# MAE 5870 – Análise de Séries temporais
## Lista #3
**data de entrega: 09/05/2017**

**3.1** For an MA(1), $x_t = w_t + \theta w_{t-1}$, show that $|\rho_x(1)| \le 1/2$ for any number $\theta$. For which values of $\theta$ does $\rho_x(1)$ attain its maximum and minimum?

**3.3** Verify the calculations made in Example 3.3:

(a) Let $x_t = \phi x_{t-1} + w_t$ where $|\phi| > 1$ and $w_t \sim$ iid $N(0, \sigma_w^2)$. Show $E(x_t) = 0$ and $\gamma_x(h) = \sigma_w^2 \phi^{-2} \phi^{-h}/(1 - \phi^{-2})$.
(b) Let $y_t = \phi^{-1} y_{t-1} + v_t$ where $v_t \sim$ iid $N(0, \sigma_w^2 \phi^{-2})$ and $\phi$ and $\sigma_w$ are as in part (a). Argue that $y_t$ is causal with the same mean function and autocovariance function as $x_t$.

**3.4** Identify the following models as ARMA$(p, q)$ models (watch out for parameter redundancy), and determine whether they are causal and/or invertible:

(a) $x_t = .80 x_{t-1} - .15 x_{t-2} + w_t - .30 w_{t-1}$.
(b) $x_t = x_{t-1} - .50 x_{t-2} + w_t - w_{t-1}$.

**3.7** For the AR(2) series shown below, use the results of Example 3.9 to determine a set of difference equations that can be used to find the ACF $\rho(h)$, $h = 0, 1, \ldots$; solve for the constants in the ACF using the initial conditions. Then plot the ACF values to lag 10 (use `ARMAacf` as a check on your answers).

(a) $x_t + 1.6 x_{t-1} + .64 x_{t-2} = w_t$.

**3.10** Let $x_t$ represent the cardiovascular mortality series (`cmort`) discussed in Chapter 2, Example 2.2.

(a) Fit an AR(2) to $x_t$ using linear regression as in Example 3.17.
(b) Assuming the fitted model in (a) is the true model, find the forecasts over a four-week horizon, $x_{n+m}^n$, for $m = 1, 2, 3, 4$, and the corresponding 95% prediction intervals.

**3.18** Fit an AR(2) model to the cardiovascular mortality series (`cmort`) discussed in Chapter 2, Example 2.2. using linear regression and using Yule–Walker.

(a) Compare the parameter estimates obtained by the two methods.
(b) Compare the estimated standard errors of the coefficients obtained by linear regression with their corresponding asymptotic approximations, as given in Property 3.10.

**3.8** Verify the calculations for the autocorrelation function of an ARMA(1, 1) process given in Example 3.13. Compare the form with that of the ACF for the ARMA(1, 0) and the ARMA(0, 1) series. Plot (or sketch) the ACFs of the three series on the same graph for $\phi = .6$, $\theta = .9$, and comment on the diagnostic capabilities of the ACF in this case.

**3.9** Generate $n = 100$ observations from each of the three models discussed in Problem 3.8. Compute the sample ACF for each model and compare it to the theoretical values. Compute the sample PACF for each of the generated series and compare the sample ACFs and PACFs with the general results given in Table 3.1.

**3.11** Consider the MA(1) series

$$x_t = w_t + \theta w_{t-1},$$

where $w_t$ is white noise with variance $\sigma_w^2$.

(a) Derive the minimum mean-square error one-step forecast based on the infinite past, and determine the mean-square error of this forecast.
(b) Let $\widetilde{x}_{n+1}^n$ be the truncated one-step-ahead forecast as given in (3.92). Show that

$$E\left[(x_{n+1} - \widetilde{x}_{n+1}^n)^2\right] = \sigma^2(1 + \theta^{2+2n}).$$

Compare the result with (a), and indicate how well the finite approximation works in this case.

**3.32** Crude oil prices in dollars per barrel are in `oil`; see Appendix R for more details. Fit an ARIMA$(p, d, q)$ model to the growth rate performing all necessary diagnostics. Comment.

**3.33** Fit an ARIMA$(p, d, q)$ model to the global temperature data `gtemp` performing all of the necessary diagnostics. After deciding on an appropriate model, forecast (with limits) the next 10 years. Comment.

**3.34** One of the series collected along with particulates, temperature, and mortality described in Example 2.2 is the sulfur dioxide series, `so2`. Fit an ARIMA$(p, d, q)$ model to the data, performing all of the necessary diagnostics. After deciding on an appropriate model, forecast the data into the future four time periods ahead (about one month) and calculate 95% prediction intervals for each of the four forecasts. Comment.