2012-2014.

Meu nome é Adriano Polpo de Campos, Bacharel em Estatística pela UNICAMP, Doutor em Estatística pelo IME-USP, Pós-doutor pela *Florida State University* e atualmente sou professor adjunto do departamento de Estatística da UFSCar. Além disso, fui secretário da ISBrA no biênio 2010-2012, bem como edtior deste boletim em 2007.

O Francisco Louzada, secretário, é Bacharel em Estatística pela UFSCar, Mestre em Ciências da Computação e Matemática Computacional pelo ICMC-USP, PhD em Estatística pela *Oxford University* e atualmente é professor titular no ICMC-USP. Além disso é também diretor de Transferência de Tecnologia do Centro de Matemática e Estatística Aplicadas à Indústria, ICMC-USP (Projeto CEPID-FAPESP), coordenador do Centro de Estudos do Risco, vice-Coordenador do Programa de Pós-graduação em Estatística da UFSCar, editor da Revista Brasileira de Estatística e do Projeto Fisher.

A Laura Leticia Ramos Rifo, tesoureira, é Bacharel e Mestre em Matemática pela *Universidad de Santiago de Chile* e Doutora em Estatística pelo IME-USP, Pós-Doutora pelo IMECC-UNICAMP, professora visitante na *Universidad de Valparaiso* e atualmente é professora doutora do IMECC-UNICAMP e Diretora Associada do Museu Exploratório de Ciências da UNICAMP.

Por fim, o editor deste boletim para o próximo biênio, Victor Fossaluza, é Bacharel, Mestre e Doutor em Estatística pelo IME-USP. Note que, apesar do editor do boletim não ser um cargo eletivo da diretoria da ISBrA, é de suma importância na divulgação das atividades da ISBrA, bem como dos assuntos de interesse de nossa comunidade.

Aproveito também para agradecer ao Márcio Alves Diniz, que fez um excelente trabalho como editor deste Boletim no biênio 2010-2012 e que irá auxiliar o Victor na edição do próximo boletim. Agradeço ao Julio e ao Marcelo pela magnífica gestão, nos deixando a ingrata tarefa de manter o que foi conquistado.

Registro aqui que esta nova diretoria conta com o apoio e colaboração de todos os membros da comunidade Bayesiana Brasileira.

Assumimos esta diretoria com a responsabilidade de dar continuidade ao excelente trabalho de todas as diretorias passadas. Nosso principal objetivo, além dar continuidade as realizações, é alcançar novas conquistas.

Em nome da nova diretoria do ISBrA, agradeço a confiança em nós depositada para levar a bom termo a gestão do biênio que ora se inicia e convidar a todos a participar com suas sugestões e efetiva colaboração.

Julio M. Stern - presidente do biênio 2010-2012
*(IME-USP)*

Caros membros da ISBrA,

A convite dos editores do boletim, faço aqui, na qualidade de *past*-presidente, um pequeno relato da gestão ISBrA 2010-2012.

Nesta gestão, organizamos três eventos principais:

- O Encontro Bayesianismo II, www.ime.usp.br/~isbra/bayes
- A Conferência em Estatística Indutiva, www.ufscar.br/~polpo/cis/en/
- O 11º Encontro Brasileiro de Estatística Bayesiana (XI EBEB 2012), www.brastex.info/ebeb2012/.

Os dois primeiros eventos já foram objeto de relatos específicos em edições anteriores deste boletim. Assim, aproveitamos esta oportunidade para fazer um pequeno sumário das atividades do EBEB 2012.

O 11º Encontro Brasileiro de Estatística Bayesiana (EBEB 2012) foi realizado em Amparo, SP, no período de 18 a 22 de março de 2012. Os objetivos do evento foram:

- Fortalecer a pesquisa em métodos Bayesianos, bem como ampliar sua aplicação na comunidade científica brasileira.
- Proporcionar um ambiente no qual pesquisadores brasileiros e internacionais pudessem colaborar, apresentar seus mais recentes desenvolvimentos e discutir problemas em aberto.
- Permitir aos alunos de pós-graduação brasileiros ter contato com pesquisadores sêniors, tanto para discutir seus trabalhos como também para iniciar possíveis contatos para projetos futuros de doutorado e pós-doutorado.
- Fortalecer a interação da comunidade Estatística com outras comunidades científicas, como Jurimetria, Econometria, Física, Astronomia, Medicina, Engenharia, etc. Destacamos o entusiasmo da Associação Brasileira de Jurimetria, através de seus vários representantes presentes no evento.

O evento teve como foco a discussão dos recentes desenvolvimentos sob os pontos de vista computacional e metodológico, com ênfase em fundamentos de probabilidade e estatística. O evento combinou um programa de ótimo nível, com palestras proferidas por pesquisadores de projeção nacional e internacional, cujos trabalhos têm marcado o cenário moderno da área.

A instituição realizadora do EBEB 2012 foi o Instituto de Matemática e Estatística da Universidade de São Paulo (IME-USP). Os membros do Comitê Organizador foram: Julio Stern (IME-USP), Adriano Polpo (UFSCar), Marcelo Lauretto (EACH-USP), Carlos Alberto de Bragança Pereira (IME-USP) e Márcio Alves Diniz (UFSCar).

O evento contou com 13 palestrantes convidados, cujos nomes e respectivas instituições são listados abaixo:
– André Rogatko (*Samuel Oschin Comprehensive Cancer Institute*, EUA)
– Alexandra Schmidt (UFRJ)
– Ariel Caticha (*State University of New York*, EUA)
– Dalia Chakrabarty (*University of Warwick*, Inglaterra)
– Debajyoti Sinha (*Florida State University*)
– Frank Lad (*University of Canterbury*, Nova Zelândia)
– Joseph Kadane (*Carnegie Mellon University*, EUA)
– Luis Raul Pericchi Guerra (*University of Puerto Rico*)
– Marco Antonio Rosa Ferreira (*University of Missouri*, EUA)
– Marlos Viana (*University of Illinois*, EUA)
– Nestor Caticha (Instituto de Física, USP)
– Rosangela Loschi (UFMG)
– Sonia Petrone (*Universita Bocconi*, Itália)

Infelizmente, os palestrantes Hedibert Freitas Lopes (*The University of Chicago*) e Sylvia Fruehwirth-Schnatter (*Vienna University of Economics and Business*), originalmente convidados e com presença confirmada, não puderam participar do evento por problemas de saúde.

O evento teve um total de 70 trabalhos apresentados, sendo:
– 28 apresentações orais, divididas em 10 sessões (paralelas duas a duas);
– 42 apresentações pôster, divididas em duas sessões.

Além dos palestrantes convidados, o evento teve um total de 76 participantes regulares, distribuídos entre as seguintes instituições:
– ABJ – Associação Brasileira de Jurimetria
– EACH - USP
– ICMC - USP
– IME - USP
– INMETRO
– PUC-RS
– UFGD
– UFMG
– UFRJ
– UFSCAR
– UNB
– UNESP
– UNICAMP
– UNIVERSIDAD DE ANTOFAGASTA
– UNIVERSIDAD DE CONCEPCION
– UNIVERSIDAD DE SANTIAGO DE CHILE
– UNIVERSIDADE FEDERAL RURAL DE PERNAMBUCO

Cabe destacar a expressiva participação de estudantes brasileiros (33 no total), em sua quase totalidade com apresentações orais e/ou pôsteres. Esse é, em nossa avaliação, um importante indicador da força da área de Inferência Bayesiana e de sua expansão futura. Todos os participantes que se inscreveram no prazo regular e pediram apoio financeiro puderam ser contemplados.

Pela primeira vez, estamos editando *proceedings* de alta qualidade para o EBEB, a serem publicados pela AIP – *The American Institute of Physics Conference Proceedings*. Um dos objetivos de termos bons *proceedings* é a internacionalização do evento, isto é, o estímulo à participação de pesquisadores estrangeiros, além daqueles especialmente convidados pelos organizadores. A exemplo do que vimos acontecer em outras áreas, esperamos ver os resultados de um trabalho consistente após duas ou três edições dos *proceedings* do evento.

Os artigos submetidos ao EBEB 2012 sofreram um processo de revisão rápida por seus pares durante o evento: participantes receberam a incumbência de revisar anonimamente dois artigos de outros participantes. As revisões foram enviadas aos respectivos autores, que já entregaram suas versões finais corrigidas. Esses artigos estão em fase de triagem e compilação final para publicação.

