

Evaluation of the “Page-Rank” Algorithm

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Table 1: The number of iterations for each value of α , the number of pages that have the same position with α_1 and $\alpha_2 = 0.99$ (reference value) and the average displacement when there was a change in position.

California.dat	$\alpha_1 = 0.85$	$\alpha_1 = 0.90$	$\alpha_1 = 0.95$
	$\alpha_2 = 0.99$	$\alpha_2 = 0.99$	$\alpha_2 = 0.99$
NI (α_1) ¹	95	146	297
NI (α_2) ¹	1511	1511	1511
NSP ¹	1961	1901	1806
AVE ¹	6.624	4.227	1.860

¹NI (α_1): Number of iterations with α_1 ; NI (α_2): Number of iterations with α_2 ; NSP: Number of pages that are not in the same position; AVE: Average of the displacements of positions.

Table 2: Ranking, the number of links and the importance obtained from the first 30 pages when $\alpha = 0.85$ and $\alpha_2 = 0.99$ (reference value).

PI ²	NL ²	$I_{(\alpha=0.85)}$ ²	$P_{(\alpha=0.85)}$ ²	$I_{(\alpha=0.99)}$ ²	$P_{(\alpha=0.99)}$ ²	DIFF ²
1489	1	0.006231	1	0.064208	1	0
4392	1	0.006085	2	0.064142	2	0
67	13	0.004773	3	0.004314	18	15
6428	0	0.004622	4	0.003031	21	17
4824	17	0.004531	5	0.003781	19	14
2079	0	0.004342	6	0.002707	24	18
1	17	0.004197	7	0.003427	20	13
1490	1	0.003965	8	0.039611	3	5
1618	3	0.003645	9	0.002717	23	14
2409	1	0.003635	10	0.039373	4	6
18	1	0.003572	11	0.030439	5	6
1807	0	0.003166	12	0.001971	28	16
998	1	0.003141	13	0.030193	6	7
42	3	0.002998	14	0.002946	22	8
212	1	0.002987	15	0.029848	7	8
1863	1	0.002931	16	0.002660	25	9
1864	0	0.002931	17	0.002660	26	9
1084	0	0.002757	18	0.002090	27	9
1080	3	0.002752	19	0.001835	30	11
127	0	0.002718	20	0.001889	29	9
8052	1	0.002595	21	0.029579	8	13
7756	5	0.002459	22	0.001725	31	9
33	13	0.002238	23	0.001724	32	9
1661	0	0.002169	24	0.001566	33	9
2476	0	0.002153	25	0.001526	35	10
10	0	0.002123	26	0.001341	37	11
103	0	0.001947	27	0.001344	36	9
7	1	0.001890	28	0.017878	9	19
2218	1	0.001855	29	0.001266	38	9
1662	6	0.001796	30	0.001563	34	4

² PI: Page identification; NL: Number of Links; $I_{(\alpha=0.85)}$: Importance with $\alpha = 0.85$; $P_{(\alpha=0.85)}$: Position with $\alpha = 0.85$; $I_{(\alpha=0.99)}$: Importance with $\alpha = 0.99$; $P_{(\alpha=0.99)}$: Position with $\alpha = 0.99$; DIFF: Difference modulus in the Positions.

Table 3: The number of the first pages analyzed, the number of pages that have the same position with $\alpha = 0.85$ and $\alpha = 0.99$, the number of pages with up to 5 difference positions when $\alpha = 0.85$ and $\alpha = 0.99$, the average and maximum number of displacements between the positions obtained when $\alpha = 0.85$ and $\alpha = 0.99$ and the positions presented in this maximum obtained displacement.

TP ³	SP ³	5D ³	AVE ³	MAX ³	POS _{MAX} ³
10	2	3	10.20	18	06/24
20	2	3	9.70	18	06/24
30	2	4	9.87	19	28/09
40	2	4	11.08	21	37/58
50	2	5	12.34	37	48/11
60	2	6	13.83	40	54/14
70	2	6	14.46	53	66/13
80	2	7	14.91	60	75/15
90	3	9	15.08	60	75/15
100	3	10	16.15	60	75/15

³ TP: Total number of pages; SP: Number of pages in the same position; 5D: Number of pages with up to 5 displacement positions; AVE: Average displacement; MAX: Maximum displacement; POS_{MAX}: Position $\alpha = 0.85$ / Position $\alpha = 0.99$ in the maximum displacement.

Table 4: The number of iterations for each value of α , the number of pages that have the same position with α_1 and $\alpha_2 = 0.99$ (reference value) and the average displacement when there was a change in position.

