Ordenação: algoritmo Heapsort

PF 10
http://www.ime.usp.br/~pf/algoritmos/aulas/hpsrt.html

Ordenação

$v[1 : n]$ é crescente se $v[1] \leq \cdots \leq v[n-1]$.

Problema: Rearranjar uma lista $v[1 : n]$ de modo que ele fique crescente.

Entra:

1										n
33	55	33	44	33	22	11	99	22	55	77

Sai:

1										n
11	22	22	33	33	33	44	55	55	77	99

Heapsort

O Heapsort ilustra o uso de estruturas de dados no projeto de algoritmos eficientes.
Rearranjar uma lista $v[1 : n]$ de modo que ele fique crescente.

Entra:

1										n
33	55	33	44	33	22	11	99	22	55	77

Sai:

1										n
11	22	22	33	33	33	44	55	55	77	99

Ordenação por seleção

$i = 5$

1				max						n
38	50	20	44	10	50	55	60	75	85	99

Ordenação por seleção

$i = 5$

1			j	max						n
38	50	20	44	10	50	55	60	75	85	99

Ordenação por seleção

$i = 5$

1			j	max						n
38	50	20	44	10	50	55	60	75	85	99

1		j	max							n
38	50	20	44	10	50	55	60	75	85	99

Ordenação por seleção

i = 5

1			j	max						n
38	50	20	44	10	50	55	60	75	85	99

1		j	max							n
38	50	20	44	10	50	55	60	75	85	99

1	j		max							n
38	50	20	44	10	50	55	60	75	85	99

Ordenação por seleção

i = 5

1			j	max						n
38	50	20	44	10	50	55	60	75	85	99

1		j	max							n
38	50	20	44	10	50	55	60	75	85	99

1	j		max							n
38	50	20	44	10	50	55	60	75	85	99

j	max									n
38	50	20	44	10	50	55	60	75	85	99

Ordenação por seleção

i = 5

1			j	max						n
38	50	20	44	10	50	55	60	75	85	99

1		j	max							n
38	50	20	44	10	50	55	60	75	85	99

1	j		max							n
38	50	20	44	10	50	55	60	75	85	99

j	max									n
38	50	20	44	10	50	55	60	75	85	99

1	max									n
38	50	20	44	10	50	55	60	75	85	99

Ordenação por seleção

1			i							n
38	10	20	44	50	50	55	60	75	85	99

Ordenação por seleção

1			i							n
38	10	20	44	50	50	55	60	75	85	99

1		i								n
38	10	20	44	50	50	55	60	75	85	99

Ordenação por seleção

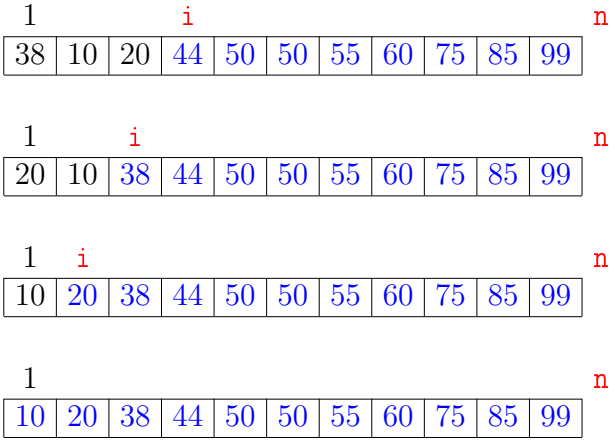
1			i							n
38	10	20	44	50	50	55	60	75	85	99

1		i								n
20	10	38	44	50	50	55	60	75	85	99

Ordenação por seleção

1	i									n
20	10	38	44	50	50	55	60	75	85	99

Ordenação por seleção



Função selecao

Algoritmo rearranja v[0 : n] em ordem crescente

```
def selecao(v):
0    n = len(v)
1    for i in range(n-1, 0, -1):  #B#
2        max = i
3        for j in range(i-1, -1, -1):
4            if v[j] > v[max]:  max = j
5        v[i], v[max] = v[max], v[i]
```

Função selecao

Algoritmo rearranja v[1 : n] em ordem crescente

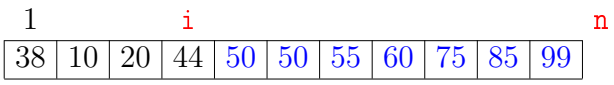
```
def selecao(v):
0    n = len(v)
1    for i in range(n-1, 1, -1):  #B#
2        max = i
3        for j in range(i-1, 0, -1):
4            if v[j] > v[max]:  max = j
5        v[i], v[max] = v[max], v[i]
```

Função selecao

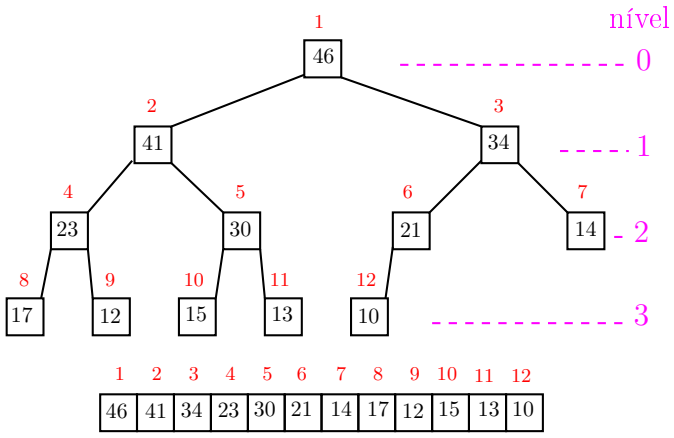
Relações invariantes: Em #B# vale que:

(i0) v[i+1 : n] é crescente;

(i1) v[1 : i+1] ≤ v[i+1];



Heapsort



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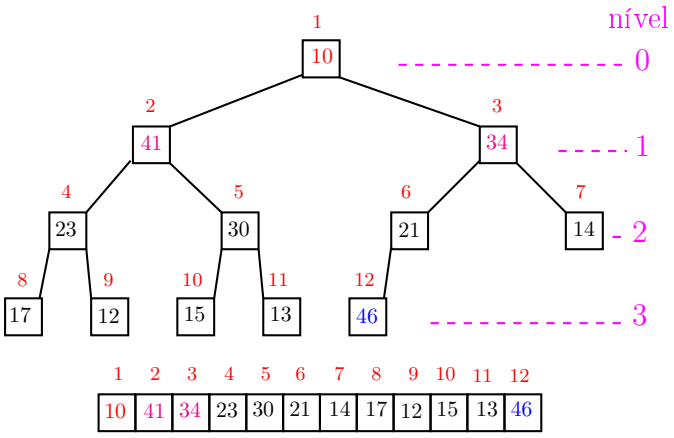

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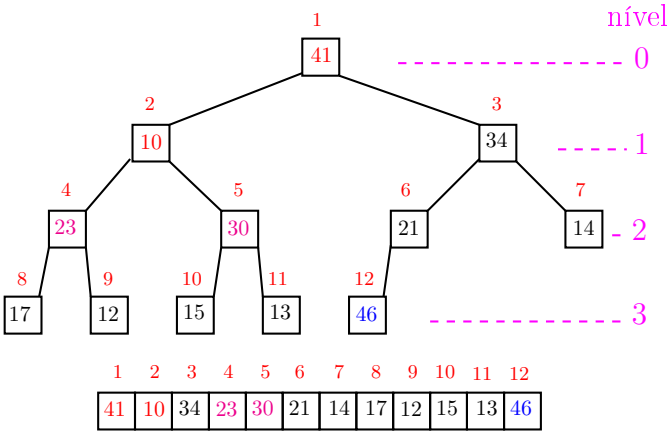