Também pela primeira vez, convidamos os ministrantes de tutoriais a escrever livros texto para o evento. A exemplo do que já faz a Associação Brasileira de Estatística (ABE), esta iniciativa disponibiliza bons recursos didáticos aos alunos/pesquisadores brasileiros, e permite aos autores um estágio intermediário (*stepping-stone*) no preparo de livros para edição comercial. Dentre todos os convidados, o Professor Ariel Caticha gentilmente atendeu a nosso convite e escreveu o livro-texto intitulado *Entropic Inference and the Foundations of Physics*, o qual foi impresso e distribuído para os participantes do evento.

Durante o EBEB, mais de 60 participantes assinaram um manifesto em solidariedade aos colegas estatísticos da Argentina, que têm sido ameaçados e coagidos de diversas formas a produzir estatísticas tendenciosas sobre a inflação naquele país. As matérias da revista *Economist* em seu fascículo 25 de 2012, "*Don't lie to me, Argentina*", p.18, e "*The price of cooking the books*", p.47-48, esclarecem a situação dramática vivida por nossos colegas, bem como as práticas anti-éticas que têm sido utilizadas para manipular estatísticas oficiais: www.ime.usp.br/∼jstern (Miscelânea).

Na reunião de eleição da nova diretoria da ISBrA, realizada ao final do evento, os membros do Comitê Organizador do EBEB 2012 aceitaram o compromisso de colaborar, na qualidade de editores, na preparação dos *proceedings* da próxima edição do evento (EBEB 2014).

Diante dos relatos acima, consideramos que a realização do evento se deu dentro das previsões iniciais, e que atingiu plenamente os objetivos propostos.

O Comitê organizador do EBEB 2012 agradece o apoio recebido das agências e institutos: FAPESP, CNPq, CAPES e Instituto Nacional de Ciência e Tecnologia de Matemática (INCTMat).

Agradecemos o patrocínio dos programas de pós-graduação de Estatística e Matemática Aplicada do IME-USP e da UFSCar. Estes recursos foram muito importantes para permitir a participação de alunos de pós-graduação e a edição de livros para os tutoriais.

Agradecemos ainda o patrocínio da Associação Brasileira de Estatística (ABE), no valor de R$ 2.000,00, e o patrocínio da Associação Brasileira de Jurimetria (ABJ), no valor de R$ 4.500,00.

Em função de descontos oferecidos na inscrição para os eventos da atual gestão, e de uma agressiva política de recrutamento, conseguimos ainda aumentar substancialmente o número de membros brasileiros ativos na ISBA através do Capítulo Brasileiro.

Embora formalmente a gestão 2010-2012 tivesse apenas 3 membros (presidente, secretário e tesoureiro), gostaríamos de agradecer nominalmente a algumas pessoas que muito nos auxiliaram:

- Prof. Márcio Diniz, que foi o editor dos nossos boletins e co-organizador dos eventos;
- Profa. Márcia Branco e Rosângela Loschi pela organização do jantar em homenagem ao Prof. Heleno Bolfarine durante o EBEB 2012;
- Sylvia Regina A. Takahashi, Lourdes Vaz da Silva Netto e Danilo Leal Mesquita pelo auxílio nos serviços de secretaria, contabilidade, informática e logística do evento.

Saudações acadêmicas,
São Paulo, 30 de julho de 2012.

# Reading Théorie Analytique des Probabilités

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**Abstract**

This note is an extended read of my read of Laplace's book *Théorie Analytique des Probabilités*, when considered from a Bayesian viewpoint but without historical nor comparative pretentions. A deeper analysis is provided in Dale (1999).

## 1 Introduction

> *"The theory of probabilities draws a remarkable distinction between observations which have been made, and those which are to be made."* A. de Morgan*, Dublin Review,* 1837.

Pierre Simon Laplace's book, *Théorie Analytique des Probabilités*, was first published in 1812, that is, exactly two centuries ago! Following a suggestion by the editor of the ISBrA Bulletin, I gladly accepted the invitation as (a) Laplace's role in Bayesisian statistics is much deeper and longlasting than Bayes' (Dale, 1982, 1999), (b) I had never looked at this book and so this was a perfect opportunity to do so, using the 1812 edition in my possession, and (c) I was curious to see how much of the book had permeated modern probability and statistics. (Note that the versions of the book evolved quite considerably from the first to the fifth edition in 1825.) The following review is not pretending at scholarly grounding the book within its academic surroundings and successors, but is to be taken as a mere Bayesian excursion along its pages. A deeper analysis of *Théorie Analytique des Probabilités* can be found in Dale (1999, pp. 250–283). In particular, Andrew Dale discusses Bayesianly relevant supplements found in later editions of *Théorie Analytique des Probabilités*, as well as connections with both Bayes' and Laplace's *Essays.*

> *"Je m'attache surtout, à déterminer la probabilité des causes et des résultats indiqués par événemens considérés en grand nombre."* P.S. Laplace*, Théorie Analytique des Probabilités,* page 3.

I must first and foremost acknowledge I found the book rather difficult to read and this for several reasons: (a) as always is the case for older books, the ratio *text-to-formulae* is very high; (b) the themes in succession are often abruptly brought (i.e. not always well-motivated) and uncorrelated with the previous ones; (c) the mathematical notations are (unsurprisingly) 18th-century, so sums are indicated by $S$, exponentials by $c$, and so on, while those symbols are also used as variables in other formulae; (d) I often missed the big picture and got mired into technical details, until they made sense or until I gave up; (e) I never understood whether or not Laplace was interested in the analytics like generating functions only to provide precise numerical approximations or for their own sake. So a certain degree of disappointment in the end, most likely due to my insufficient investment in the project (on which I only spent an Amsterdam/Calgary flight and a few sleepless nights in Banff...), even though I got excited by finding the bits and pieces about Bayesian estimation and testing.

## 2 Contents of Théorie Analytique des Probabilités

> *"Sa théorie est une des choses les plus curieuses et les plus utiles que l'on ait trouvées sur les suites."* P.S. Laplace*, Théorie Analytique des Probabilités,* page 8.

The *Livre Premier* is about generating functions (Calcul des Fonctions géneratrices). As such, it is not directly of interest, focusing on finite difference equations, even though the techniques developped therein will be exploited in the second part. (There is an interesting connection with Abraham de Moivre, incidentally, since this older mathematical giant used generating functions to derive binomial formulas. He is acknowledged in Laplace's preface by the above quote, Bellhouse, 2011.)

> *"La théorie des probabilités consiste à réduire tous les événemens qui peuvent avoir lieu dans une circonstance donnée à un certain nombre de cas également possibles."* P.S. Laplace*, Théorie Analytique des Probabilités,* page 178.

The *Livre Second* is about probability theory, first about urn type problems, then about asymptotic approximations. The introduction to this second part reflects the famous (almost mythical!) determinism of Laplace, where randomness is simply *l'expression de notre ignorance* (yes, our ignorance as so expressed, page 177)... The intial pages contain the basics of probability like the chain rule, the product rule, the conditional probability and what we now call Bayes' rule, even though it is not called as such in *Théorie Analytique des Probabilités.* I did not find any mention of Thomas Bayes in the book. However, when looking at the on-line version of the book, I realised to my dismay that the 1814 edition has changed quite significantly, with an historical introduction to the theory of probability, incl. the mention of Bayes. (Thus, the changes were not restricted to the removal of the dedication to Napoléon-le-Grand [not longer appropriate after Waterloo and the restauration of the monarchy!] and the change from Chancellier du Sénat [an honorific title under Napoléon I] to Pair du Royaume [an honorific title under Louis XVIII], reflecting the well-known turncoat politics of Laplace!) An interesting syntactic point is the paragraph where Laplace introduces the notion of *expectation* (in the sense of Dicken's *Great Expectations*), along with *fears* ("*crainte*"), and as in Laplace's *Essai philosophique*, he distinguishes between mathematical expectation and moral expectation. (He later acknowledge Bernoulli's priority, as discussed below.)

> *"Nous traiterons d'abord les questions dans lesquelles les probabilités des événemens simples, sont données; nous considérerons ensuite celles dans lesquelles ces probabilités sont inconnues, et doivent être déterminées par les événemens observés."* P.S. Laplace*, Théorie Analytique des Probabilités,* page 188.