	$\alpha_1 = 0.85$	$\alpha_1 = 0.90$	$\alpha_1 = 0.95$
Epa.dat	$\alpha_2 = 0.99$	$\alpha_2 = 0.99$	$\alpha_2 = 0.99$
NI (α_1) ⁴	81	123	246
NI (α_2) ⁴	1238	1238	1238
NSP ⁴	1930	1799	1469
AVE ⁴	8.856	5.432	1.965

⁴ NI (α_1): Number of iterations with α_1 ; NI (α_2): Number of iterations with α_2 ; NSP: Number of pages that are not in the same position; AVE: Average of the displacements of positions

Table 5: Ranking, the number of links and the importance obtained from the first 30 pages when $\alpha = 0.85$ and $\alpha_2 = 0.99$ (reference value).

PI ⁵	NL ⁵	$I_{(\alpha=0.85)}$ ⁵	$P_{(\alpha=0.85)}$ ⁵	$I_{(\alpha=0.99)}$ ⁵	$P_{(\alpha=0.99)}$ ⁵	DIFF ⁵
1248	1	0.019116	1	0.194546	1	0
2839	1	0.018932	2	0.194391	2	0
968	0	0.006302	3	0.004125	8	5
709	0	0.005561	4	0.004251	7	3
288	0	0.005123	5	0.004314	6	1
222	0	0.004115	6	0.002610	12	6
2176	1	0.003960	7	0.002853	9	2
1577	0	0.003747	8	0.002737	11	3
276	11	0.003654	9	0.002810	10	1
2800	0	0.003634	10	0.002575	13	3
711	0	0.003409	11	0.002128	17	6
4250	1	0.003354	12	0.002182	16	4
1322	7	0.003339	13	0.002363	14	1
4478	1	0.003163	14	0.028444	3	11
1195	1	0.003150	15	0.002283	15	0
4314	2	0.002864	16	0.028254	4	12
1232	1	0.002769	17	0.001708	20	3
840	0	0.002528	18	0.001655	21	3
943	1	0.002516	19	0.001899	19	0
2041	5	0.001974	20	0.001478	22	2
47	8	0.001946	21	0.001447	23	2
4674	0	0.001946	22	0.001277	26	4
791	0	0.001941	23	0.001282	25	2
2828	1	0.001871	24	0.014396	5	19
1509	1	0.001868	25	0.001178	30	5
46	0	0.001808	26	0.001082	33	7
88	8	0.001795	27	0.001120	32	5
1029	8	0.001736	28	0.001191	28	0
252	0	0.001715	29	0.001048	35	6
3841	0	0.001681	30	0.001155	31	1

⁵ PI: Page identification; NL: Number of Links; $I_{(\alpha=0.85)}$: Importance with $\alpha = 0.85$; $P_{(\alpha=0.85)}$: Position with $\alpha = 0.85$; $I_{(\alpha=0.99)}$: Importance with $\alpha = 0.99$; $P_{(\alpha=0.99)}$: Position with $\alpha = 0.99$; DIFF: Difference modulus in the Positions.

Table 6: The number of the first pages analyzed, the number of pages that have the same position with $\alpha = 0.85$ and $\alpha = 0.99$, the number of pages with up to 5 difference positions when $\alpha = 0.85$ and $\alpha = 0.99$, the average and maximum number of displacements between the positions obtained when $\alpha = 0.85$ and $\alpha = 0.99$ and the positions presented in this maximum obtained displacement.

TP ⁶	SP ⁶	5D ⁶	AVE ⁶	MAX ⁶	POS _{MAX} ⁶
10	2	8	2.40	6	06/12
20	4	14	3.30	12	16/04
30	5	18	3.90	19	24/05
40	5	21	4.53	19	24/05
50	6	24	4.88	19	24/05
60	6	25	5.88	21	55/34
70	6	26	6.77	27	65/38
80	6	27	7.26	27	65/38
90	6	29	7.61	27	65/38
100	6	31	7.99	27	65/38

⁶ TP: Total number of pages; SP: Number of pages in the same position; 5D: Number of pages with up to 5 displacement positions; AVE: Average displacement; MAX: Maximum displacement; POS_{MAX}: Position $\alpha = 0.85$ / Position $\alpha = 0.99$ in the maximum displacement.

Table 7: The average number of iterations and their confidence interval for $\alpha = 0.85$ and $\alpha = 0.99$, varying the size of the closed set, the existence of the linked set and the number of dangling nodes, for a total of 1,000, 10,000 and 100,000 pages in the closed sets.

