MAE 5870 – Análise de Séries temporais
Lista #1
Data de entrega: 06/04/2017

Shumway, R.H. and Stoffer, D.S. Time Series Analysis and Its Applications With R Examples. 2nd or 3rd Edition. Springer.

1.2 Consider a signal plus noise model of the general form $x_t = s_t + w_t$, where w_t is Gaussian white noise with $\sigma_w^2 = 1$. Simulate and plot $n = 200$ observations from each of the following two models (*Save the data generated here for use in Problem 1.21*):

(a) $x_t = s_t + w_t$, for $t = 1, ..., 200$, where

$$s_t = \begin{cases} 0, & t = 1, ..., 100 \\ 10\exp\{-\frac{(t-100)}{20}\}\cos(2\pi t/4), & t = 101, ..., 200. \end{cases}$$

(b) $x_t = s_t + w_t$, for $t = 1, ..., 200$, where

$$s_t = \begin{cases} 0, & t = 1, ..., 100 \\ 10\exp\{-\frac{(t-100)}{200}\}\cos(2\pi t/4), & t = 101, ..., 200. \end{cases}$$

(c) Compare the general appearance of the series (a) and (b) with the earthquake series and the explosion series shown in Figure 1.7. In addition, plot (or sketch) and compare the signal modulators (a) $\exp\{-t/20\}$ and (b) $\exp\{-t/200\}$, for $t = 1, 2, ..., 100$.

1.7 For a moving average process of the form

$$x_t = w_{t-1} + 2w_t + w_{t+1},$$

where w_t are independent with zero means and variance σ_w^2, determine the autocovariance and autocorrelation functions as a function of lag $h = s - t$ and plot.

1.5 For the two series, x_t, in Problem 1.2 (a) and (b):

(a) compute and sketch the mean functions $\mu_x(t)$; for $t = 1, \ldots, 200$.

(b) calculate the autocovariance functions, $\gamma_x(s,t)$, for $s, t = 1, \ldots, 200$.

1.3 (a) Generate $n = 100$ observations from the autoregression

$$x_t = -.9x_{t-2} + w_t$$

with $\sigma_w = 1$, using the method described in Example 1.10. Next, apply the moving average filter

$$v_t = (x_t + x_{t-1} + x_{t-2} + x_{t-3})/4$$

to x_t, the data you generated. Now plot x_t as a line and superimpose v_t as a dashed line. Comment on the behavior of x_t and how applying the moving average filter changes that behavior.

(b) Repeat (a) but with

$$x_t = \cos(2\pi t/4).$$

(c) Repeat (b) but with added N(0, 1) noise,

$$x_t = \cos(2\pi t/4) + w_t.$$

(d) Compare and contrast (a)–(c).

1.10 Suppose we would like to predict a single stationary series x_t with zero mean and autocorrelation function $\gamma(h)$ at some time in the future, say, $t+\ell$, for $\ell > 0$.

(a) If we predict using only x_t and some scale multiplier A, show that the mean-square prediction error

$$MSE(A) = E[(x_{t+\ell} - Ax_t)^2]$$

is minimized by the value

$$A = \rho(\ell).$$

(b) Show that the minimum mean-square prediction error is

$$MSE(A) = \gamma(0)[1 - \rho^2(\ell)].$$

(c) Show that if $x_{t+\ell} = Ax_t$, then $\rho(\ell) = 1$ if $A > 0$, and $\rho(\ell) = -1$ if $A < 0$.

1.13 Consider the two series

$$x_t = w_t$$

$$y_t = w_t - \theta w_{t-1} + u_t,$$

where w_t and u_t are independent white noise series with variances σ_w^2 and σ_u^2, respectively, and θ is an unspecified constant.

(a) Express the ACF, $\rho_y(h)$, for $h = 0, \pm 1, \pm 2, \ldots$ of the series y_t as a function of σ_w^2, σ_u^2, and θ.

(b) Determine the CCF, $\rho_{xy}(h)$ relating x_t and y_t.

(c) Show that x_t and y_t are jointly stationary.

1.19 (a) Simulate a series of $n = 500$ Gaussian white noise observations as in Example 1.8 and compute the sample ACF, $\widehat{\rho}(h)$, to lag 20. Compare the sample ACF you obtain to the actual ACF, $\rho(h)$. [Recall Example 1.19.]

(b) Repeat part (a) using only $n = 50$. How does changing n affect the results?

1.24 A real-valued function $g(t)$, defined on the integers, is non-negative definite if and only if

$$\sum_{s=1}^{n}\sum_{t=1}^{n} a_s g(s-t) a_t \geq 0$$

for all positive integers n and for all vectors $\boldsymbol{a} = (a_1, a_2, \ldots, a_n)'$. For the matrix $G = \{g(s-t), s, t = 1, 2, \ldots, n\}$, this implies that $\boldsymbol{a}'G\boldsymbol{a} \geq 0$ for all vectors $\boldsymbol{a}$.

(a) Prove that $\gamma(h)$, the autocovariance function of a stationary process, is a non-negative definite function.

(b) Verify that the sample autocovariance $\widehat{\gamma}(h)$ is a non-negative definite function.

9. Consider a contrived set of data generated by tossing a fair coin, letting $x_t = 1$ when a head is obtained and $x_t = 0$ when a tail is obtained. Construct y_t as

$$y_t = 5 + x_t - 0.65\, x_{t-1}$$

a) Compare the sample ACF you obtain to the actual ACF, $\rho(h)$, with n=10, 100, 200, 500 and 1000.

b) For each n (10, 100, 200, 500 and 1000), simulate 1000 replications, and compute the sample ACF to lag 10, verify the Large Sample Distribution of the ACF.