The above quote is the introduction to Chapter II which essentially consists in a sequence of combinatorial problems solved by polynomial decompositions and approximated by the finite difference formulae of the first *Livre.* (Despite this enticing quote, the chapter does not cover the statistical part.) While the accumulation of lottery and urn problems is not exactly fascinating, to say the least, some entries highlight Laplace's analytical skills. For instance, a convoluted urn problem leads to an equally convoluted integral (page 222)

$$\frac{\int_0^\infty x^{rn-n}dx \cdot (x-r)^n e^{-x}}{\int_0^\infty x^{rn-n}dx \cdot e^{-x}} \qquad (0)$$

where Laplace uses a Laplace approximation to replace (0) with

$$\frac{(1-1/n)^{n+1}}{\sqrt{(1-1/n)^2 + \frac{2}{rn} - \frac{1}{rn^2}}}$$

for $n$ and $rn$ large. The cdf is used in a convoluted (if labeled as "*très-simple*" on page 264!) derivation of an expectation of several variables. The chapter concludes with reflections on an optimal voting system that relates to Condorcet's (although no mention is made of this political scientist in the book, even though Laplace owed his position [at the age of 24!] in the Académie Royale des Sciences to his intervention).

> *"On peut encore, par l'analyse des probabilités, vérifier l'existence ou l'influence de certaines causes dont on a cru remarquer l'action sur les êtres organisés."* P.S. Laplace*, Théorie Analytique des Probabilités,* page 358.

Chapter III moves to asymptotic approximations and the law of large numbers for frequencies, "*cet important théorème*" (page 275). The beginning of the chapter shows that the variation of the empirical frequency around the corresponding probability is of order $1/\sqrt{n}$, with a normal approximation to the coverage of the confidence interval. Dale (1999) makes the crucial point (and I missed it!) that Laplace defines there a confidence interval on a probability parameter $p$, by a Bayesian argument, i.e. by using a flat prior on the probability parameter (page 254).

> *"On peut reconnaître l'effet très-petit d'une cause constante, par une longue suite d'observations dont les erreurs peuvent excéder cette effet lui-même."* P.S. Laplace*, Théorie Analytique des Probabilités,* page 352.

Chapter IV extends the above law of large numbers to a sum of iid variables. It then remarks that the most likely error is zero (which simply means that the mode of the standard normal distribution is indeed zero). It also contains a derivation of (a) the posterior median as minimising the absolute error loss and (b) the empirical average as minimising the squared error error or being the least square estimator (page 321). I think Laplace uses a Fourier transform to derive the distribution of a weighted sum (page 314). Laplace then proceeds to generalise this optimality result to a bivariate quantity, obtaining again the least square estimate and computing a bivariate Gaussian density on the way. And then comes the major step,! namely Laplace's derivation of a posterior distribution (page 334):

$$\frac{\prod_i \varphi(x_i - \theta)}{\int \prod_i \varphi(x_i - \theta)\,\mathrm{d}\theta}$$

(with my notations), thus using a flat prior on the location parameter! This fundamental step is compounded by the introduction of a (not yet) Bayes estimator minimising posterior absolute error loss and found to be the median of the posterior. In the next pages, Laplace attempts to find the MAP (which is also the maximum likelihood estimator in this case), as an approximation to the posterior median (page 336). From therein, he moves to identify the distribution for which the MAP is also the (arithmetic) average, ending up with the normal distribution (page 338). (This result was to be extended by J.M. Keynes, see Keynes, 1920, to different types of estimators.) The chapter concludes with a defense of the arithmetic mean as a limiting Bayes estimator that does not depend on the law of the errors.

> *"Pour déterminer avec quelle probabilité cette cause est indiquée, concevons que cette cause n'existe point."* P.S. Laplace, *Théorie Analytique des Probabilités,* page 350.

Chapter V starts with the computation of a p-value, nothing less! Laplace analyses the likelihood (*vraisemblance*) of a non-zero effect by looking at the cdf of the observation under the null (page 361). The following pages discuss Laplace's analysis of the irregularities in celestial trajectories, like the perturbations between Saturn and Jupiter. It argues in a philosophical if un-Popperian way about the importance of probabilistic analysis (read statistics) for uncovering scientific facts (page 358).

> *"Laplace actually used the theory of probabilities as a method of discovery."* A. de Morgan, *Dublin Review,* 1837.

In Chapter VI, *De la probabilité des causes et des événemens futurs, tirés des événemens observés*, Laplace develops his Bayesian (or Laplacian) perspective for drawing inference about unknown probabilities. He uses a uniform prior (with an interesting argument transferring the prior into the likelihood as to always consider this case, see page 364).[1] He then derives a normal approximation to the posterior (first term of the Laplace approximation!, page 367). This chapter also contains the famous study on the proportion $\varrho$ of female births in Paris, using an approximation to the beta integral to show that the (posterior) probability that is larger than 1/2 is negligible ("*d'une petitesse excessive*", page 380). Laplace also computes the posterior probability that the probability of a male birth in London is larger than in Paris, which he finds equal to 1-1/328269 (using a double integral and a continued fraction approximation!). He then moves to the applications of these techniques to mortality tables and insurances, exhibiting there a thematic connection (Bellhouse, 2011) with Abraham de Moivre (and maybe even Bayes!). The chapter concludes by a computation of the posterior (or predictive!) probability that $1-\varrho$ will remain larger than 1/2 in the next century, obtaining a value of 0.782.

Chapter VII is a short chapter on biased coins and compounded experiments, not directly related with Bayesian perspectives (Dale, 1999 extrapolates on this point, since the imprecision on the coin biasedness can be seen as a prior). Chapter VIII is similarly short, reproducing earlier normal approximations on averages of life durations. It also contains an interesting study on the impact of removing the impact of smallpox on the death rate. Chapter IX deals with expectations of simple functions for binomial experiments and with their normal approximation, again exhibiting the above link with de Moivre's on life insurances.

Chapter X returns to the notion of moral expectation mentioned both earlier and in Laplace's *Essai Philosophique*. The core (to solving the Saint Petersburg paradox) is to use $\log(x)$ instead of $x$ as a utility function, following Bernoulli's derivation (now mentioned on page 439).

## 3 Reflections

> *"In reviewing the general design of the work of Laplace, we desire to make the description of a book mark the present state of a science."* A. de Morgan, *Dublin Review,* 1837.

In conclusion, *Théorie Analytique des Probabilités* provides a fascinating historical perspective on Laplace's genius in framing probability and statistics within mathematical analysis and in deriving numerical approximations to intractable integrals. As put by Augustus de Morgan in a praising if sometimes hilarious review of the book, "Théorie des Probabilités is the Mont Blanc of mathematical analysis". (Morgan considers that the French national school of mathematics neglects to credit predecessors. It is quite true that it is impossible to gather which results are original and which are not in *Théorie Analytique des Probabilités.* He similarly thinks that the first part on generating functions is mostly useless for the second part. And that the introduction [in the 1814 edition] is the *Essai Philosophique*, whose final version is much enlarged compared with this introduction. Interestingly, de Morgan also spends quite some time on the notion of moral expectation.) As opposed

[1] As pointed out by Jean-Louis Foulley (personnal communication), this idea of representing the non-uniform prior as an additional set of data independent of the observation is very innovative. In modern Bayesian statistics language, it leads to easy and useful interpretations for conjugate priors and may even be viewed as the basic idea behind partial (intrinsic and fractional) Bayes Factors.

to Thomas Bayes' 1763 short essay,[2] the book by Laplace leads to a global vision of the role and practice of probability theory, as it was then understood at the beginning of the 19th Century, and it can be argued the Théorie Analytique des Probabilités shaped the field (or fields) for close to a hundred years.[3]

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[2]Dale (1999) compares Bayes' and Laplace's input, making the significant remark that Bayes considers "a single urn" while "Laplace entertained the idea of a *population* of urns" (p.277). This is a very powerful distinction, in that it highlights how closer Laplace was from the notion of prior distribution.

[3]It thus came as a surprise to read that Laplace was so much scorned and despised by the statisticians of the mid-1800's and even far into the 1900's, see McGrayne (2011).

# The Legacy of the Théorie Analytique des Probabilités

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*"It was necessary therefore that the centenary of the death of Newton was marked by the end of one of his most illustrious successors, of the one that England and France have so often named the French Newton, so as to express at the same time the glory of the two nations!"* S. D. Poisson, *Discours prononcé aux obsèques de M. le marquis de Laplace.*[1]

Isaac Newton died 20 March 1727; Pierre-Simon Laplace died 5 March 1827. This year marks the 200th anniversary of the publication of the *Théorie Analytique des Probabilités* of Pierre Laplace (1749–1827). A work, one of the most famous in western mathematics, one which was quite influential, yet one which was little read, if at all. Nonetheless, it defined the theory of probability—including what we today would call methods of statistical inference—as well as the content of textbooks for decades to come.