				$\alpha=0.85$			$\alpha=0.99$		
NCS ⁷	TPCS ⁷	TPLS ⁷	TNDP ⁷	NI ⁷	LL ⁷	UL ⁷	NI ⁷	LL ⁷	UL ⁷
1	1,000	0	0	24.86	24.78	24.94	31.17	31.05	31.30
1	1,000	100	0	24.84	24.76	24.92	31.15	31.03	31.28
1	1,000	0	50	24.76	24.69	24.84	31.03	30.91	31.14
1	1,000	100	50	24.77	24.70	24.85	31.08	30.95	31.21
1	1,000	100	2,000	25.07	25.00	25.14	31.06	30.94	31.18
1	10,000	0	0	23.81	23.76	23.85	29.73	29.67	29.78
1	10,000	1,000	0	23.79	23.74	23.83	29.71	29.65	29.77
1	10,000	0	500	23.80	23.75	23.84	29.68	29.63	29.73
1	10,000	1,000	500	23.80	23.75	23.84	29.66	29.60	29.71
1	10,000	1,000	20,000	23.82	23.77	23.86	29.72	29.67	29.78
1	100,000	0	0	22.54	22.50	22.59	28.16	28.12	28.20
1	100,000	10,000	0	22.52	22.48	22.57	28.14	28.10	28.18
1	100,000	0	5,000	22.49	22.45	22.53	28.14	28.09	28.18
1	100,000	10,000	5,000	22.54	22.50	22.58	28.18	28.14	28.23
1	100,000	10,000	200,000	22.50	22.46	22.55	28.15	28.11	28.19
2	500	0	0	77.44	77.35	77.52	959.90	958.64	961.16
2	500	100	0	77.36	77.28	77.45	959.02	957.72	960.31
2	500	0	50	77.36	77.28	77.44	958.93	957.68	960.19
2	500	100	50	77.37	77.29	77.45	959.10	957.87	960.33
2	500	100	2,000	77.43	77.35	77.51	960.19	958.99	961.40
2	5,000	0	0	64.91	64.84	64.97	757.23	756.27	758.18
2	5,000	1,000	0	64.98	64.92	65.05	757.95	757.01	758.90
2	5,000	0	500	64.96	64.89	65.02	758.38	757.43	759.33
2	5,000	1,000	500	64.96	64.90	65.02	758.34	757.39	759.29
2	5,000	1,000	20,000	64.96	64.90	65.03	758.24	757.30	759.18
2	50,000	0	0	51.89	51.84	51.94	546.62	545.94	547.30
2	50,000	10,000	0	51.92	51.87	51.97	547.17	546.45	547.89
2	50,000	0	5,000	51.89	51.84	51.95	546.82	546.05	547.59
2	50,000	10,000	5,000	51.92	51.87	51.98	547.25	546.52	547.98
2	50,000	10,000	200,000	51.90	51.85	51.95	547.01	546.31	547.71
10	100	0	0	88.13	88.04	88.22	1,132.88	1,131.46	1,134.30
10	100	100	0	88.17	88.09	88.26	1,133.77	1,132.42	1,135.11
10	100	0	50	88.17	88.07	88.26	1,133.51	1,132.03	1,135.00
10	100	100	50	88.23	88.13	88.33	1,134.70	1,133.12	1,136.28
10	100	100	2,000	88.15	88.06	88.25	1,133.22	1,131.70	1,134.75
10	1,000	0	0	76.90	76.81	76.98	951.36	950.05	952.66
10	1,000	1,000	0	76.85	76.77	76.94	950.68	949.40	951.95
10	1,000	0	500	76.84	76.77	76.92	950.09	948.93	951.24
10	1,000	1,000	500	76.87	76.79	76.95	950.80	949.61	951.99
10	1,000	1,000	20,000	76.95	76.87	77.03	951.99	950.78	953.21
10	10,000	0	0	64.30	64.24	64.36	747.59	746.67	748.52
10	10,000	10,000	0	64.32	64.25	64.38	747.53	746.62	748.43
10	10,000	0	5,000	64.25	64.19	64.31	747.02	746.13	747.92
10	10,000	10,000	5,000	64.36	64.30	64.42	748.04	747.09	748.99
10	10,000	10,000	200,000	64.23	64.16	64.29	746.76	745.82	747.70
50	20	0	0	93.82	93.75	93.89	1,224.62	1,223.57	1,225.67
50	20	100	0	93.80	93.73	93.87	1,224.90	1,223.88	1,225.92
50	20	0	50	93.82	93.75	93.89	1,224.74	1,223.74	1,225.74
50	20	100	50	93.82	93.75	93.88	1,224.91	1,223.91	1,225.91
50	20	100	2,000	93.84	93.77	93.90	1,225.35	1,224.33	1,226.37
50	200	0	0	85.55	85.46	85.65	1,091.35	1,089.89	1,092.82
50	200	1,000	0	85.60	85.51	85.69	1,092.06	1,090.64	1,093.48
50	200	0	500	85.56	85.47	85.66	1,091.23	1,089.78	1,092.68
50	200	1,000	500	85.52	85.42	85.61	1,090.54	1,089.13	1,091.94
50	200	1,000	20,000	85.53	85.44	85.63	1,090.87	1,089.40	1,092.34
50	2,000	0	0	73.69	73.61	73.76	899.10	897.95	900.24
50	2,000	10,000	0	73.74	73.67	73.82	900.05	898.94	901.16
50	2,000	0	5,000	73.71	73.64	73.79	899.91	898.75	901.06
50	2,000	10,000	5,000	73.68	73.61	73.75	899.30	898.23	900.37
50	2,000	10,000	200,000	73.70	73.63	73.77	899.29	898.17	900.42