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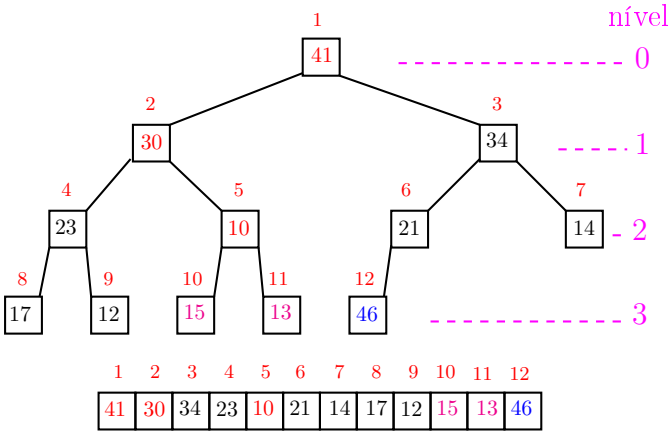

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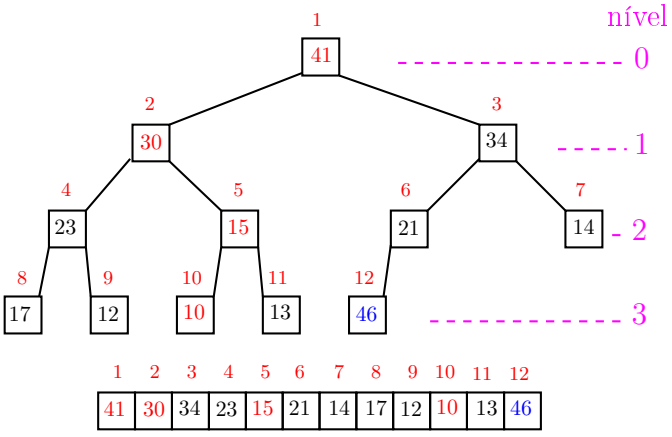

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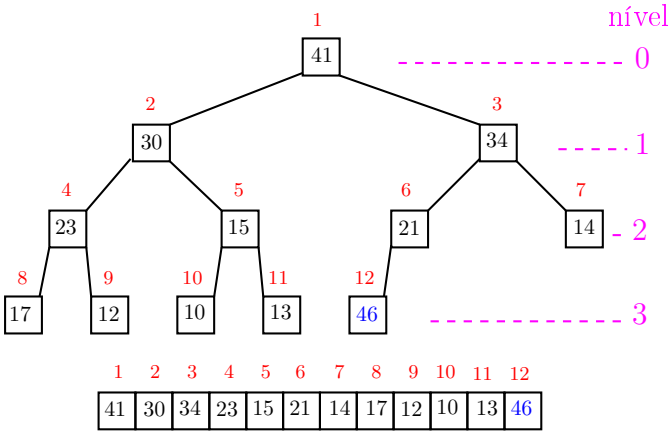

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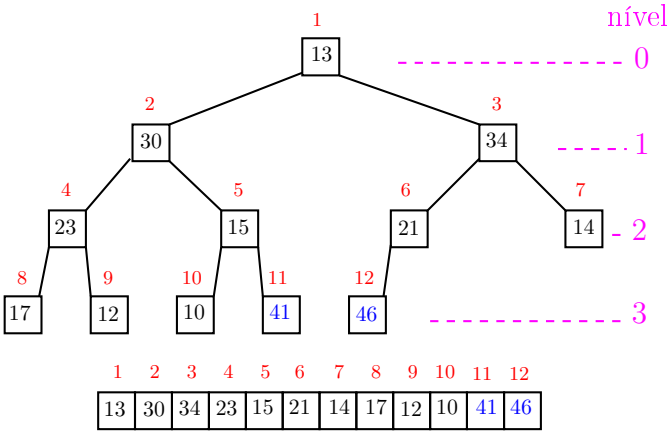

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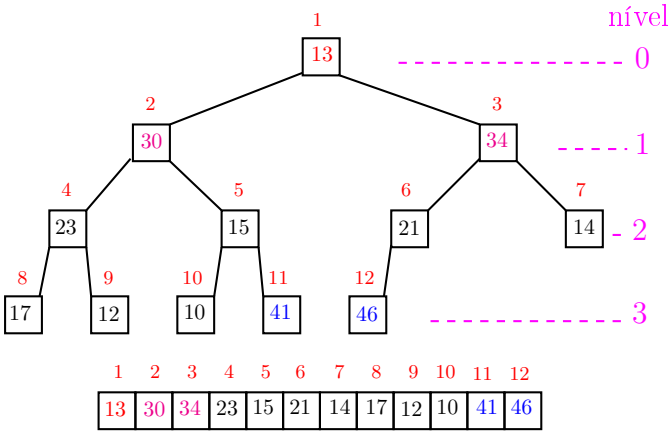

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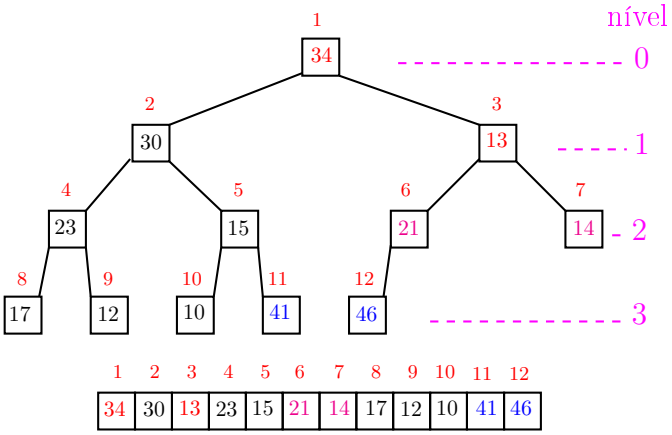

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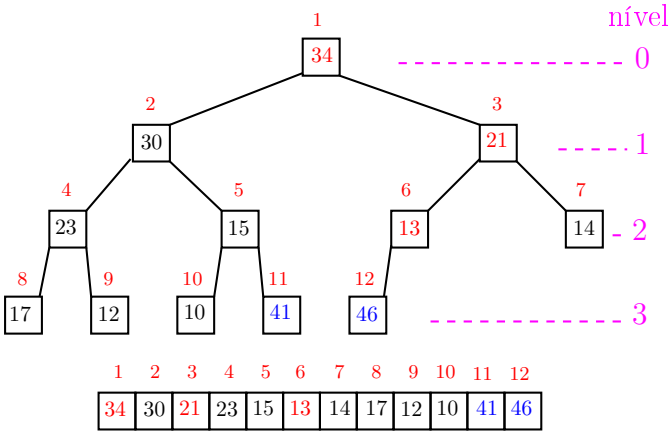

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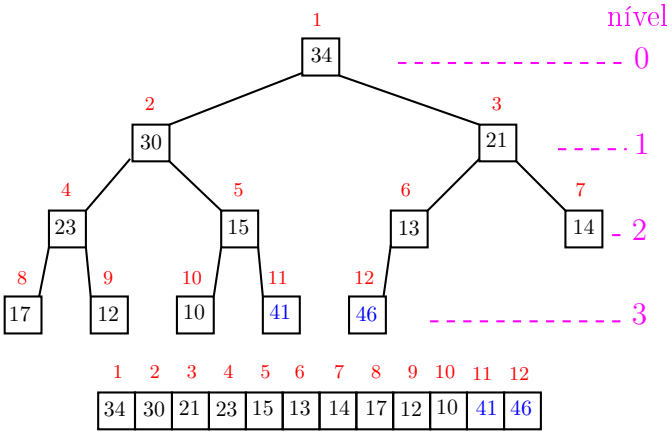