## Introduction

The first edition in two books appeared in 1812. One part was issued 23 March, another 29 June. A second edition, published in 1814, is distinguished by the inclusion of a version of the famous *Essai Philosophique sur les Probabilités* [31] and the cancellation and substitution of seven pages of the first due to errata. This was followed in 1820 by a third edition which included three supplements published in 1816, 1818 and 1820 respectively. In 1825 it was published again with a fourth supplement likely written by his son. The third edition is that which appears in the seventh volumes of *Oeuvres de Laplace* (1847) and *Oeuvres Complète de Laplace* (1886) [32]. [2]

On account of the difficulty of the work, several mathematicians, including Sylvestre La Croix in France (1816), Augustus de Morgan in England (1838), and Mikhail Buniakovsky in Russia (1846) created their own more accessible treatments.

The literature concerning Laplace and the *Théorie Analytique des Probabilités* (henceforth denoted as TAP) is vast. It is neither possible to do justice to him nor to that work in only one short survey. We can do no better than to sketch in rather broad strokes the most important contributions of Laplace to the theory of probability and statistics.

## The *Théorie Analytique des Probabilités*

It is unreasonable to study TAP in isolation as it is the synthesis of nearly 40 years of research. During the period from 1774 to 1784, there were published nine memoirs written by Laplace on the subject of probability. After devoting the next period of his life to writing the *Mécanique Céleste*, he resumed work again in 1809 and produced three very important papers before the publication of TAP. Laplace had two goals in mind: (1) to unite under the theory of generating functions all the analytical techniques used previously and (2) to apply to all the known problems concerning probabilities this one method.

The work is very much a compilation of these memoirs. This is especially true of Book I, itself substantially a reprint, but with some modifications, of his two memoirs, "Mémoire sur les suites" [24] and "Mémoire sur les approximations des formules qui sont fonctions de très grands nombres" [25] published in 1782 and 1785 respectively. Yet it has a kind of unity in that it is the first work of its kind to treat the theory of probability as a whole. Laplace applies the theory to demographics, interpolation, analysis of tribunals and the credibility of witnesses.

Let us first consider the outline of the work.

**Book I** Concerning the calculus of generating functions.

**First Part** General considerations on the elements of magnitudes.

**Chapter I** Concerning generating functions of one variable.

[1] All translations are by this author.
[2] See `cerebro.xu.edu/math/Sources/Laplace` for a provisional English translation of TAP Book II.

**Chapter II** Concerning generating functions of two variables.

**Second Part** Theory of the approximations of formulas which are functions of large numbers.

**Chapter I** Concerning the integration by approximation of the differentials which contain some factors raised to high powers.

**Chapter II** Concerning the integration by approximation of linear equations in finite and infinitely small differences.

**Chapter III** Application of the preceding methods to the approximation of diverse function of very large numbers.

**Book II** General Theory of Probabilities

**Chapter I.** General principles of this theory.

**Chapter II.** Concerning the probability of events composed of simple events of which the respective probabilities are given.

**Chapter III.** Concerning the laws of probability, which result from the indefinite multiplication of events.

**Chapter IV.** Concerning the probability of the errors of the mean results of a great number of observations, and of the most advantageous mean results.

**Chapter V.** Application of the Calculus of Probabilities, to the research on phenomena and of their causes.

**Chapter VI.** Concerning the probability of causes and of future events, drawn from observed events.

**Chapter VII.** Concerning the influence of unknown inequalities which can exist among the chances that one supposes perfectly equal.

**Chapter VIII.** Concerning the mean duration of life, of marriages and of any associations.

**Chapter IX.** Concerning benefits depending on the probability of future events.

**Chapter X.** Concerning moral expectation.

**Chapter XI.** Concerning the probability of witnesses. (Added to 1814 edition.)

We quote the introduction of the first edition [31] in full:

*"I myself propose to expose in this work, the analysis and the principles necessary in order to resolve the problems concerning probabilities. This analysis is composed of two theories that I have given, thirty years ago, in the Mémoires de l'Académie des Sciences. One of them is* **the Theory of generating Functions***; the other is* **the Theory of the approximation of Formulas functions of very great numbers***. They are the object of the first Book, in which I present them in a manner yet more general than in the Memoirs cited. Their union shows evidently, that the second is only an extension of the first, and that they are able to be considered as two branches of one same calculus, that I designate by the name of* **Calculus of generating Functions***. This calculus is the foundation of my* **Théorie des Probabilités***, which is the object of my second Book. The questions relative to events due to chance, amount most often with facility, to some linear equations in simple or partial differences: the first branch of the calculus of generating functions gives the most general method to integrate this kind of equations. But when the events that we consider, are in great number, the expressions to which we are led, are composed of a so great multitude of terms and factors, that their numerical calculation becomes impractical; it is therefore then indispensable to have a method which transforms them into convergent series. It is this that the second branch of the Calculus of generating Functions does with so much more advantage, as the method becomes more necessary.*

*"My object being to present here the methods and the general results of the theory of probabilities, I treat especially the most delicate questions, the most difficult, and at the same time the most useful of this theory. I apply myself especially, to determine the probability of the causes and of the results indicated by the events considered in great number, and to seek the laws according to which that probability approaches its limits, in measure as the events are multiplied. This research merits the attention of the Geometers, by the analysis that it requires: it is there principally that the theory of approximation of the formulas functions of large numbers, finds its most important applications. This research interests observers, by indicating to them the means that they must choose among the results of their observations, and the probability of the errors that they have yet to fear. Finally, it merits the attention of the philosophers, by showing how the regularity completes by being established in the same things which appear to us entirely delivered by chance, and by revealing the hidden, but constant causes, on which this regularity depends. It is on this regularity of the mean results of the events considered in great number, that diverse establishments repose, such as life annuities, tontines, assurances, etc. The questions which are related to them, such as inoculation of vaccine, and to the decisions of electoral assemblies, offer no difficulty according to my theory. I limit myself here to resolve the most general; but the importance of these objects in civil life, the moral considerations of which they complicate themselves, and the numerous observations that they suppose, require a work apart.*

*"If one considers the analytical methods to which the theory of probabilities has already given birth, and those that it is able to yet give birth; the justice of the principles which serve as foundation to it, the rigorous and delicate logic that their use requires in the solution of the problems; the establishments of public utility which depend on it: if one observes next that in the same things which are not able to be submitted to the*

> *calculation, this theory gives the most certain outline which is able to guide us in our judgments, and that it teaches to guard against illusions which often mislead us; we will see that there is no science more worthy of our meditations, and of which the results are more useful. It owes birth to two French Geometers of the seventeenth century, so fecund in great men and in great discoveries, and perhaps of all the centuries the one which gives most honor to the human spirit. Pascal and Fermat proposed and resolved some problems on probabilities. Huygens united these solutions, and extended them in a small treatise on this matter, which next had been considered in a more general manner by Bernoulli, Montmort, Moivre, and by many celebrated Geometers of these last times."*

## Essai Philosophique sur les Probabilités

The *Essai Philosophique sur les Probabilités* is the most enduring piece. It is an expansion of the tenth lecture given by Laplace at the *École Polytechnique* during the year 1795. The essay summarizes the mathematical content in language better suited to the non-mathematical reader. Not always successful, nonetheless Laplace discussed essentially all applications of probability made to that time. First printed in February 1814 and included with the *Théorie Analytique des Probabilités* since its second edition, the essay itself has gone through several editions: a second also in 1814, a third in 1816, a fourth in 1819, a fifth in 1825 and a sixth in 1840. There are significant differences among these. The table of contents of the *Essai* presented in the 1840 edition is as follows:

- Philosophical Essay on Probabilities
  - Concerning probability.
  - General Principles of the Calculus of Probabilities.
  - Concerning expectation.
  - Concerning analytic methods of the Calculus of Probabilities.
- Application of the Calculus of Probabilities.
  - Concerning games.
  - Concerning unknown inequalities that can exist among the chances that one supposes equal.
  - Concerning the laws of probability, which result from the indefinite multiplication of events.
  - Application of the Calculus of Probabilities to natural philosophy.
  - Application of the Calculus of Probabilities to the moral sciences.
  - Concerning the probability of witnesses.
  - Concerning the choices and decisions of assemblies.
  - Concerning the probability of judgments of tribunals.
  - Concerning Tables of mortality and of the mean durations of life, of marriages, and of unspecified associations.
  - Concerning benefits of the establishments which depend on the probability of events.
  - Concerning illusions in the estimation of probabilities.
  - Concerning diverse means to approach certainty.
  - Historical notice on the Calculus of Probabilities.