⁷ NCS: Number of closed sets; TPCS: Total number of pages in the closed set; TPLS: Total number of pages in the linked set; TNDP: Total number of dangling nodes; NI: Number of iterations; LL: Lower limit; UL: Upper Limit.

Table 8: The average number of iterations and their confidence interval for $\alpha = 0.85$ and $\alpha = 0.99$, varying the size of the closed set, for closed sets of 1,000 and 10,000 pages.

				$\alpha=0.85$			$\alpha=0.99$		
NCS ⁸	TPCS ⁸	TPLS ⁸	TNDP ⁸	NI ⁸	LL ⁸	UL ⁸	NI ⁸	LL ⁸	UL ⁸
1	1,000	100	1,000	24.84	24.77	24.92	31.09	30.97	31.21
2	1,000	200	2,000	73.74	73.66	73.82	900.48	899.28	901.69
3	1,000	300	3,000	75.07	74.98	75.15	921.95	920.61	923.29
4	1,000	400	4,000	75.72	75.64	75.80	932.42	931.21	933.63
5	1,000	500	5,000	76.24	76.16	76.32	940.58	939.34	941.83
10	1,000	1,000	10,000	76.94	76.86	77.02	951.73	950.48	952.97
15	1,000	1,500	15,000	77.17	77.09	77.24	955.52	954.39	956.66
20	1,000	2,000	20,000	77.20	77.12	77.28	956.01	954.79	957.23
1	10,000	1,000	10,000	23.74	23.69	23.79	29.68	29.62	29.73
2	10,000	2,000	20,000	61.07	61.02	61.13	695.25	694.41	696.09
3	10,000	3,000	30,000	62.41	62.35	62.47	717.43	716.50	718.36
4	10,000	4,000	40,000	63.21	63.15	63.27	729.80	728.83	730.77
5	10,000	5,000	50,000	63.50	63.44	63.56	734.91	734.01	735.81
10	10,000	10,000	100,000	64.32	64.26	64.38	748.10	747.22	748.97
15	10,000	15,000	150,000	64.51	64.45	64.58	750.96	749.99	751.94
20	10,000	20,000	200,000	64.61	64.55	64.68	752.83	751.92	753.74

⁸ NCS: Number of closed sets; TPCS: Total number of pages in the closed set; TPLS: Total number of pages in the linked set; TNDP: Total number of dangling nodes; NI: Number of iterations; LL: Lower limit; UL: Upper Limit.

Table 9: The average number of iterations and their confidence interval for $\alpha = 0.85$ and $\alpha = 0.99$, varying the size of the closed set, for closed sets of 500 to 2,000 pages.

				$\alpha=0.85$			$\alpha=0.99$		
NCS ⁹	TPCS ⁹	TPLS ⁹	TNDP ⁹	NI ⁹	LL ⁹	UL ⁹	NI ⁹	LL ⁹	UL ⁹
1	500	50	500	25.07	24.97	25.17	31.52	31.34	31.69
1	1,000	100	1,000	24.84	24.77	24.92	31.09	30.97	31.21
1	1,500	150	1,500	24.65	24.58	24.72	30.88	30.78	30.98
1	2,000	200	2,000	24.53	24.48	24.59	30.70	30.62	30.78
2	500	100	1,000	77.39	77.31	77.47	959.04	957.81	960.28
2	1,000	200	2,000	73.74	73.66	73.82	900.48	899.28	901.69
2	1,500	300	3,000	71.60	71.52	71.67	865.52	864.42	866.62
2	2,000	400	4,000	70.00	69.93	70.07	839.44	838.37	840.51
3	500	150	1,500	78.58	78.49	78.66	978.66	977.38	979.94
3	1,000	300	3,000	75.07	74.98	75.15	921.95	920.61	923.29
3	1,500	450	4,500	72.89	72.81	72.97	886.52	885.30	887.73
3	2,000	600	6,000	71.34	71.27	71.42	861.60	860.44	862.76
4	500	200	2,000	79.38	79.29	79.46	991.11	989.77	992.45
4	1,000	400	4,000	75.72	75.64	75.80	932.42	931.21	933.63
4	1,500	600	6,000	73.59	73.52	73.67	897.90	896.74	899.06
4	2,000	800	8,000	72.07	72.00	72.14	872.84	871.67	874.02
5	500	250	2,500	79.79	79.69	79.88	998.24	996.82	999.65
5	1,000	500	5,000	76.24	76.16	76.32	940.58	939.34	941.83
5	1,500	750	7,500	74.06	73.98	74.13	905.22	904.06	906.38
5	2,000	1,000	10,000	72.45	72.37	72.52	879.36	878.21	880.51
6	500	300	3,000	79.95	79.86	80.03	1,000.88	999.58	1,002.18
6	1,000	600	6,000	76.41	76.34	76.49	943.23	942.08	944.39
6	1,500	900	9,000	74.19	74.11	74.27	907.69	906.49	908.88
6	2,000	1,200	12,000	72.73	72.65	72.80	884.10	882.94	885.27
10	500	500	5,000	80.50	80.41	80.59	1,009.39	1,008.00	1,010.78
10	1,000	1,000	10,000	76.94	76.86	77.02	951.73	950.48	952.97
10	1,500	1,500	15,000	74.74	74.66	74.82	916.26	915.07	917.46
10	2,000	2,000	20,000	73.18	73.11	73.25	891.13	890.04	892.22