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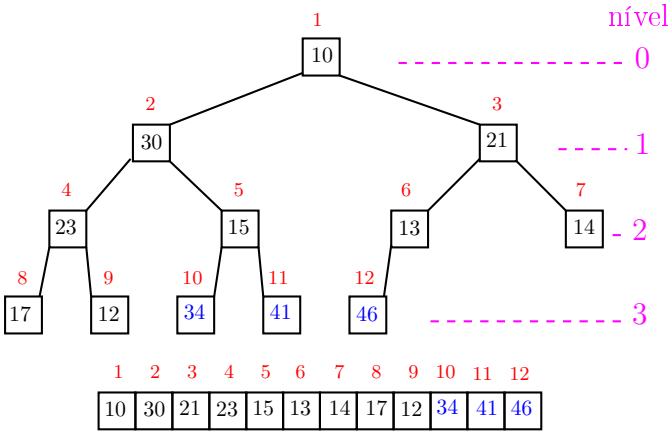

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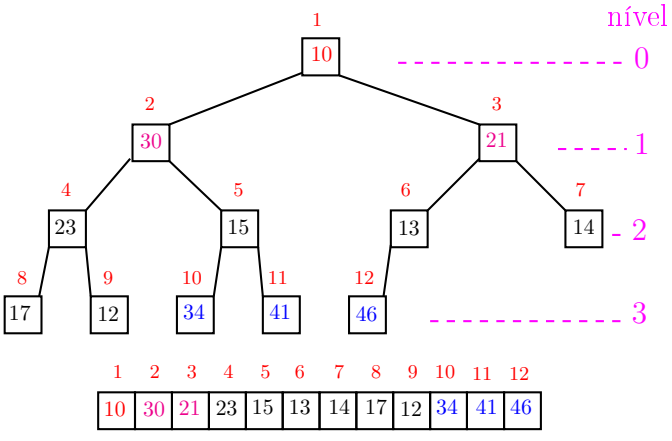

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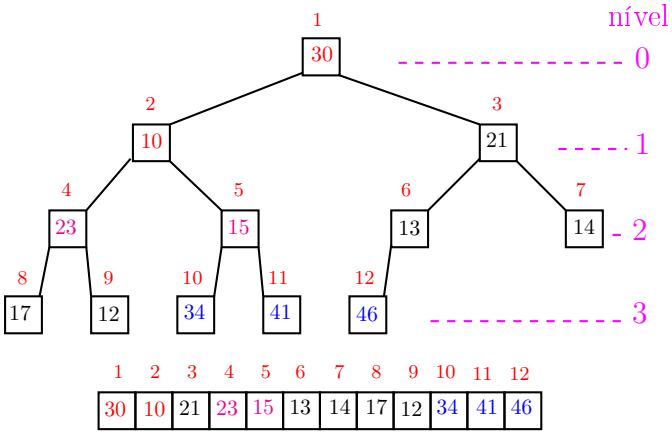

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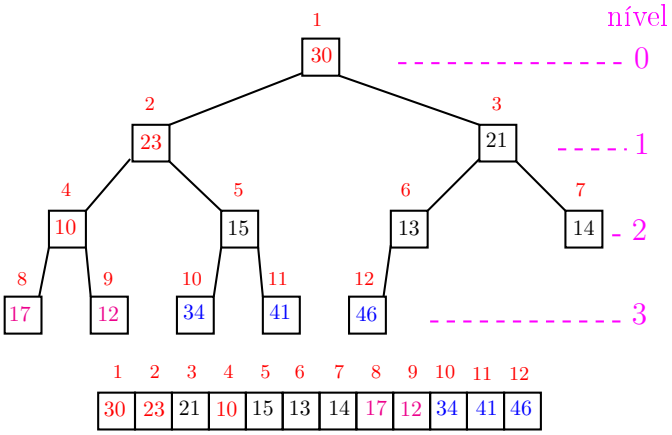

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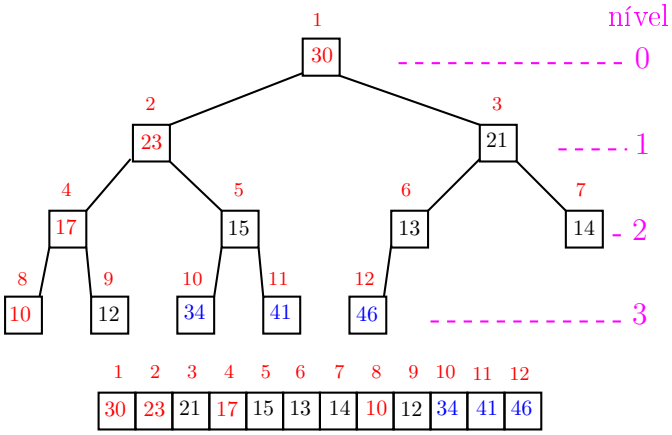

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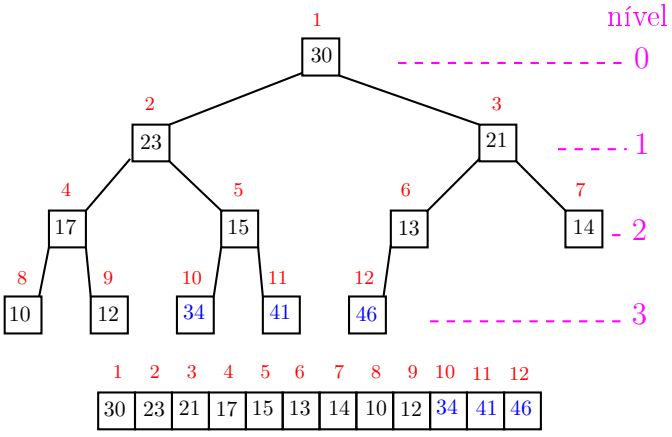

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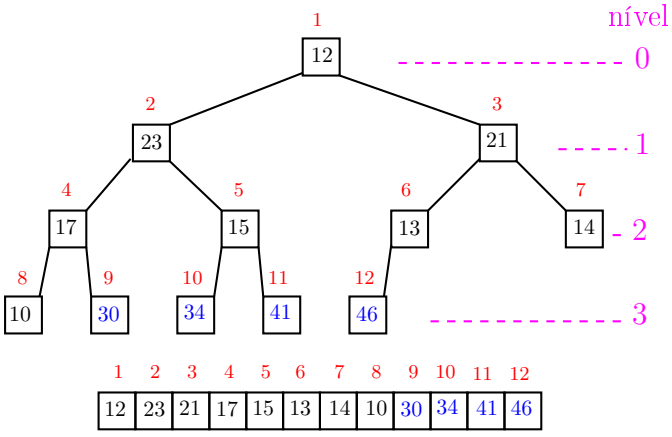

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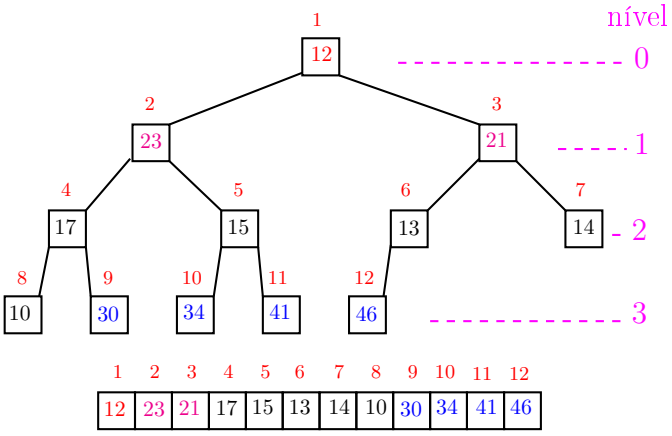