Shortly after its first publication, we have a German translation by Friederich Tönnies, *Philosophicher Versuch über Wahrscheinlichkeiten* (1819). Others were made by Norbert Schwaiger *Philosophischer Versuch über die Wahrscheinlichkeit* (1886), H. Löwy *Philosophischer Versuch über die Wahrscheinlichkeit* (1932), by Alfredo B. Besio and José Banfi, *Ensayo Filosófico sobre las Probabilidades* (1947), by S. Oliva, *Saggio Filosofico sulle Probabilitatà*, and by A.I. Dale, *Pierre-Simon Laplace. Philosophical Essay on Probabilities* (1995). A Russian translation appeared in 1908.

Laplace took a deterministic view of reality:

> *"We must therefore envision the present state of the universe as the effect of its previous state, and as the cause of the one which follows. An intelligence which, for a given instant, knew all the forces of which nature is animated, and the respective situation of the beings which compose it, if moreover it was vast enough to submit these data to analysis, would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom: nothing would be uncertain for it, and the future as the past would be present to its eyes."* [18], pages vi–vii.

He believed that the theory of probabilities could be a method of discovery. In the section of the essay concerning natural philosophy, he remarks:

> *"All the time therefore that we see that a cause of which the march is regular, can influence on a kind of events; we can seek to recognize its influence by multiplying the observations; and, when this influence appears to manifest itself, the analysis of probabilities determines the probability of its existence and that of its intensity."* [18], page lxxxiii.

Near the end of this same section, he proposes an experimental design:

> *"The calculus of probabilities can make us appreciate the advantages and disadvantages of the methods employed in the conjectural sciences. Thus, in order to recognize the better treatments in use in the cure of a disease, it suffices to test each of them on a like number of patients, by rendering all the circumstances similar; the superiority of the most advantageous treatment will be manifest more and more, in measure as this number will increase, and the calculation will make known the corresponding probability of its advantage and of the ratio according to which it is superior to the others."* [18], page lxxxv.

Nor is use limited to natural science. Introducing the next section, he remarks

> *"We just saw the advantages that the analysis of probabilities offers, in the research of the laws of natural phenomena of which the causes are unknown, or too complicated in order that their effects be able to be submitted to calculation. This is the case of nearly all the objects of the moral sciences."* [18], page lxxxvi.

# Reviews of TAP

Fellow countrymen praise TAP. A review in the *Connaissance des Temps pour l'année* 1815 (1812) states

> *"The work that we announce contains all that which has been done of importance on this branch of human knowledge, that the author appears to us to have perfected, either by the generality of his analysis, or by the novelty and the difficulty of the problems that he has resolved."* page 217.

Poisson, writing a review of TAP in the *Nouveau Bulletin des Sciences, par la Societé Philomatique* (1812), says

> *"Mr. Laplace has united in this work, the memoirs that he has published elsewhere on probabilities, and the two memoirs that he has given lately on the same subject...*
>
> *There results from it a complete Treatise on the theory of chances, in which one will find uniform and general methods to resolve the questions relative to the theory, and the application of these methods to the most important problems."* pages 160–1.

In the Eloge of Laplace composed by Baron Fourier [9], we have

> *"Laplace has united and fixed the principles of [the analysis of probabilities]. In his hands it has become a new science, submitted to a single analytical method, and of prodigious extent. Fertile in useful applications, it will one day throw a brilliant light over all the branches of natural philosophy."* page 376.

Augustus de Morgan reviewed the third edition of TAP in the *Dublin Review* 2 (1836) and 3 (1837). This edition is substantially the same as the first with the exception of the *Essai* and a small amount of additional material in Book II. We begin with selections of the first half of the review.

> *"The* ***Théorie des Probabilités*** *is the Mont Blanc of mathematical analysis; but the mountain has this advantage over the book, that there are guides always ready near the former, whereas the student has been left to his own method of encountering the latter.*
>
> *The genius of Laplace was a perfect sledge hammer in bursting purely mathematical obstacles; but, like that useful instrument, it gave neither finish nor beauty to the results."* pages 347–8.

Regarding the accessibility of the work, we have

> *"The subject of the work is, in its highest parts, comparatively isolated and detached, though admitted to be of great importance in the sciences of observation. The pure theorist has no immediate occasion for the results, as results, and therefore contents himself in many instances with a glance at the processes, sufficient for admiration, though hardly so for use. The practical observer and experimenter obtains a knowledge of results and nothing more, well knowing in most cases, that the analysis is above his reach. We could number upon the finders of one hand, all the men we know* ***in Europe*** *who have* ***used*** *the results in their* ***published*** *writings in a manner which makes it clear that they could both* ***use*** *and* ***demonstrate****."* page 350.

Noting that TAP is a collection of research papers, de Morgan says

> *"Here the reader may begin to suspect that the difficulty of this work does not lie entirely in the subject, but is to be attributed in great part to the author's method. That such difficulty is in part wholesome, may be very true; but it is also discouraging unless the student be distinctly informed upon its cause and character."* page 354.

Indeed, in the preface to his *Essay on Probabilities*, [4] de Morgan wrote:

> *"Laplace, armed with the mathematical aid given by De Moivre, Stirling, Euler, and others, and being in possession of the inverse principle already mentioned, succeeded both in the application of this theory to more useful species of questions, and in so far reducing the difficulties of calculation that very complicated problems may be put, as to method of solution, within the reach of an ordinary arithmetician. His contribution to the science was a general method (the analytical beauty and power of which would alone be sufficient to give him a high rank among mathematicians) for the solution of all questions in the theory of chances which would otherwise require large numbers of operations."* pages vii and viii.

Later, in the article on the "Theory of Probabilities" contributed to the *Encyclopedia Metropolitana* [5], he writes in a footnote to §52:

> *"His* **Théorie des Probabilités** *is by very much the most difficult mathematical work we have ever met with. . . "*

Herschell, in his review of Quetelet's *Lettres sur la Théorie des Probabilités* [16], notes

> *"In all these respects the great work of Laplace ('Théorie Analytique des Probabilités') stands deservedly preeminent; occupying in this department of science the same rank and position which the 'Méchanique Analytique' of his illustrious rival Lagrange holds in that of force and motion, and marking (we had almost said) the* **ne plus ultra** *of mathematical skill and power. So completely has this sublime work been held to embody the subject in its utmost extent, and to satisfy every want of the theorist, that an interval of a quarter century elapsed before from the date of its appearance (1812) before any further original contribution*[3] *of moment was made to the theory. . .*
>
> *"It may easily be imagined that a work like this of Laplace, followed at a short interval by an admirable* **exposé** *of its contents by himself ('Essai Philosophique sur les Prob.'), could not fail to make a lively impression and to excite general attention."*

De Morgan's criticisms are quite justified. In 1873, Laurent writes in the preface to his textbook *Traité du Calcul des Probabilités* [33]

> *"Persons who desire to study the Calculus of Probabilities generally experience some difficulties which hold less to the nature of the subject than to the absence of really classic Treatises. And, in fact, in order to approach the celebrated* **Théorie analytique des probabilités** *of Laplace, it is already necessary to be, to a certain point, familiarized with the analysis of chances, the author treating, as he himself swears, only the most difficult questions; . . . "*

Similarly, Bertrand writes in the preface of his own *Calcul des Probabilités* (1889), [1]

> *"The Calculus of probabilities is one of the most attractive of the mathematical Sciences and however one of the most neglected. The beautiful book of Laplace is perhaps one of the causes. Two opinions, in fact, are formed, without encountering scarcely opponents: one is able to understand well the Calculus of probabilities without having read the book of Laplace; one is not able to read the book of Laplace without being prepared by the deepest mathematical studies."*

## Contributions of Laplace

In Chapter I of the Second Part of Book I are found series expansions of the following integrals, developed previously in the memoir published in 1785,

$$\int_0^x e^{-u^2} du \text{ and } \int_x^\infty e^{-u^2} du,$$

these being useful in the computation of probabilities.

We noted previously that the second book of TAP concerns the development of tools and their application. Laplace can be credited with several important contributions to the fields of probability and mathematical statistics. These are

[3]Poisson's *Recherches sur la probabilité des jugements.*

1. A theory of inverse probability,
2. A central limit theorem,
3. Justification of the method of least squares,
4. Origin of mathematical statistics.

We take up each in order.