⁹ NCS: Number of closed sets; TPCS: Total number of pages in the closed set; TPLS: Total number of pages in the linked set; TNDP: Total number of dangling nodes; NI: Number of iterations; LL: Lower limit; UL: Upper Limit.

Table 10: The average number of iterations and their confidence interval for $\alpha = 0.85$ and $\alpha = 0.99$, varying the size of the closed set, for closed sets of 5,000 to 20,000 pages.

NCS ¹⁰	TPCS ¹⁰	TPLS ¹⁰	TNDP ¹⁰	$\alpha=0.85$			$\alpha=0.99$		
				NI ¹⁰	LL ¹⁰	UL ¹⁰	NI ¹⁰	LL ¹⁰	UL ¹⁰
1	5,000	500	5,000	24.14	24.10	24.19	30.16	30.09	30.23
1	10,000	1,000	10,000	23.74	23.69	23.79	29.68	29.62	29.73
1	15,000	1,500	15,000	23.54	23.49	23.58	29.42	29.37	29.47
1	20,000	2,000	20,000	23.44	23.39	23.48	29.27	29.23	29.32
2	5,000	1,000	10,000	64.96	64.89	65.02	758.05	757.12	758.99
2	10,000	2,000	20,000	61.07	61.02	61.13	695.25	694.41	696.09
2	15,000	3,000	30,000	58.77	58.71	58.83	658.15	657.31	658.99
2	20,000	4,000	40,000	57.14	57.08	57.19	631.46	630.65	632.28
3	5,000	1,500	15,000	66.37	66.30	66.44	781.04	780.00	782.08
3	10,000	3,000	30,000	62.41	62.35	62.47	717.43	716.50	718.36
3	15,000	4,500	45,000	60.14	60.08	60.20	680.06	679.19	680.93
3	20,000	6,000	60,000	58.50	58.44	58.56	653.70	652.86	654.54
4	5,000	2,000	20,000	67.07	67.00	67.13	792.30	791.36	793.25
4	10,000	4,000	40,000	63.21	63.15	63.27	729.80	728.83	730.77
4	15,000	6,000	60,000	60.92	60.86	60.98	692.56	691.69	693.43
4	20,000	8,000	80,000	59.30	59.24	59.36	666.66	665.80	667.52
5	5,000	2,500	25,000	67.43	67.36	67.50	798.06	797.01	799.10
5	10,000	5,000	50,000	63.50	63.44	63.56	734.91	734.01	735.81
5	15,000	7,500	75,000	61.32	61.27	61.38	699.32	698.50	700.14
5	20,000	10,000	100,000	59.67	59.61	59.73	672.56	671.68	673.44
6	5,000	3,000	30,000	67.71	67.64	67.78	802.60	801.59	803.61
6	10,000	6,000	60,000	63.82	63.76	63.89	739.71	738.80	740.62
6	15,000	9,000	90,000	61.60	61.54	61.66	703.84	702.95	704.72
6	20,000	12,000	120,000	59.93	59.87	59.99	676.79	675.88	677.70
10	5,000	5,000	50,000	68.20	68.13	68.27	810.50	809.44	811.57
10	10,000	10,000	100,000	64.32	64.26	64.38	748.10	747.22	748.97
10	15,000	15,000	150,000	62.02	61.96	62.08	710.96	710.07	711.85
10	20,000	20,000	200,000	60.39	60.33	60.45	684.65	683.78	685.52

¹⁰ NCS: Number of closed sets; TPCS: Total number of pages in the closed set; TPLS: Total number of pages in the linked set; TNDP: Total number of dangling nodes; NI: Number of iterations; LL: Lower limit; UL: Upper Limit.