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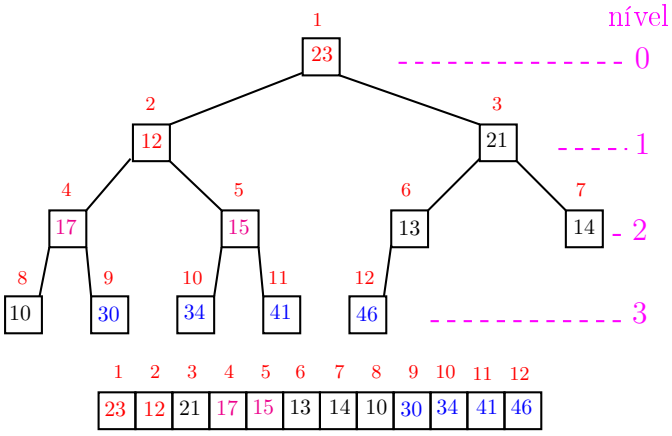

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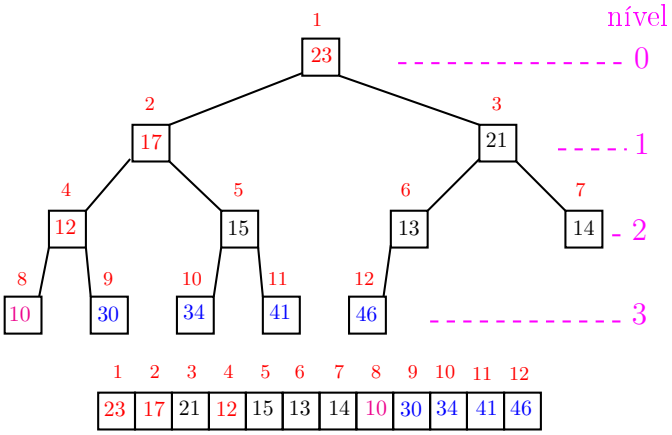

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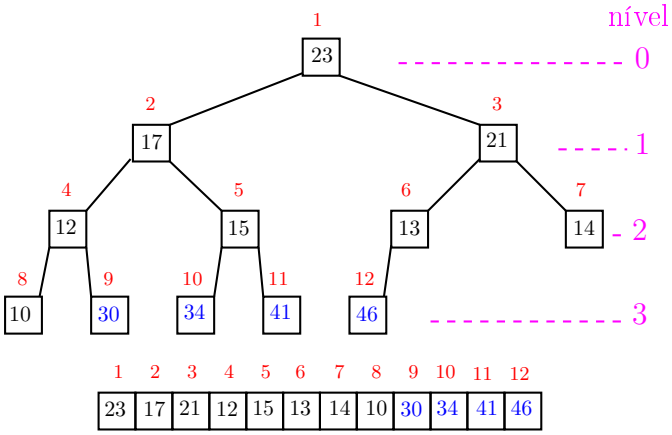

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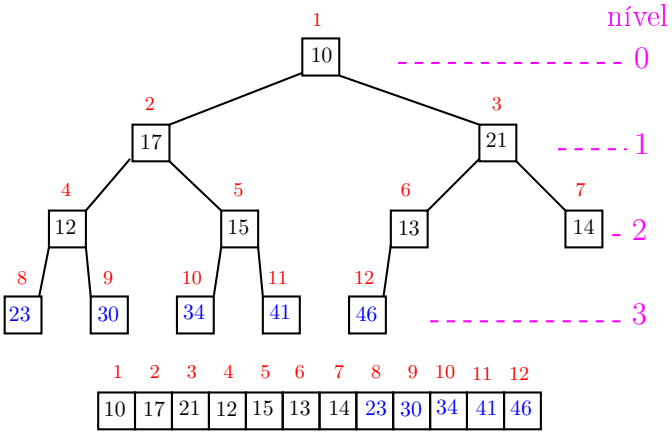

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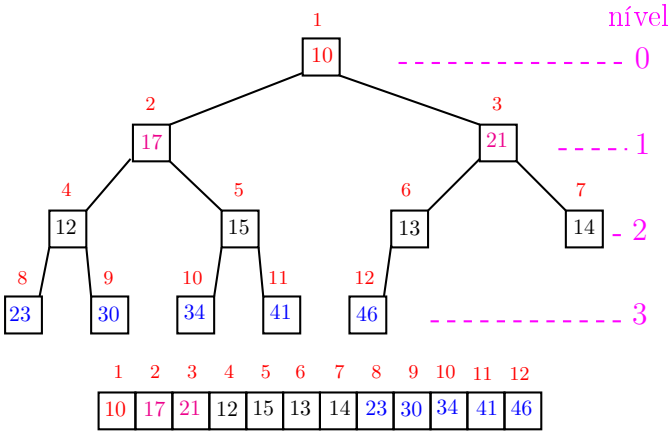

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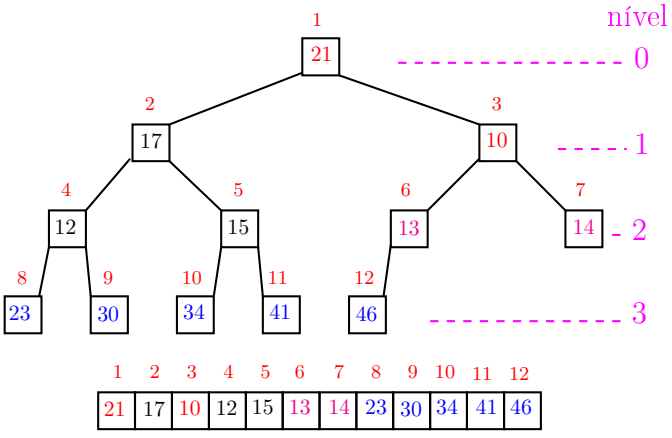

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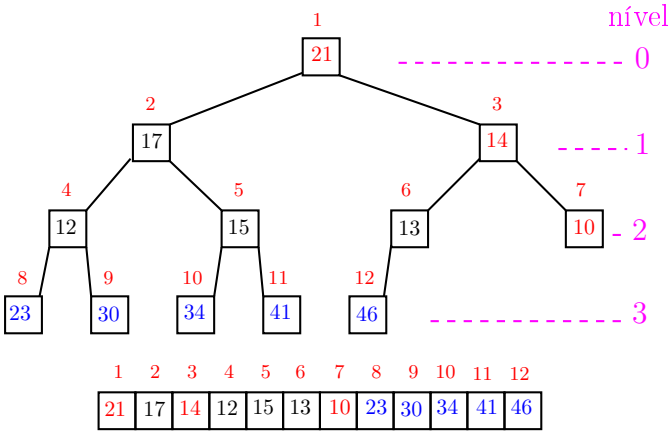

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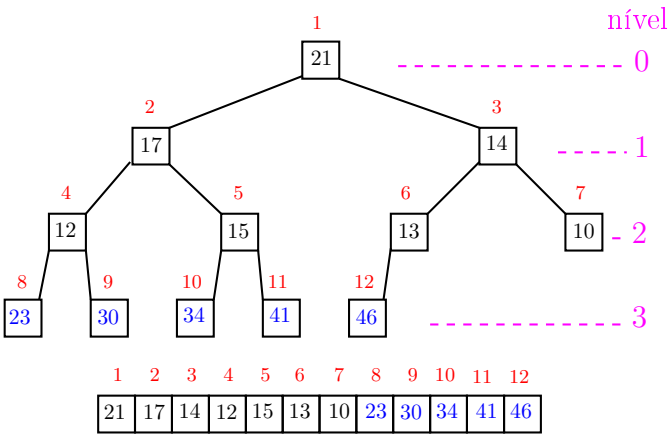