## Inverse Probability

A detailed study of Laplace's theory of inverse probability may be found in Dale [3]. In the 1774 memoir "Mémoire sur la probabilité des causes par les événements," [21], Laplace notes that questions of probability are of two types: Direct (the cause is known, but event uncertain) and Inverse (the event is known, but the cause is uncertain). He presents his version of the principle of inverse probability as

> Principle.—*If an event [E] is able to be produced by a number n of different causes [$H_i$], the probabilities of the existence of these causes taken from the event are between them as the probabilities of the event taken from the causes, and the probability of the existence of each of them is equal to the probability of the event taken from that cause, divided by the sum of all the probabilities of the event taken from each of these causes.*

We may express this statement more succinctly as

$$\frac{P(H_i|E)}{P(H_j|E)} = \frac{P(E|H_i)}{P(E|H_j)}, \qquad i,j = 1,2,\dots n, \quad i \neq j$$

and

$$P(H_i|E) = \frac{P(E|H_i)}{\sum_{j=1}^{n} P(E|H_j)}, \qquad i = 1,2,\dots n$$

where it is clear that Laplace is treating all causes as equiprobable.

Laplace was apparently unaware of Bayes' paper. Discrete versions of what we now call "Bayes' Theorem" are demonstrated in his papers "Mémoir sur les probabilités," [23] and "Mémoire sur les Approximations des Formules qui sont fonctions de très grands nombres (Suite)," [26]. Throughout his work, Laplace uses the method of inverse probability to solve problems while typically, but not exclusively, assuming a uniform prior distribution. It is certainly worthwhile noting that the first methods of statistical inference developed are based on inverse probability and not frequentist methods.

To Laplace, this concept had broad applicability. The applications of inverse probability to the social sciences, particularly the study of witnesses and judgments, were very important to him. His analyses served to demonstrate that claims of extraordinary facts must weaken testimony. In the case of tribunals, the size and the majority required for condemnation of a defendant are considered. Here he compares the probabilities of making an error of judgment under different compositions of them. Indeed, this matter was developed further by Poisson. However, this application was quite controversial and often omitted from later texts on the theory of probability.

Problems with the use of inverse probability were noted by Ellis [6], Boole [2] and Venn [44]. Laplace's *Rule of Succession*, as it was named by Venn, was often the focus of criticism. This rule asserts that if an event has occurred $n$ times in succession, the probability that it will recur is $\frac{n+1}{n+2}$. Its derivation is as follows. Let $X_i$, $i = 1,2,\dots$ be i.i.d. Bernoulli random variables with common probability of success $p$ and let $Y = \sum_{i=1}^{n} X_i$. It follows from Laplace's theory of inverse probability that

$$P(X_{n+1} = 1|Y = r) = \frac{\int_0^1 p^{r+1}(1-p)^{n-r}dp}{\int_0^1 p^{r}(1-p)^{n-r}dp} = \frac{r+1}{n+2}$$

Let us note that Bayesian methods continued to be included in textbooks on probability until the 1920s. Even so, Zabell [45] remarks that few used the technique in practice. Ultimately, criticisms by Jerzy Neyman and Ronald Fisher in the 1920s caused the theory of inverse probability to fall from favor until revived several decades later.

## The Central Limit Theorem

Laplace's first study related to what will become the central limit theorem appears in his "Mémoir sur les approximations des formules qui sont fonctions de très grands nombres" [25], published in 1785. Here he introduces a simple form of the characteristic function and inversion formula.

He returns to the study in 1810 in "Mémoir sur les approximations des formules qui sont fonctions de très grands nombres et sur leur application aux probabilités." , where he analyses the mean inclination of comets. Next, most likely motivated by Gauss's work on least squares, he returns to it in "Supplement au Mémoir sur les approximations des formules qui sont fonctions de très grands nombres" also published in 1810, [28,29]. Another version next appears in "Mémoire sur les intégrales définies et leur application aux probabilités, et spécialment à la recherche du milieu qu'il faut choisir entre les résultats des observations," [30]. Through the use of the characteristic function, these papers generalize the 1785 proof and extend it to the case where the random variables have arbitrary distribution with compact support.

Chapter 4 of TAP contains the exposition of these results. However, at no place does Laplace demonstrate a general theorem. In fact, what he does do is repeat arguments successively under more and more general conditions. In more modern notation adapted from Fischer [8], Laplace shows that if the random variables $X_i$ are i.i.d. with mean $\mu$ and variance $\sigma^2$, $w_i$ a series of weights and $a$ a constant, then

$$P\left(|\Sigma w_i(X_i - \mu)| \leq a\sqrt{\Sigma w_i}\right) \approx \frac{2}{\sigma\sqrt{\pi}}\int_0^a e^{-\frac{x^2}{2\sigma^2}}dx.$$

Laplace's demonstrations are cumbersome. Todhunter does not bother to give them, but instead presents the simplification of the exposition due to Poisson. See Hald [14] for an explanation of Laplace and see Fischer [8] for the subsequent history of the Central Limit Theorem.

## Least Squares

Legendre was the first to publish the method of least squares in 1806. Three years later, Gauss published his *Theoria motus corporum celestium in sectionibus conicis solem ambientium* [10] in which he showed that if the arithmetic mean is the most probable value of an unknown, then the probability is maximized when the distribution of errors in normal. Conversely, if the errors are normally distributed, the least squares estimates of coefficients are the most probable values.

Laplace realized that by his theorem, the distribution of the mean for large samples is approximately Gaussian. He first gave in 1810 a Bayesian demonstration that estimates can be improved by the method of least squares under the assumption of a large number of observations with the same error law. He showed that the least squares estimate minimizes the posterior error. In 1811, he followed this with a non-Bayesian argument in which he finds the limiting distribution of a weighted sum of observed errors.

The results of Laplace's studies are presented in sections 20–24 of Chapter IV of Book II of TAP. He closes that chapter with the following remarks:

> *"When we have only one element to determine, this method leaves no difficulty; but, when we must correct at the same time many elements, it is necessary to have as many final equations formed by the union of many equations of condition, and by means of which we determine by elimination the corrections of the elements. But what is the most advantageous manner to combine the equations of condition, in order to form the final equations? It is here that the observers abandoned themselves to some arbitrary gropings, which must lead them to some different results, although deduced from the same observations. In order to avoid these gropings, Mr. Legendre had the simple idea to consider the sum of the squares of the errors of the observations, and to render it a* **minimum**, *that which furnishes directly as many final equations as there are elements to correct. This scholarly geometer is the first who has published this method; but we owe to Mr. Gauss the fairness to observe that he had had, many years before this publication, the same idea of which he made a habitual usage, and that he had communicated to many astronomers. Mr. Gauss, in his* **Theory of Elliptic Movement**, *has sought to connect this method to the Theory of Probabilities, by showing that the same law of errors of the observations, which give generally the rule of the arithmetic mean among many observations, admitted by the observers, gives similarly the rule of the least squares of the errors of the observations, and it is this which we have seen in n° 23. But, as nothing proves that the first of these rules gives the most advantageous result, the same uncertainty exists with respect to the second. Research on the most advantageous manner to form the final equations is without doubt one of the most useful of the Theory of Probabilities: its importance in physics and astronomy moves me to occupy myself with it. For this, I will consider that all the ways to combine the equations of condition, in order to form a final linear equation, returned to multiplying them respectively by some factors which were null relative to the equations that we did not employ, and to make a sum of all these products, this which gives a first final equation. A second system of factors gives a second final equation, and thus consecutively, until one has as many final equations as elements to correct. Now it is clear that it is necessary to choose the system of factors, such that the mean error to fear to more or to less respecting each element is a* **minimum**; *the mean error being the sum of the products of each error by its probability. When the observations are in small number, the choice of these systems depends on the law of errors of each observation. But, if one considers a great number of observations, that which holds most often in astronomical researches, this choice becomes independent of this law, and we have seen, in that which precedes, that Analysis leads then directly to the results of the method of least squares of the errors of the observations. Thus this method which offered first only the advantage to*

> *furnish, without groping, the final equations necessary to the correction of the elements, gives at the same time the most precise corrections, at least when we wish to employ only final equations which are linear, an indispensable condition, when one considers at the same time a great number of observations; otherwise, the elimination of the unknowns and their determination would be impractical."*

We note, moreover, if an observed error is itself an accumulation of independent small errors—what became known as the "hypothesis of elementary errors"—it would have approximately a normal distribution as a consequence of the Central Limit Theorem. The method of least squares would then hold for small samples as well.