Table 11: The average number of iterations and their confidence interval for $\alpha = 0.85$ and $\alpha = 0.99$, varying the size of the closed set, for a total of 500, 5,000 and 50,000 pages in the closed sets.

				$\alpha=0.85$			$\alpha=0.99$		
NCS ¹¹	TPCS ¹¹	TPLS ¹¹	TNDP ¹¹	NI ¹¹	LL ¹¹	UL ¹¹	NI ¹¹	LL ¹¹	UL ¹¹
1	500	50	500	25.07	24.97	25.17	31.52	31.34	31.69
2	250	50	500	80.91	80.83	81.00	1,016.40	1,015.05	1,017.75
5	100	50	500	87.51	87.42	87.61	1,123.20	1,121.71	1,124.69
8	63	50	504	89.85	89.76	89.95	1,161.05	1,159.52	1,162.58
10	50	50	500	90.94	90.86	91.03	1,178.54	1,177.18	1,179.90
20	25	50	500	93.27	93.20	93.34	1,215.89	1,214.87	1,216.91
50	10	50	500	94.25	94.18	94.32	1,231.72	1,230.74	1,232.70
1	5,000	500	5,000	24.16	24.12	24.20	30.13	30.07	30.19
2	2,500	500	5,000	68.79	68.72	68.87	820.08	819.01	821.15
5	1,000	500	5,000	76.24	76.16	76.32	940.58	939.34	941.83
8	625	500	5,000	79.22	79.13	79.31	988.81	987.44	990.19
10	500	500	5,000	80.49	80.40	80.57	1,009.35	1,008.04	1,010.66
20	250	500	5,000	84.38	84.29	84.47	1,072.09	1,070.71	1,073.46
50	100	500	5,000	88.64	88.55	88.73	1,141.16	1,139.72	1,142.61
80	63	500	5,040	90.67	90.57	90.76	1,174.02	1,172.51	1,175.53
100	50	500	5,000	91.58	91.49	91.67	1,188.65	1,187.27	1,190.04
1	50,000	5,000	50,000	22.96	22.93	23.00	28.60	28.55	28.65
2	25,000	5,000	50,000	55.85	55.79	55.90	610.93	610.16	611.69
5	10,000	5,000	50,000	63.50	63.44	63.56	734.91	734.01	735.81
8	6,250	5,000	50,000	66.72	66.65	66.78	787.15	786.17	788.12
10	5,000	5,000	50,000	68.20	68.13	68.27	810.50	809.44	811.57
20	2,500	5,000	50,000	72.29	72.21	72.36	876.43	875.35	877.52
50	1,000	5,000	50,000	77.43	77.34	77.51	959.70	958.47	960.94
80	625	5,000	50,000	79.91	79.83	80.00	999.87	998.60	1,001.15
100	500	5,000	50,000	81.00	80.92	81.08	1,017.86	1,016.53	1,019.20

¹¹ NCS: Number of closed sets; TPCS: Total number of pages in the closed set; TPLS: Total number of pages in the linked set; TNDP: Total number of dangling nodes; NI: Number of iterations; LL: Lower limit; UL: Upper Limit.

Table 12: The average number of iterations and their confidence interval for $\alpha = 0.85$ and $\alpha = 0.99$, varying the size of the closed set, for combinations of closed sets of 50 and 500 pages and 450 and 500 pages.