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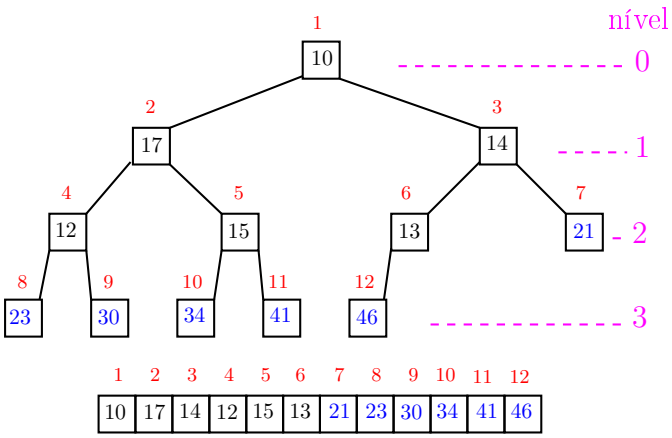

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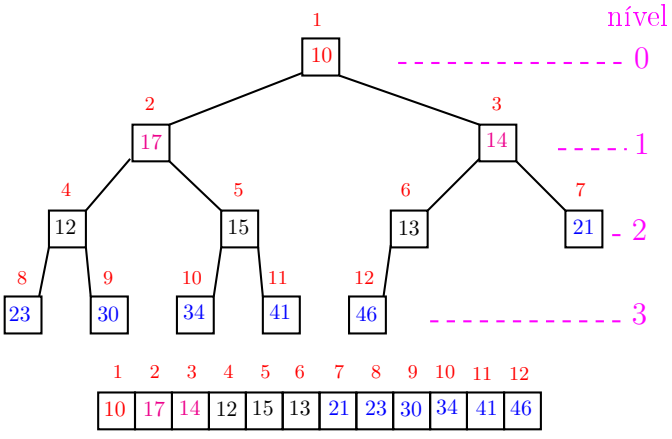

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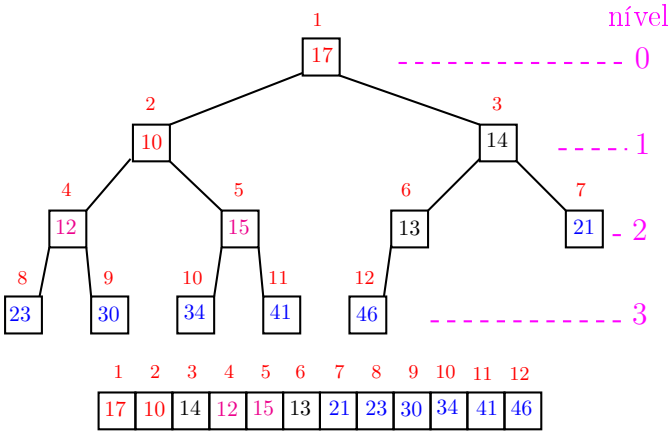

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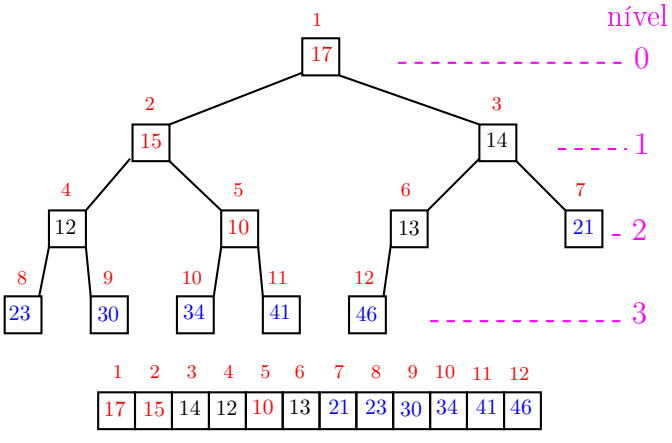

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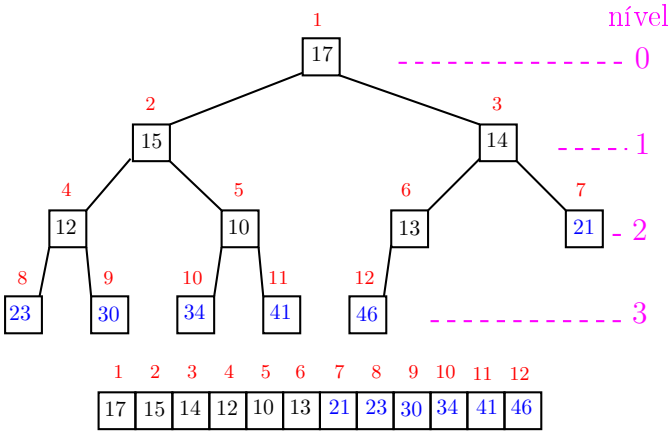

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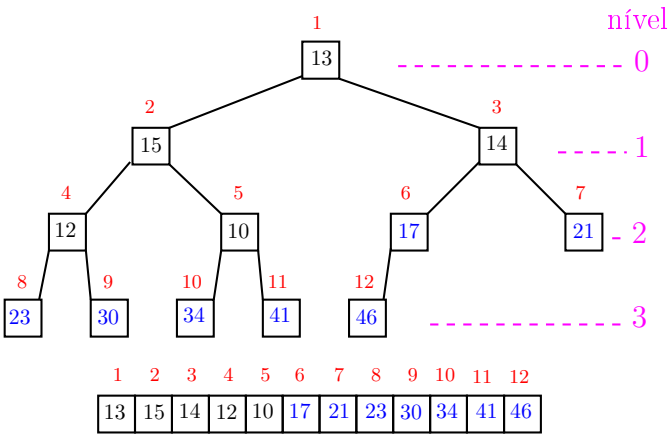

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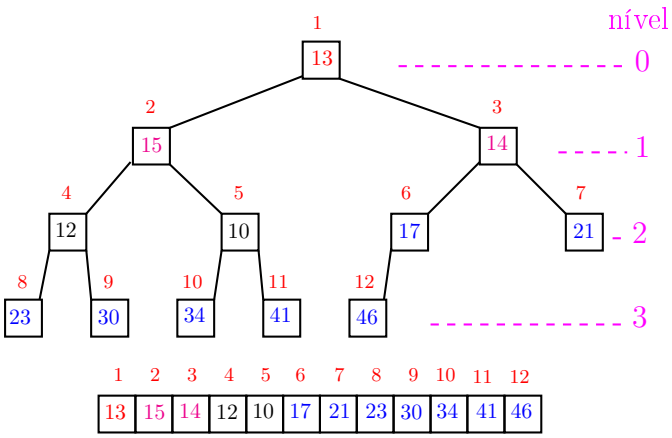

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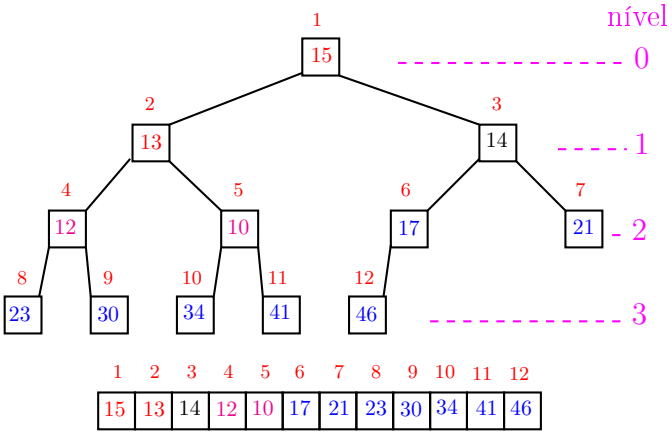