Laplace continued work with least squares in the first three supplements to TAP in which he applied the method to geodesy in particular. He is now able to compute an estimate of the precision of the last quantity estimated by the method. Gauss followed his work with another paper in 1823 [11] in which he gave his second proof of the method. Neither the work of Laplace nor of Gauss settled the matter as the repeated attempts to give a "proof" of the method of least squares throughout the nineteenth century attest. It is perhaps appropriate here to note that a substantial statistical literature produced during the nineteenth century concerns the application of the method of least squares and the theory of errors. Merriman [34] constructed a non-exhaustive list of 408 memoirs, books and parts of books concerning the method of least squares and the theory of errors of which 386 were published in or after the year 1805 to 1874. See also Harter [15] for a summary of the early development of the method of least squares and an extension of the list. We mention also that Bienaymé and Cauchy engaged in a vigorous debate in 1853 regarding the method. Bienaymé defended Laplace. Cauchy, arguing for his method of interpolation, showed that the method failed under the distribution named for him as the law of error while noting the method of least squares was only appropriate when the law was normal.

## Mathematical Statistics

Laplace [19] conducts a test of significance using a direct probability calculation in his 1776 paper on comets.

> *"I suppose an indefinite number of bodies launched at random into space and circulating about the Sun; the question is to find the probability that the mean inclination of their orbits on a given plane, such as the ecliptic, will be contained between two given limits, as* $40°$ *and* $50°$*."*

Suppose $\bar{X}$ is the mean inclination and $\alpha$ is an arbitrary angle. He first argues that if $P(\bar{X} < 45 + \alpha)$ is large, then there is evidence the comets tend to lie in the same plane. Laplace then proceeds to estimate the probability using data of the 12 most recently observed comets.

Laplace is responsible for the modern theory of testing statistical hypotheses. These are the large sample tests based upon the normal approximation to the actual distribution. While developed according to the method of inverse probability, Laplace seems to have intuited that the posterior distribution is largely independent of the prior distribution and therefore direct probability will yield quite similar results.

In the 1786 paper [26] Laplace investigates the proportion of male births and comparison of the proportion of males births in London to those in Paris.

In Section 39, he writes

> *"we suppose that, out of* $p+q$ *observed births, there have been* $p$ *boys and* $q$ *girls,* $p$ *being greater than* $q$*, and we seek the probability that the possibility of the births of the boys not surpass any quantity* $\theta$*".*

This is solved by the method of inverse probability using a uniform prior. Laplace finds

> *"that we can regard as certain that the excess of the births of the boys over those of the girls, observed at Paris, is due to a greater possibility in the births of the boys."*

That is, for $\theta$ the true proportion of male births, he compares $P(\theta > \frac{1}{2})$ to $P(\theta \leq \frac{1}{2})$. With the number of boys $p = 251527$ and the number of girls $q = 241945$, he computes an estimate of

$$P(\theta > \tfrac{1}{2}) = \frac{\int_{1/2}^{1} x^p (1-x)^q dx}{\int_0^1 x^p (1-x)^q dx}$$

and finds $P(\theta > \frac{1}{2}) = 1 - \epsilon$ where $\epsilon$ is exceedingly small.

Another test using uniform priors is conducted in Section 40.

> *"We have seen, in the preceding section, that the ratio of the births of boys to that of girls is around* $\frac{19}{18}$ *at London, while it is at Paris around* $\frac{26}{25}$ *; this difference seems to indicate, in the first city, a possibility in the births of boys greater than in the second city. We determine with what likelihood the observations indicate this result."*

Laplace compares $P(p_1 > p_2)$ to $P(p_1 \leq p_2)$ where $p_1$, $p_2$ are the probabilities a boy is born in London and Paris respectively. He comes to the conclusion:

> *"and that thus there are odds of more than 400000 against 1 that there exists at London a cause more than at Paris, which facilitates the births of boys."*

This material is also included in Chapter 6 of TAP sections 28 and 29.

## Conclusion

Direct probability large sample theory is Laplace's main contribution from 1811 to 1827. By 1812 he no longer used the method of inverse probability for fitting functions to data. The asymptotic equivalence of results found by direct probability and by inverse probability show that either may be used as convenient. Farebrother [7] credits Laplace with anticipating the Gauss-Markov Theorem in the second supplement published in 1818.

For a comprehensive biography of Laplace, one should consult Gillespie's *Pierre-Simon Laplace*, 1749–1827 [12].

For a summary of the content of TAP and his papers related to probability, there is Todhunter's *History of the Theory of Probability* [43]. Hald in his *A History of Mathematical Statistics from 1750 to 1930* [14] gives a comprehensive treatment of Laplace's theory of statistical inference.

Of other more recent works, one may consult Stigler's early paper entitled "Napoleonic Statistics: The Work of Laplace" [41], his *The History of Statistics* [42] and his chapter on TAP in *Landmark Writings in Western Mathematics* 1640–1940 [13]. See also Dale's *A History of Inverse Probability from Thomas Bayes to Karl Pearson* [3], and Fischer's *A History of the Central Limit Theorem* [8]. Karl Pearson's lecture notes [37] as well as the separate article "Laplace" [36] also may be read with profit.

Among the journal literature, we must note Molina's "The Theory of Probability: Some Comments on Laplace's Théorie Analytique" [35], three comprehensive papers by Sheynin: "Finite Random Sums [38], "P. S. Laplace's Work on Probability" [39] and "Laplace's Theory of Errors" [40] and Schneider's "Laplace and thereafter: The status of the probability calculus in the nineteenth century" [17].

We close with the last paragraph of the *Essai* (1840), [18]:

> *"We see by this Essay, that the theory of probabilities is, at base, only good sense reduced to calculus: it makes us estimate with exactitude that which the right-minded sense by a sort of instinct, without them often being able to render account of it. It leaves nothing arbitrary in the choice of opinions and of the decisions to take, all the time that one can, by its means, determine the most advantageous choice. Thence, it becomes the happiest supplement to ignorance and to the weakness of the human spirit. If one considers the analytic methods to which this theory has given birth, the truth of its principles which serve as foundation to it, the fine and delicate logic that require their use in the solution of problems, the establishments of public utility that are supported on it, and the extension that it has received and that it can receive yet, by its application to the most important questions of natural philosophy and moral sciences; if one observes next that in the same things that cannot be submitted to the calculus, it gives the most certain outlines which can guide us in our judgments, and that it teaches to be guarded from the illusions that often mislead us, we see that there is no science more worthy of our meditations, and that it is more useful to make it enter into the system of public instruction."*

## Acknowledgment

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# XI EBEB

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No período de 18 a 22 de Março de 2012 ocorreu o 11º Encontro Brasileiro de Estatística Bayesiana - XI EBEB. O evento é organizado pelo capítulo brasileiro da *International Society for Bayesian Analysis* (ISBrA) e foi realizado no paradisíaco Canto da Floresta Hotel Resort, localizado na cidade de Amparo, no interior paulista.

O comitê organizador foi composto por Julio Stern (IME-USP), Adriano Polpo (DEs-UFSCar), Marcelo Lauretto (EACH-USP), Carlos Alberto de Bragança Pereira (IME-USP) e Márcio Alves Diniz (DEs-UFSCar). Além da ISBrA, o evento contou com auxílio financeiro do CNPq, CAPES, FAPESP, ABE, INCTMat (Instituto Nacional de Ciência e Tecnologia de Matemática), ABJur (Associação Brasileira de Jurimetria) e dos programas de pós-graduação do IME-USP e DEs-UFSCar. Os amplos recursos obtidos permitiram que todos os autores de trabalhos que solicitaram ajuda de custo fossem contemplados com algum auxílio.

Nessa edição, o evento contou com 14 palestrantes convidados e cerca de 80 participantes regulares, incluindo professores, pesquisadores e cerca de 30 estudantes de graduação e pós-graduação. O evento teve um total de 68 trabalhos apresentados, sendo 28 apresentações orais e 42 apresentações pôster, divididas em duas sessões.

Os conferencistas internacionais convidados foram André Rogatko (*Samuel Oschin Comprehensive Cancer Institute*), Ariel Caticha (*The State University of New York, Albany*), Dalia Chakrabarty (*University of Warwick*), Debajyoti Sinha (*Florida State University*), Frank Lad (*University of Canterbury*, Nova Zelândia), Joseph Kadane (*Carnegie Mellon University*), Luis Raul Pericchi Guerra (*University of Puerto Rico*), Marco Antonio Rosa Ferreira (*University of Missouri*), Marlos Viana (*University of Illinois*) e Sonia Petrone (*Universita Bocconi*), e os conferencistas nacionais foram Alexandra Schmidt (UFRJ), Nestor Caticha (Instituto de Física, USP), e Rosangela Loschi (UFMG).

