FISHER 0

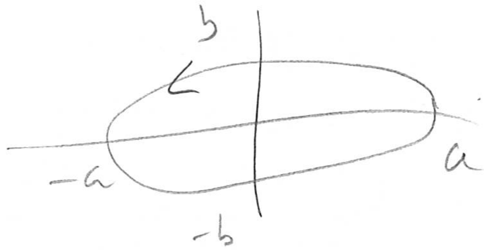
PROVINHA

1) Ache: $\displaystyle\int_\gamma x\,dx + y\,dy$

$\gamma(t) = (\arctan t, \operatorname{sen} t^3)$

$\gamma : [0,1] \to \mathbb{R}^2$

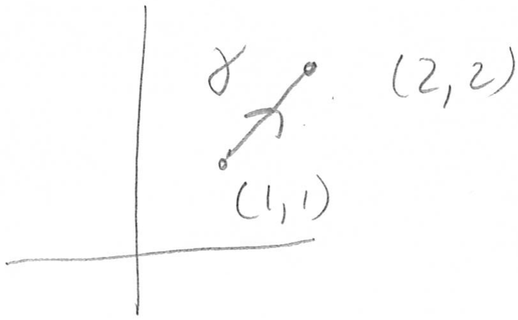
2) Ache: $\displaystyle\int_\gamma x\,dy$ onde γ é elipse



3) $\gamma(t) = (\cos t, \operatorname{sen}(t))$

$$\int_\gamma (x^4 - y^3)\,dx + (x^3 + y^5)\,dy = ?$$

4) Ache: $\displaystyle\int_\gamma y\,dx + x^2\,dy$



GABARTO PROVINHA: (1)

(1) $\int x\,dx + y\,dy$ $\quad F = (x,y) = (P,Q)$

OBS: $\text{rot}\, F \cdot k = \dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = 0 - 0 = 0$

dai é possivel que F seja conservativo.
Tambem, F e definido em $\mathbb{R}^2$, que é simplesmente conexo, dai é de fato conservativo. Isto é, $\exists\, \varphi$ tal que $\nabla \varphi = F$. Procurando:

$\dfrac{\partial \varphi}{\partial x} = x$, $\quad \varphi(x,y) = \dfrac{x^2}{2} + c$

$c = c(y)$ pois este constante depende de y!

$\dfrac{\partial \varphi}{\partial y} = \dfrac{\partial}{\partial y}\left(\dfrac{x^2}{2} + c(y)\right) = \dfrac{\partial c}{\partial y} = y$

$c(y) = \dfrac{y^2}{2} + d(x)$

$\varphi(x,y) = \dfrac{x^2}{2} + \dfrac{y^2}{2} + d(x)$ $\quad \dfrac{\partial \varphi}{\partial x} = x + d'(x) = x$

$\Rightarrow d'(x) = 0$

$d(x) = d.$

(2)

Daí, $\varphi(x,y) = \frac{x^2}{2} + \frac{y^2}{2} + \underline{const.}$

Sendo que F é conservativo com

$\nabla\varphi = F$,

$$\int_\gamma F\, d\gamma = \varphi(B) - \varphi(A)$$

$B = \gamma(b) = \gamma(1) = (\arctan 1, (\operatorname{sen} 1)^3$

$= \left(\frac{\pi}{4}, \operatorname{sen}^3(1)\right)$

$A = \gamma(a) = (0,0)$.

$\varphi(A) = 0$, $\varphi(B) = \frac{\left(\frac{\pi}{4}\right)^2}{2} + \frac{(\operatorname{sen}^3 1)^2}{2}$

$$\int_\gamma x\,dx + y\,dy = \boxed{\frac{\pi^2}{32} + \frac{\operatorname{sen}^6(1)}{2}}$$