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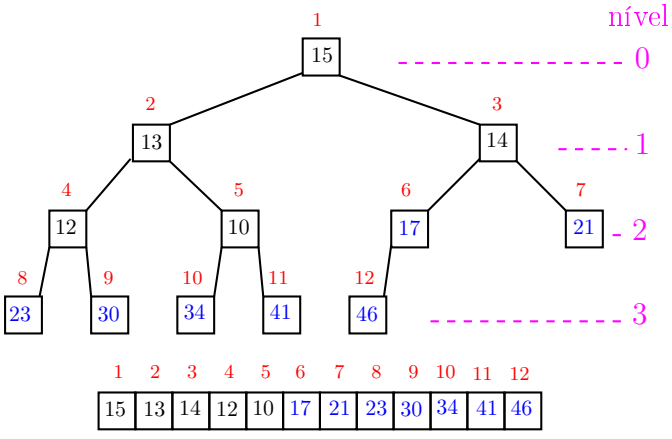

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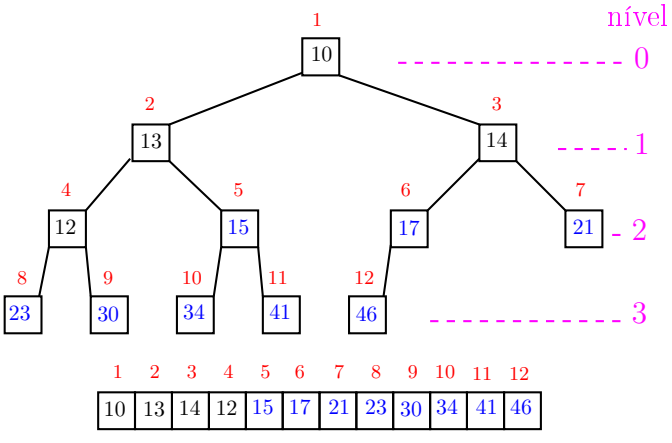

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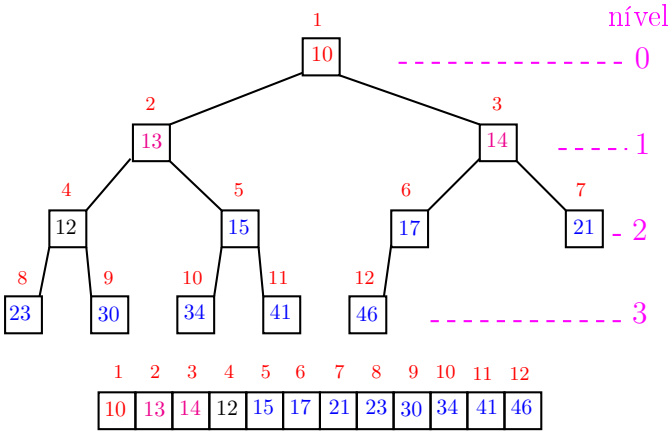

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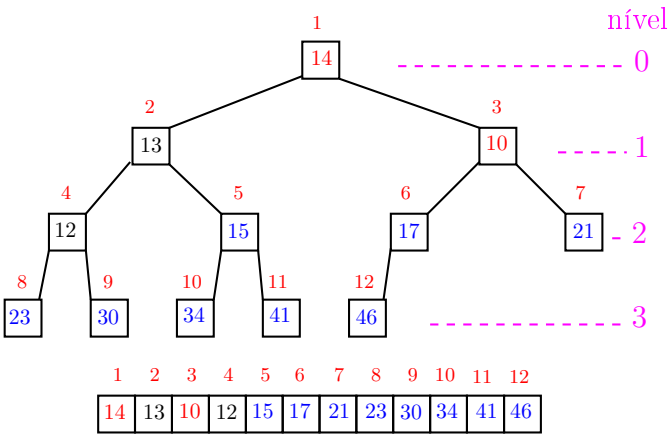

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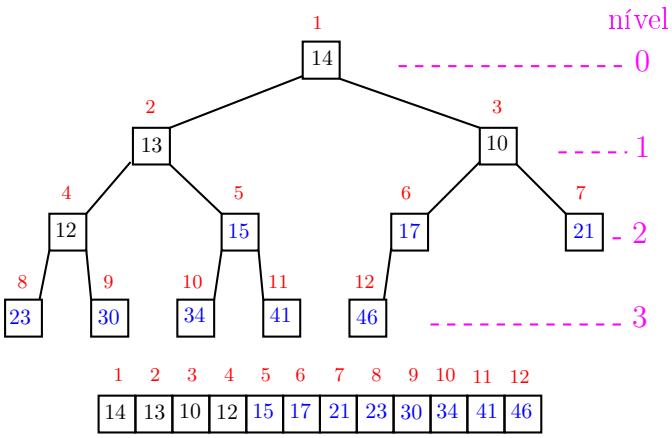

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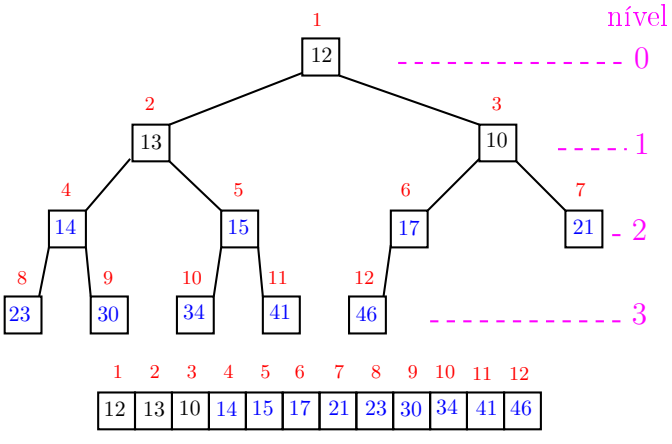

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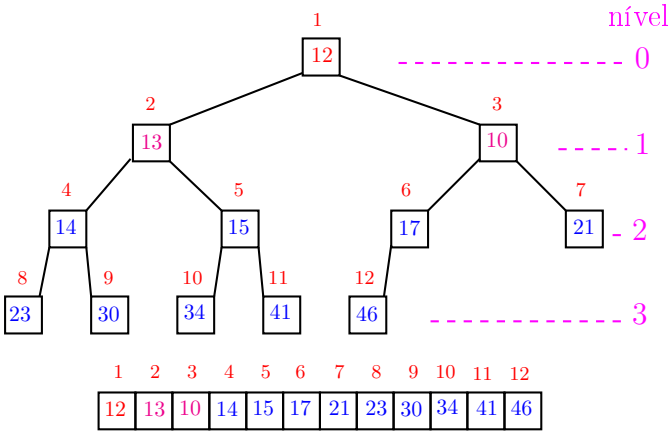

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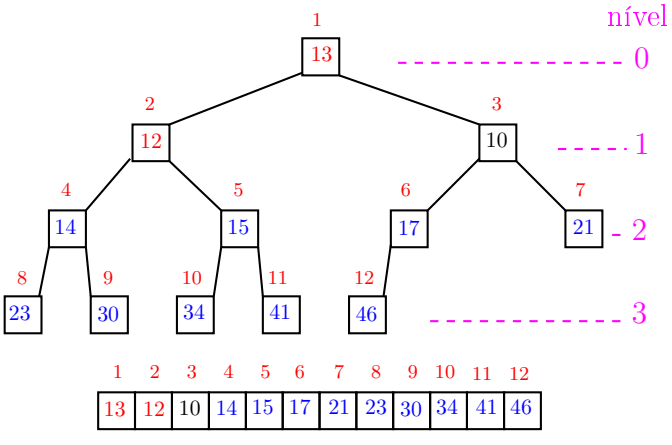