É importante destacar também a valorosa contribuição das secretárias Sylvia Regina A. Takahashi (IME-USP), Lourdes Vaz da Silva Netto (IME-USP) e Elvira Cerniavskis que participaram de forma ímpar para a perfeita organização do evento.

Uma das maiores novidades desse EBEB é que, pela primeira vez, os trabalho serão publicados pela AIP (*The American Institute of Physics Conference Proceedings*), dando assim visibilidade internacional ao congresso e incentivando a participação de pesquisadores brasileiros e estrangeiros. Outra novidade bastante interessante foi o convite aos participantes na revisão (anônima) de dois artigos de outros participantes. Também, pela primeira vez os ministrantes de tutoriais foram convidados a escrever um livro. O livro foi escrito pelo Prof. Dr. Ariel Caticha e é entitulado *Entropic Inference and the Foundations of Physics.*

A noite de quarta-feira, além de um delicioso jantar de confraternização com música ao vivo, foi marcada por uma bela homenagem ao Prof. Dr. Heleno Bolfarine. Como é conhecido de nossa comunidade, Heleno ocupa a posição de professor titular no IME-USP e é autor de livros amplamente utilizados em cursos de estatística em todo o país, além de mais de 150 artigos em importantes periódicos da área de estatística.

Essa edição do EBEB teve como objetivos fortalecer a pesquisa em métodos Bayesianos, bem como ampliar sua aplicação na comunidade científica brasileira, proporcionar um ambiente no qual pesquisadores brasileiros e internacionais pudessem colaborar, apresentar seus mais recentes desenvolvimentos e discutir problemas em aberto. Também permitiu aos alunos de pós-graduação brasileiros ter contato com pesquisadores sêniors, tanto para discutir seus trabalhos como também para iniciar possíveis contatos para projetos futuros de doutorado e pós-doutorado e fortaleceu a interação da comunidade Estatística com outras comunidades científicas, como Jurimetria, Econometria, Física, Astronomia, Medicina, Engenharia e outras. Sob minha visão, todos os objetivos foram plenamente alcançados. A excelente organização aliada com o ambiente extramamente agradável do hotel permitiu ver pesquisadores internacionais de alto nível como Joseph ("Jay") Kadane ou Sonia Petrone interagindo com alunos nas mesas do hotel e até aulas sobre os *Teoremas de De Finetti* sendo ministradas na piscina do hotel por Frank Lad. Além disso, o entusiasmo da Associação Brasileira de Jurimetria, através de seus vários representantes presentes no evento, ou a marcante participação dos professores Nestor e Ariel Caticha (físicos) e André Rogatko (biólogo), são alguns exemplos do sucesso que a inferência bayesiana tem feito nas mais diversas áreas do conhecimento.

# Eventos

- **Bayes Lectures 2012**, Edimburgo – Escócia, 29 e 30 de agosto de 2012. (*http://conferences.inf.ed.ac.uk/bayeslectures/*)

Dando continuidade às palestras realizadas no ano passado em razão dos 250 anos da morte do Reverendo Thomas Bayes, as faculdades de Matemática e Informática da Universidade de Edimburgo, onde Bayes estudou entre 1719 e 1722, organizarão uma nova série de palestras sobre inferência Bayesiana.

Os palestrantes convidados para essa nova série são: M. J. Bayarri da *Universitat de València*, Espanha; Peter Grünwald do *Centrum voor Wiskunde en Informatica*, Holanda; Jesper Møller da *Aalborg University*, Dinamarca; e Aad van der Vaart da *Leiden University*, Holanda.

Além das palestras, haverá sessões de discussão e existe a possibilidade - desde que haja um bom número de trabalhos submetidos - de ocorrer uma sessão pôster no segundo dia do evento.

A participação no evento será possível apenas por convite. Para pedir um convite, os interessados devem enviar um email para *bayes.lectures.edinburgh@gmail.com* com informações básicas requeridas no sítio do evento, disponível acima.

- **2012 Applied Bayesian Statistics School - Stochastic Modelling for Systems Biology**, Pavia - Itália, 3 a 7 de setembro de 2012. (*www.mi.imati.cnr.it/conferences/abs12.html*)

Essa série de cursos tem ocorrido desde 2004 e já abordou diversos temas como: *machine learning* com aplicações em biomedicina, modelagem hierárquica aplicada à ecologia, expressão gênica, modelagem de decisão em assistência à saúde, dentre outros.

O objetivo é convidar especialistas dos temas escolhidos para apresentar as aplicações Bayesianas de fronteira. Em 2012 o tema escolhido foi a modelagem estocástica de sistemas biológicos e o palestrante convidado é Darren Wilkinson, da *Newcastle University*, Inglaterra.

Uma breve descrição do curso e do público-alvo podem ser encontrados no endereço do evento disponibilizado acima.

- **European Seminar of Bayesian Econometrics**, Viena - Áustria, 1º e 2 de novembro de 2012.
(*esobe2012.wu.ac.at)*

Esta série de seminários foi lançada em 2010 e tem por objetivo reunir pesquisadores e profissionais interessados em aplicações de inferência Bayesiana em economia, finanças, marketing e áreas correlatas. Também pretende servir como fórum de discussão sobre novos métodos capazes de enfrentar os desafios associados à aplicação da inferência Bayesiana aos modelos de crescente complexidade e aos conjuntos de dados de elevada dimensão.

Nessa edição serão discutidos, principalmente, trabalhos relacionados a: econometria financeira, microeconometria e avaliação de políticas públicas, métodos semi-paramétricos baseados em misturas infinitas, estimação *shrinkage* e seleção de variáveis em conjuntos de dados de elevada dimensão, computação em paralelo em modelos aplicados à economia e finanças e métodos MCMC eficientes.

- **Bayes on the Beach 2012**, Sunshine Coast – Austrália, 6 a 8 de novembro de 2012.
(*bragqut.wordpress.com/beachbayes2012/*)

Nesse encontro ocorrerão o 9º *Workshop* Internacional da seção da ISBA na região Australásia e o encontro anual da seção Bayesiana da Sociedade Australiana de Estatística.

A conferência vai incluir seminários, uma sessão pôster, tutoriais e *workshops*. Existe a possibilidade de se oferecerem mini-cursos, sobre temas ainda não definidos.

Entre os palestrantes convidados estão Robert Wolpert, da *Duke University*, EUA; e Matt Wand, da *University of Technology*, Austrália.

- **2012 The Alan Turing Year**
(*www.mathcomp.leeds.ac.uk/turing2012/*)

O ano de 2012 marca o centenário do nascimento de Alan Turing. Sua importância é amplamente reconhecida na ciência da computação. Porém, pouco se comenta sobre seus trabalhos em matemática e estatística. Isso pode ser devido ao teor secreto de seus resultados nessas duas últimas áreas, desenvolvidos principalmente durante a II Guerra Mundial para auxiliar os Aliados a quebrar os códigos da famigerada Enigma nazista.

Nesse período, tão bem retratado pela jornalista Sharon McGrayne em seu livro *The Theory that Would Not Die*[4], Turing trabalhou ao lado de I. J. Good e fez amplo uso de inferência bayesiana para quebrar os códigos nazistas. Em razão disso, nada mais justo que lembrar aqui a série de eventos promovidos em diversos países, inclusive no Brasil, lembrando a importância de suas idéias para diversas áreas do conhecimento. No endereço do sítio eletrônico dado acima, é possível encontrar *links* para vários eventos que ocorrerão no segundo semestre deste ano.

A Universidade Federal do Rio Grande do Sul promoverá o evento **Alan Turing Brasil 2012**. Entre 28 de agosto e 4 de janeiro de 2013 o museu da universidade realizará uma mostra em homenagem a Turing. A mostra destacará a contribuição de Alan Turing, tanto para a computação como para a humanidade, através do trabalho de decodificação da Enigma. Os visitantes terão acesso a painéis, móbiles, projeções de vídeos e atividades complementares como um ciclo de cinema com filmes biográficos. Uma atração especial será uma réplica da Enigma.

Também ocorrerá um ciclo de palestras com a participação de pesquisadores brasileiros e estrangeiros. Dentre os palestrantes convidados, estão: S. Barry Cooper, da *University of Leeds*, Inglaterra; Sue Black, da *University College*, Inglaterra e Luís da Cunha Lamb, do Instituto de Informática da UFRGS.

[4]Para mais detalhes a respeito da relação de Turing com a inferência Bayesiana, veja o próximo número deste boletim.