			$\alpha=0.85$			$\alpha=0.99$		
NTPCS ¹²	TPLS ¹²	TNDP ¹²	NI ¹²	LL ¹²	UL ¹²	NI ¹²	LL ¹²	UL ¹²
1×50;1×500	55	550	91.04	90.95	91.13	1,179.89	1,178.49	1,181.29
1×450;1×500	95	950	78.14	78.06	78.22	971.22	969.95	972.49
1×50;2×500	105	1,050	91.36	91.27	91.45	1,185.29	1,183.87	1,186.70
1×450;2×500	145	1,450	79.42	79.33	79.51	992.47	991.09	993.86
2×50;1×500	60	600	91.07	90.98	91.15	1,180.12	1,178.77	1,181.47
2×450;1×500	140	1,400	79.30	79.21	79.39	990.40	988.99	991.81
2×50;2×500	110	1,100	91.44	91.35	91.53	1,186.27	1,184.95	1,187.58
2×450;2×500	190	1,900	80.02	79.92	80.11	1,001.45	1,000.01	1,002.89
1×50;4×500	205	2,050	91.48	91.39	91.57	1,187.23	1,185.88	1,188.59
1×450;4×500	245	2,450	80.39	80.30	80.48	1,008.16	1,006.81	1,009.50
4×50;1×500	70	700	91.24	91.15	91.32	1,183.12	1,181.76	1,184.48
4×450;1×500	230	2,300	80.39	80.30	80.48	1,008.02	1,006.57	1,009.47
4×50;4×500	220	2,200	91.40	91.30	91.49	1,185.91	1,184.46	1,187.35
4×450;4×500	380	3,800	80.89	80.81	80.98	1,015.60	1,014.27	1,016.92
1×50;8×500	405	4,050	91.56	91.47	91.65	1,188.42	1,187.02	1,189.83
1×450;8×500	445	4,450	81.05	80.97	81.14	1,018.37	1,017.05	1,019.69
3×50;6×500	315	3,150	91.54	91.46	91.63	1,188.21	1,186.85	1,189.57
3×450;6×500	435	4,350	81.04	80.95	81.13	1,018.36	1,016.99	1,019.73
8×50;1×500	90	900	91.24	91.15	91.33	1,182.90	1,181.52	1,184.28
8×450;1×500	410	4,100	81.06	80.96	81.15	1,018.57	1,017.17	1,019.98
6×50;3×500	180	1,800	91.34	91.25	91.43	1,185.11	1,183.78	1,186.44
6×450;3×500	420	4,200	81.01	80.92	81.09	1,017.31	1,015.95	1,018.68
7×50;7×500	385	3,850	91.64	91.55	91.74	1,189.69	1,188.26	1,191.11
7×450;7×500	665	6,650	81.24	81.16	81.33	1,021.46	1,020.09	1,022.82
10×50;10×500	550	5,500	91.63	91.54	91.72	1,189.29	1,187.92	1,190.67
10×450;10×500	950	9,500	81.36	81.27	81.45	1,023.54	1,022.18	1,024.89

¹² NTPCS: Number of closed sets and total number of pages in the closed sets; TPLS: Total number of pages in the linked set; TNDP: Total number of dangling nodes; NI: Number of iterations; LL: Lower limit; UL: Upper Limit.

Table 13: The average number of iterations and their confidence interval for $\alpha = 0.85$ and $\alpha = 0.99$, varying the size of the closed set, for combinations of closed sets of 500 and 5,000 pages and 4,500 and 5,000 pages.

			$\alpha=0.85$			$\alpha=0.99$		
NTPCS ¹³	TPLS ¹³	TNDP ¹³	NI ¹³	LL ¹³	UL ¹³	NI ¹³	LL ¹³	UL ¹³
1×500;1×5000	550	5,500	80.59	80.50	80.67	1,010.74	1,009.38	1,012.10
1×4500;1×5000	950	9,500	65.60	65.54	65.66	768.60	767.68	769.53
1×500;2×5000	1,050	10,500	80.83	80.75	80.92	1,014.67	1,013.37	1,015.97
1×4500;2×5000	1,450	14,500	67.14	67.07	67.21	793.15	792.14	794.17
2×500;1×5000	600	6,000	80.68	80.59	80.77	1,012.28	1,010.83	1,013.72
2×4500;1×5000	1,400	14,000	67.02	66.96	67.09	791.58	790.58	792.58
2×500;2×5000	1,100	11,000	80.86	80.77	80.95	1,015.23	1,013.84	1,016.62
2×4500;2×5000	1,900	19,000	67.65	67.59	67.72	801.64	800.65	802.63
1×500;4×5000	2,050	20,500	81.04	80.95	81.13	1,018.02	1,016.70	1,019.33
1×4500;4×5000	2,450	24,500	68.15	68.08	68.21	809.69	808.72	810.65
4×500;1×5000	700	7,000	80.67	80.59	80.76	1,012.52	1,011.23	1,013.82
4×4500;1×5000	2,300	23,000	68.09	68.02	68.16	808.91	807.84	809.97
4×500;4×5000	2,200	22,000	81.04	80.96	81.12	1,018.12	1,016.80	1,019.44
4×4500;4×5000	3,800	38,000	68.60	68.53	68.67	817.16	816.18	818.14
1×500;8×5000	4,050	40,500	81.14	81.05	81.23	1,019.49	1,018.08	1,020.90
1×4500;8×5000	4,450	44,500	68.74	68.67	68.81	819.10	818.05	820.16
3×500;6×5000	3,150	31,500	80.99	80.91	81.08	1,017.65	1,016.35	1,018.95
3×4500;6×5000	4,350	43,500	68.66	68.59	68.73	817.89	816.89	818.89
8×500;1×5000	900	9,000	80.81	80.72	80.90	1,014.50	1,013.16	1,015.84
8×4500;1×5000	4,100	41,000	68.70	68.63	68.76	818.56	817.54	819.58
6×500;3×5000	1,800	18,000	81.00	80.91	81.09	1,017.45	1,016.07	1,018.83
6×4500;3×5000	4,200	42,000	68.69	68.62	68.76	818.65	817.64	819.65
7×500;7×5000	3,850	38,500	81.08	80.99	81.16	1,018.49	1,017.15	1,019.82
7×4500;7×5000	6,650	66,500	68.97	68.90	69.04	823.32	822.27	824.36
10×500;10×5000	5,500	55,000	81.16	81.07	81.25	1,020.18	1,018.75	1,021.61
100×4500;100×5000	9,500	95,000	69.07	69.00	69.14	824.75	823.71	825.78

¹³ NTPCS: Number of closed sets and total number of pages in the closed sets; TPLS: Total number of pages in the linked set; TNDP: Total number of dangling nodes; NI: Number of iterations; LL: Lower limit; UL: Upper Limit.