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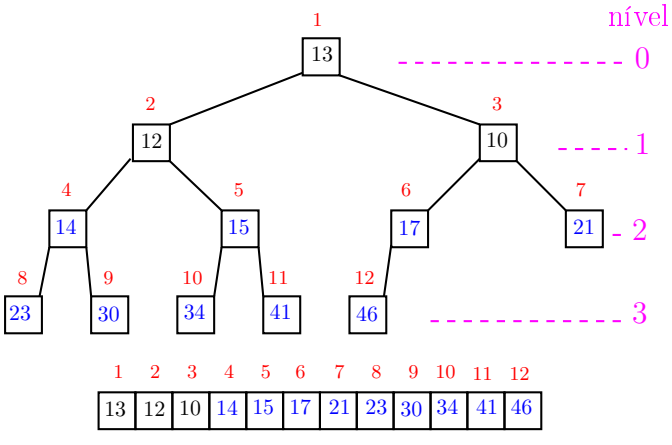

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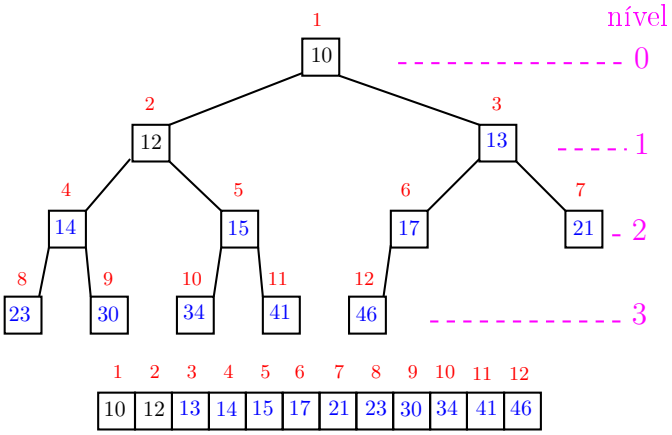

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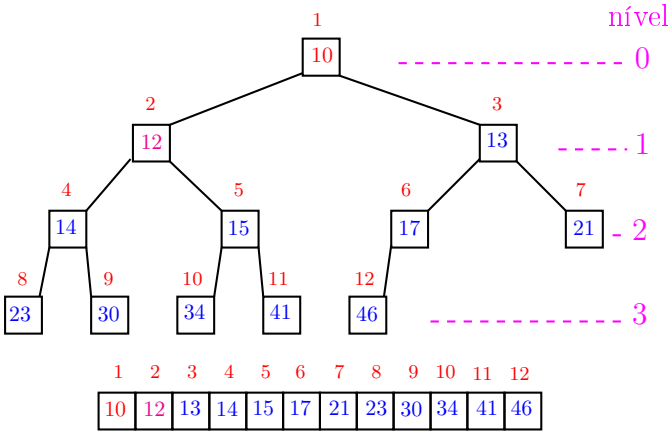

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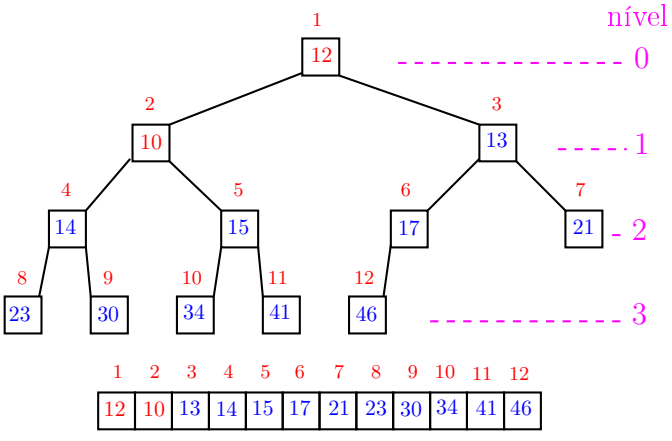
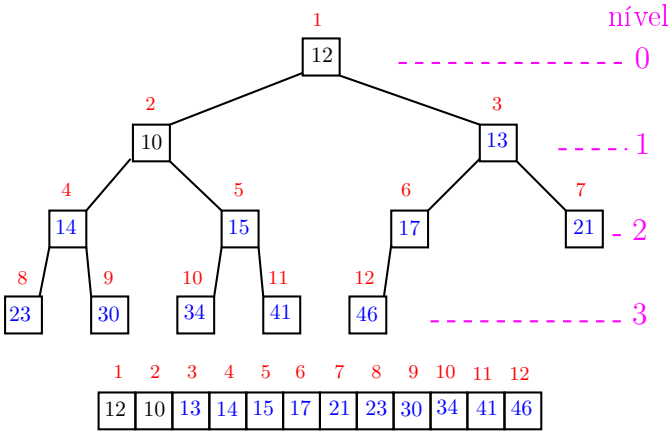
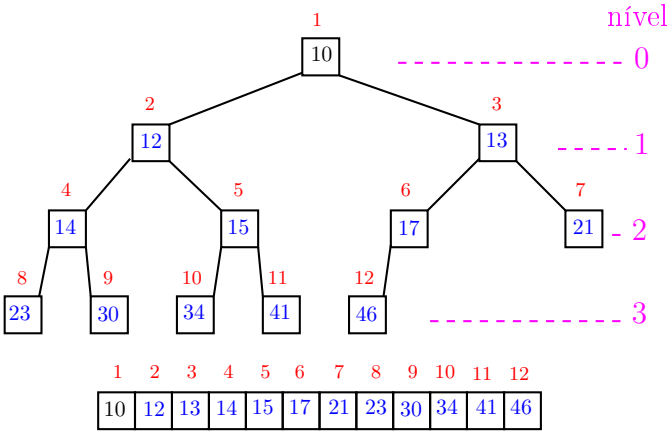


