Teorema de STOKES:

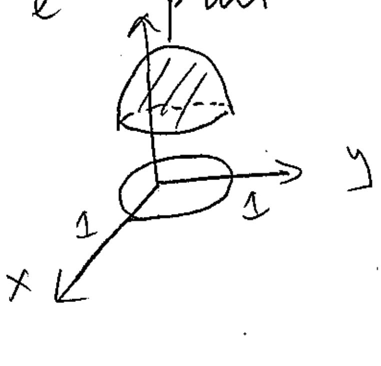
Se σ é uma superfície parametrizada C^2 e regular (isto é, $\frac{\partial \sigma}{\partial u} \wedge \frac{\partial \sigma}{\partial v} \neq \underline{0}$), com F um campo de vetores C^1, definido num aberto que contem $Im(\sigma) = S$, com $\sigma : K \to S$ e ∂K uma curva $\gamma(t)$, e $\Gamma = \sigma \circ \gamma$ a fronteira $\partial(S)$, então

$$\int_\Gamma F \cdot d\Gamma = \iint_\sigma rot.F \cdot \underline{n}\, dS.$$

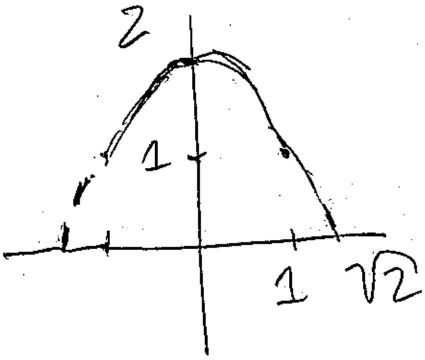
<u>Exemplo</u>: $K = \{(u,v) : u^2 + v^2 \leq 1\} \subseteq \mathbb{R}^2$

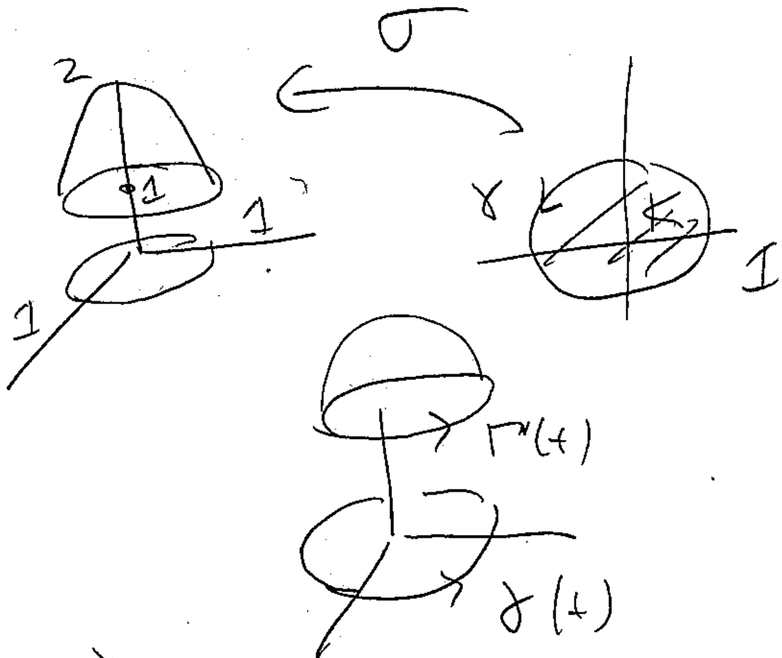
$\sigma(u,v) = (x, y, z) = (u, v, 2 - u^2 - v^2)$

S é parte de uma paraboloide:



(o exemplo #1,3 do livro) ②





$$\gamma(t) = (\cos t, \operatorname{sen} t)$$

$$\Gamma(t) = (\cos t, \operatorname{sen} t, 1)$$

$$\Gamma = \sigma \circ \gamma$$

$$0 \le t \le 2\pi$$

$$\int_\Gamma F \cdot d\Gamma = \int_0^{2\pi} F(\Gamma(t)) \cdot \Gamma'(t)\, dt$$

$$\Gamma'(t) = (-\operatorname{sen} t, \cos t, 0)$$

$$F(x, y, z) = (y, 0, x+y) \; ; \; F \circ \Gamma(t) =$$

$$= (\operatorname{sen} t, 0, \cos t + \operatorname{sen} t)$$

$$\int_0^{2\pi} (\operatorname{sen} t, 0, \cos t + \operatorname{sen} t) \cdot (-\operatorname{sen} t, \cos t, 0)\, dt$$

$$= -\int_0^{2\pi} \operatorname{sen}^2 t \, dt = -\int_0^{2\pi} \frac{1}{2} - \frac{1}{2}\cos 2t \, dt$$

$= \boxed{-\pi}$ Por outro lado, (3)

$$\iint_K \operatorname{rot} F \cdot \underline{n}\, dS = \iint_K$$

$$\operatorname{rot} F = \begin{vmatrix} \hat{i} & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & 0 & x+y \end{vmatrix} = i - j + k$$

$= (1, -1, -1)$ $\qquad \sigma = (u, v, 2-u^2-v^2)$

$$D\sigma = \begin{bmatrix} \frac{\partial \sigma}{\partial u} & \frac{\partial \sigma}{\partial v} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2u & -2v \end{bmatrix}$$

$$\frac{\partial \sigma}{\partial u} \wedge \frac{\partial \sigma}{\partial v} = \begin{vmatrix} i & j & k \\ 1 & 0 & -2u \\ 0 & 1 & -2v \end{vmatrix} = (2u, 2v, 1)$$

(4)

$$\iint_{\sigma} \operatorname{rot} F \cdot \underline{n}\, dS' = \iint_{K} (\operatorname{rot} F \circ \sigma) \cdot \frac{\partial \sigma}{\partial u} \wedge \frac{\partial \sigma}{\partial v}\, du\, dv$$

$$= \iint_{K} (1,-1,-1) \cdot (2u, 2v, 1)\, du\, dv$$

$$= \iint_{K} 2u - 2v - 1\, du\, dv$$

polar

$u = r\cos\theta \quad , \quad v = r\,\mathrm{sen}\,\theta$

$$\int_{\theta=0}^{2\pi} \int_{r=0}^{1} (2r\cos\theta - 2r\,\mathrm{sen}\,\theta - 1)\, r\, dr\, d\theta$$

$$\boxed{2\int_0^1 r^2\, dr = \frac{2}{3}}$$

$$= \int_0^{2\pi} \frac{2}{3}\cos\theta - \frac{2}{3}\,\mathrm{sen}\,\theta - \frac{1}{2}\, d\theta$$

$$-\int_0^1 r\, dr = -\frac{r^2}{2}\Big|_0^1 = -\frac{1}{2}$$

$$= 0 + 0 - \frac{1}{2}\theta\Big|_0^{2\pi} = \boxed{-\pi}$$

verificando Stokes, que

$$\int_\Gamma F \cdot d\Gamma = \iint_\sigma \text{rot}\, F \cdot \underline{n}\, dS.$$