Table 14: The average number of iterations and their confidence interval for $\alpha = 0.85$ and $\alpha = 0.99$, varying the size of the closed set, for combinations of closed sets of various sizes.

NTPCS ¹⁴	TPLS ¹⁴	TNDP ¹⁴	$\alpha=0.85$			$\alpha=0.99$		
			NI ¹⁴	LL ¹⁴	UL ¹⁴	NI ¹⁴	LL ¹⁴	UL ¹⁴
1×500	50	500	25.02	24.91	25.13	31.43	31.25	31.61
1×20;1×480	50	500	93.66	93.59	93.72	1,222.73	1,221.76	1,223.69
1×200;1×300	50	500	82.56	82.48	82.65	1,042.62	1,041.23	1,044.00
2×250	50	500	80.93	80.85	81.01	1,016.73	1,015.46	1,017.99
1×10;1×190;1×300	50	500	94.30	94.23	94.37	1,232.36	1,231.36	1,233.37
2×10;1×480	50	500	94.25	94.18	94.32	1,231.61	1,230.65	1,232.57
1×20;2×120;1×240	50	500	93.71	93.65	93.78	1,223.10	1,222.10	1,224.10
3×100;1×200	50	500	87.50	87.41	87.60	1,122.61	1,121.14	1,124.08
1×20;4×120	50	500	93.73	93.66	93.79	1,223.30	1,222.28	1,224.32
5×10;1×150;1×300	50	500	94.28	94.21	94.35	1,232.26	1,231.22	1,233.31
1×5000	500	5,000	24.16	24.11	24.21	30.19	30.12	30.25
1×200;1×4800	500	5,000	85.37	85.27	85.47	1,088.63	1,087.11	1,090.15
1×2000;1×3000	500	5,000	70.65	70.58	70.73	850.35	849.16	851.53
2×2500	500	5,000	68.76	68.69	68.83	819.74	818.71	820.78
1×100;1×1900;1×3000	500	5,000	88.74	88.65	88.83	1,142.52	1,141.09	1,143.95
2×100;1×4800	500	5,000	88.72	88.62	88.82	1,142.67	1,141.14	1,144.19
1×200;2×1200;1×2400	500	5,000	85.47	85.37	85.56	1,089.68	1,088.24	1,091.12
3×1000;1×2000	500	5,000	76.16	76.08	76.24	939.40	938.16	940.64
4×50;1×4800	500	5,000	91.55	91.47	91.64	1,188.23	1,186.90	1,189.56
1×200;4×1200	500	5,000	85.45	85.35	85.55	1,089.51	1,088.01	1,091.01
5×100;1×1500;1×3000	500	5,000	88.77	88.67	88.87	1,142.90	1,141.39	1,144.41
1×50000	5,000	50,000	23.00	22.97	23.04	28.63	28.58	28.67
1×2000;1×48000	5,000	50,000	73.53	73.46	73.60	896.57	895.49	897.65
1×20000;1×30000	5,000	50,000	57.96	57.90	58.02	645.03	644.10	645.95
2×25000	5,000	50,000	55.85	55.79	55.90	610.93	610.16	611.69
1×1000;1×19000;1×30000	5,000	50,000	77.50	77.41	77.58	960.63	959.29	961.96
2×1000;1×48000	5,000	50,000	77.46	77.38	77.54	960.53	959.29	961.78
1×2000;2×12000;1×24000	5,000	50,000	73.60	73.52	73.67	898.01	896.85	899.17
3×10000;1×20000	5,000	50,000	63.59	63.53	63.65	735.81	734.88	736.73
4×500;1×48000	5,000	50,000	81.16	81.07	81.25	1,020.13	1,018.74	1,021.53
1×2000;4×12000	5,000	50,000	73.57	73.50	73.64	897.00	895.90	898.10
5×1000;1×15000;1×30000	5,000	50,000	77.38	77.29	77.46	959.12	957.81	960.43

¹⁴ NTPCS: Number of closed sets and total number of pages in the closed sets; TPLS: Total number of pages in the linked set; TNDP: Total number of dangling nodes; NI: Number of iterations; LL: Lower limit; UL: Upper Limit.