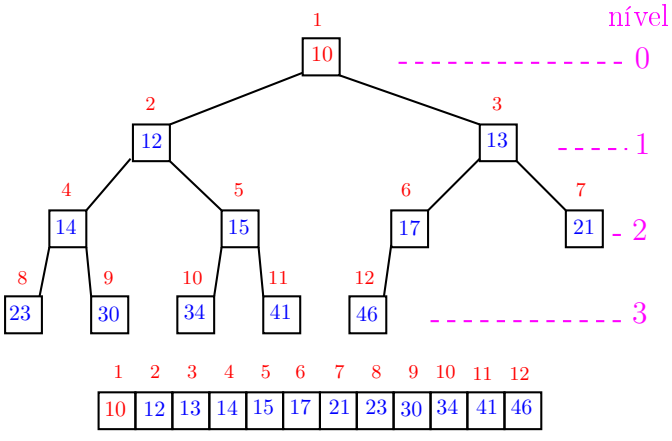
Heapsort



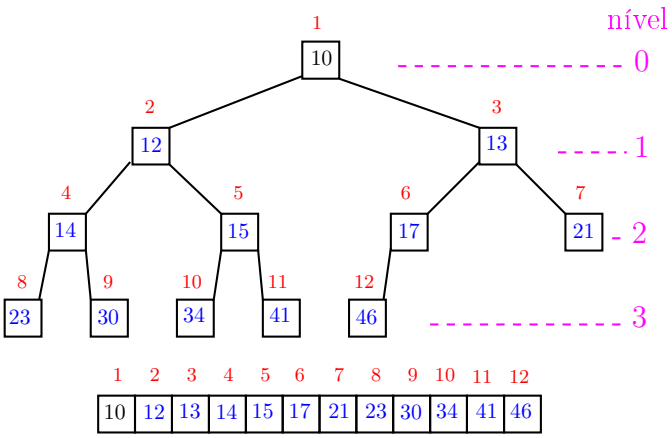
Heapsort



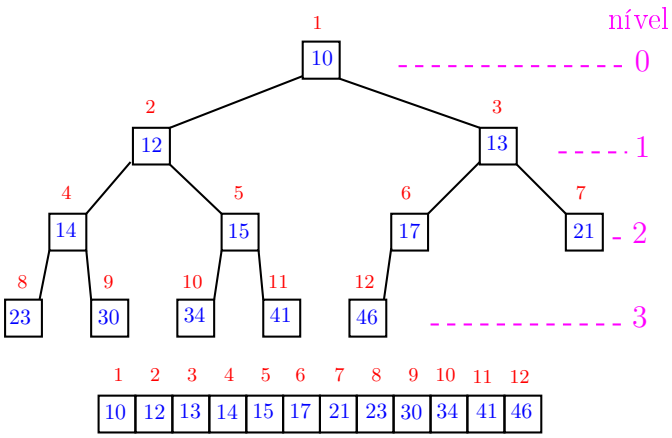
Heapsort



Heapsort



Heapsort



Função heap_sort

Algoritmo rearranja v[1 : n] em ordem crescente

```
def heap_sort(v):
0    n = len(v)
     # pre-processamento
1    for i in range((n-1)//2, 0, -1):
2        peneire(i,n,v)

3    for i in range(n-1, 1, -1):  #C#
4        v[i], v[1] = v[1], v[i]
5        peneire(1,i,v)
```

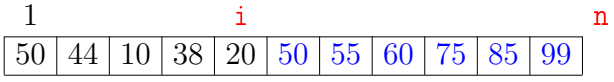
Função heap_sort

Relações invariantes: Em #C# vale que:

(i0) v[i+1 : n] é crescente;

(i1) v[1 : i+1] ≤ v[i+1];

(i2) v[1 : i+1] é um max-heap.



Consumo de tempo

linha	consumo de tempo das execuções da linha	
1-2	$\approx n \lg n$	$= O(n \lg n)$
3	$\approx n$	$= O(n)$
4	$\approx n$	$= O(n)$
5	$\approx n \lg n$	$= O(n \lg n)$
total	$= 2n \lg n + 2n$	$= O(n \lg n)$

Conclusão

O consumo de tempo da função heap_sort é proporcional a $n \lg n$.

O consumo de tempo da função heap_sort é $O(n \lg n)$.

Mais análise experimental

Algoritmos implementados:

- `mergeR` `merge_sort` recursivo.
- `mergeI` `merge_sort` iterativo.
- `quick` `quick_sort` recursivo.
- `heap` `heap_sort`.

Mais análise experimental

A plataforma utilizada nos experimentos foi um computador rodando Ubuntu GNU/Linux 4.4.0-40

Python: `Python 3.5.2`.

Computador:

```
 model name:  AMD FX(tm)-4300 Quad-Core Processor
cpu MHz :  3800.000
cache size:  2048 KB
MemTotal :  4095920 kB
```

Aleatório: média de 10

n	mergeR	mergeI	quick	heap
512	0.00	0.00	0.00	0.00
1024	0.01	0.01	0.00	0.01
2048	0.01	0.01	0.01	0.01
4096	0.03	0.02	0.02	0.03
8192	0.05	0.05	0.04	0.06
16384	0.11	0.11	0.09	0.13
32768	0.24	0.24	0.18	0.27
65536	0.50	0.50	0.40	0.60
131072	1.07	1.06	0.88	1.34
262144	2.27	2.27	1.87	2.96
524288	4.85	4.85	4.08	6.45
1048576	10.31	10.43	8.63	14.05

Tempos em segundos.

Decrescente

n	mergeR	mergeI	quick	heap
512	0.00	0.00	0.03	0.00
1024	0.01	0.00	*	0.01
2048	0.01	0.01	*	0.01
4096	0.02	0.02	*	0.02
8192	0.05	0.05	*	0.06
16384	0.11	0.10	*	0.12
32768	0.22	0.21	*	0.26
65536	0.48	0.46	*	0.56
131072	1.00	0.97	*	1.20
262144	2.10	2.05	*	2.57
524288	4.36	4.26	*	5.51

Tempos em segundos.

Para n=1024 `quick_sort` dá `RecursionError: maximum recursion depth exceeded in comparison`

Crescente

n	mergeR	mergeI	quick	heap
512	0.00	0.00	0.04	0.00
1024	0.01	0.00	*	0.01
2048	0.01	0.01	*	0.01
4096	0.02	0.02	*	0.03
8192	0.05	0.05	*	0.06
16384	0.11	0.10	*	0.13
32768	0.23	0.22	*	0.30
65536	0.49	0.47	*	0.62
131072	1.07	1.02	*	1.37
262144	2.17	2.09	*	2.77
524288	4.54	4.52	*	6.07

Tempos em segundos.

Para n=1024 `quick_sort` dá `RecursionError: maximum recursion depth exceeded in comparison`

Resumo

função	consumo de tempo	observação
`bubble`	$O(n^2)$	todos os casos
`insercao`	$O(n^2)$	pior caso
	$O(n)$	melhor caso
`insercao_binaria`	$O(n^2)$	pior caso
	$O(n \lg n)$	melhor caso
`selecao`	$O(n^2)$	todos os casos
`merge_sort`	$O(n \lg n)$	todos os casos
`quick_sort`	$O(n^2)$	pior caso
	$O(n \lg n)$	melhor caso
`heap_sort`	$O(n \lg n)$	todos os casos

Animação de algoritmos de ordenação

Criados por Nicholas André Pinho de Oliveira:
http://nicholasandre.com.br/sorting/

Criados na Sapientia University (Romania):
https://www.youtube.com/channel/UCIqiLefbVHsOAXDAxQJH7

Fila de prioridades

PF 10
http://www.ime.usp.br/~pf/algoritmos/aulas/hpsrt.html

Fila de prioridades

Uma **fila de prioridades** (= *priority queue*) é qualquer tipo-de-dados abstrato dotado de duas operações:

- retirar um elemento de valor máximo;
- acrescentar um novo elemento.

É fácil implementar a fila de prioridades utilizando um max-heap.
A definição acima é a de uma fila de prioridades "de máximo". Não é difícil adaptar essa definição para filas de prioridades "de mínimo".

Função remova_heap

Remoção de um elemento x de um max-heap $v[1 : n]$

```
def remova_heap(v):
1    x = v.pop()
2    peneire(1,len(v),v)
3    return x
```

Função insira_heap

Inseção de um elemento x em um max-heap $v[1 : n]$

```
def insira_heap(x, v):
0    n = f = len(v) # f = filho
1    v.append(x)
2    p = f // 2 # p = pai
3    while f > 1 and v[p] < v[f]:   #D#
         # troca o filho com o pai
4        v[p], v[f] = v[f], v[p]
         # pai no papel de filho
5        f = p
6        p = f // 2
```

Função insira_heap

Relações invariantes: Em /*D*/ vale que:

(i0) $v[1 : n]$ é uma permutação da lista original
(i1) $v[i//2] \geq v[i]$ para todo $i = 2, \ldots, n-1$ diferente de f.

1				f						n
83	75	25	68	99	15	10	60	57	65	79

Conclusões

O consumo de tempo da função `remova_heap` é proporcional a $\lg n$, onde `n` é o número de elementos no `max-heap`.

O consumo de tempo da função `insira_heap` é proporcional a $\lg n$, onde `n` é o número de elementos no `max-heap`.