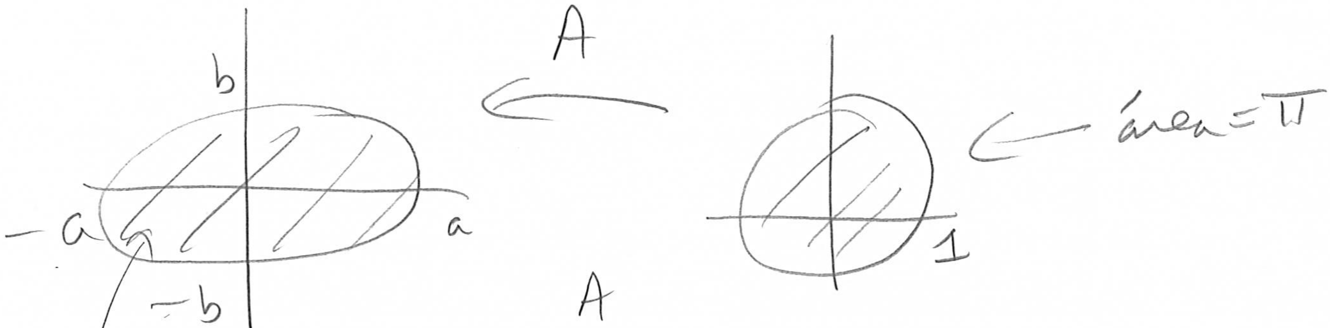
(2) $$\int_\gamma x\,dy = \int_\gamma P\,dx + Q\,dy = \int_\gamma F\cdot d\gamma$$

$P(x,y)=0, \quad Q(x,y)=x \qquad F=(P,Q)$

$$= \iint_B \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dx\,dy = \iint_B 1\,dx\,dy$$

$\dfrac{\partial Q}{\partial x} = 1, \quad \dfrac{\partial P}{\partial y} = 0$

$=$ área (ellipse) $= \boxed{\pi ab}$



A

$$\det \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = ab$$

área $= \det A \cdot \pi = \boxed{\pi ab}$

3 $\gamma(t) = (\cos t, \text{sen}\, t)$

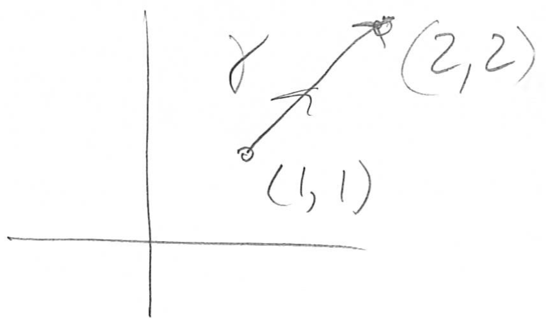
$$\int_\gamma (x^4 - y^3)\,dx + (x^3 + y^5)\,dy = \int_\gamma P\,dx + Q\,dy$$

$$= \iint_B \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dx\,dy = \iint_B 3x^2 + 3y^2\,dx\,dy$$

$$= 3\int_0^{2\pi}\int_0^1 r^2\, r\,dr\,d\theta = 3\int_0^{2\pi} \left.\frac{r^4}{4}\right|_0^1 d\theta$$

$$= \frac{3}{4}\cdot 2\pi = \boxed{\frac{3\pi}{2}}$$

$$\int_\gamma y\,dx + x^2\,dy$$



$\gamma(t) = (t, t) \quad t \in [1, 2]$

$\gamma'(t) = (1, 1)$

$$\int_\gamma \cancel{y}\, dx + x^2 dy = \int_\gamma F \cdot d\gamma \qquad F = (P, Q)$$

$P(x,y) = y$

$Q(x,y) = x^2$

$$= \int_1^2 F(\gamma(t)) \cdot \gamma'(t)\,dt$$

$$= \int_1^2 (t, t^2) \cdot (1,1)\,dt = \int_1^2 t + t^2\,dt$$

$$= \frac{t^2}{2} + \frac{t^3}{3}\Big|_1^2 = \frac{4}{2} + \frac{8}{3} - \frac{1}{2} - \frac{1}{3}$$

$$= \frac{3}{2} + \frac{7}{3} = \frac{9+14}{6} = \boxed{\frac{23}{6}}$$