Aula 04: 24/03/2017

Tópico

- *union-find*

Union-Find

Leitura: Case Study: Union-Find, S&W

Vídeos: *Union-find* e Kruskal, Gabriel Russo, canal BCC e *Union-find*, Robert Sedgewick.

Considere uma coleção de conjuntos disjuntos $S_1, S_2, \ldots, S_n$.

Conjuntos são modificados ao longo do tempo.

Terminologia utiliza metáfora de redes: sítios/*sites*, conexão,...

API

```
public class UF
-----------------------------------------------------------------------
            UF(int n)               inicialize n sites com nomes inteiros 0,...,n-1
       void union(int p, int q)     acrescenta ligação os sítios  p e q
        int find(int p)             retorna nome (=id) do componente contendo p
    boolean connected(int p, int q) true se p e q estão em um mesmo componente
        int count()                 número de componentes
```

Uso

```
% java Cliente
3
1 2
1 2
2 components
1 0
1 0
1 components
2 0
1 components
% java Cliente < tinyUF.txt
4 3
9 components
3 8
8 components
6 5
7 components
9 4
```

```
6 components
2 1
5 components
8 9
5 components
5 0
4 components
7 2
3 components
6 1
2 components
1 0
2 components
6 7
2 components
```

Estrutura

```
QuickFindUF uf = new QuickFindUF(10);

uf ----+
       |
       V
+-----------------------------------------------------+
| id (private)                   count: 0 (private)   |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-----------------------------------------------------+
```

Quick-find

```
UF uf = new UF(10);

uf ----+
       |
       V
+-----------------------------------------------------+
| id (private)                   count: 10 (private)  |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
```

```
|         0   1   2   3   4   5   6   7   8   9      |
|                                                    |
| Métodos: cont(), connected(), find(), union()      |
|                                                    |
+----------------------------------------------------+
```

```
uf.find(3) retorna 3
uf.find(0) retorna 0

uf.union(4, 3);
```

```
uf ----+
       |
       V
+----------------------------------------------------+
| id (private)                  count:  9 (private)  |
|  |                                                 |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 3 | 3 | 5 | 6 | 7 | 8 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                    |
| Métodos: cont(), connected(), find(), union()      |
|                                                    |
+----------------------------------------------------+
```

```
uf.find(3) retorna 3
uf.find(4) retorna 3

uf.union(3, 8);
```

```
uf ----+
       |
       V
+----------------------------------------------------+
| id (private)                  count:  8 (private)  |
|  |                                                 |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 8 | 8 | 5 | 6 | 7 | 8 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                    |
| Métodos: cont(), connected(), find(), union()      |
|                                                    |
+----------------------------------------------------+
```

```
uf.union(6, 5);

uf ----+
```

```
       |
       V
+-----------------------------------------------------+
| id (private)                    count:  7 (private) |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 8 | 8 | 5 | 5 | 7 | 8 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-----------------------------------------------------+

uf.union(9, 4);

uf ----+
       |
       V
+-----------------------------------------------------+
| id (private)                    count:  6 (private) |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 8 | 8 | 5 | 5 | 7 | 8 | 8 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-----------------------------------------------------+

uf.union(2, 1);

uf ----+
       |
       V
+-----------------------------------------------------+
| id (private)                    count:  5 (private) |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 1 | 8 | 8 | 5 | 5 | 7 | 8 | 8 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-----------------------------------------------------+

uf.union(8, 9);
```

```
uf ----+
       |
       V
+-----------------------------------------------------+
| id (private)                  count:  5 (private)   |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 1 | 8 | 8 | 5 | 5 | 7 | 8 | 8 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-----------------------------------------------------+
```

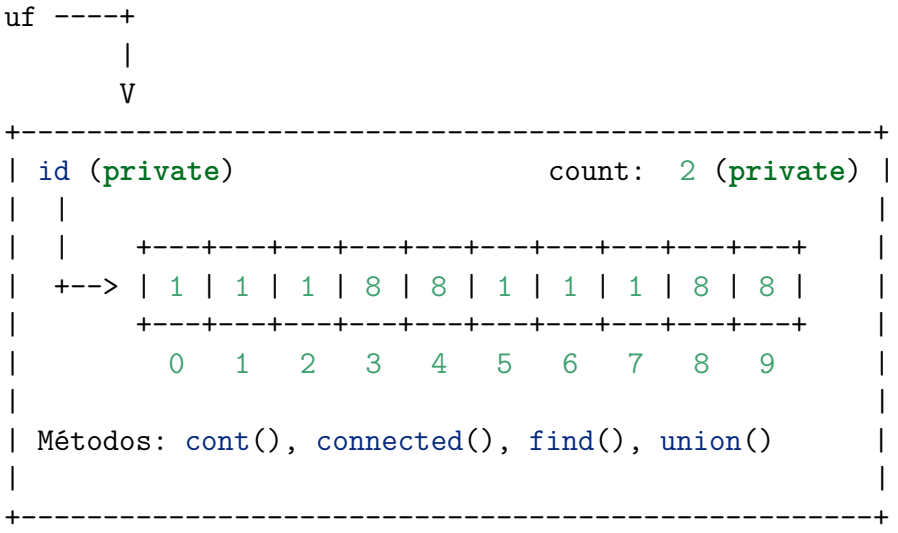
```
uf.union(5, 0);
```

```
uf ----+
       |
       V
+-----------------------------------------------------+
| id (private)                  count:  5 (private)   |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 1 | 8 | 8 | 0 | 0 | 7 | 8 | 8 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-----------------------------------------------------+
```

```
uf.union(7, 2);
```

```
uf ----+
       |
       V
+-----------------------------------------------------+
| id (private)                  count:  3 (private)   |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 1 | 8 | 8 | 0 | 0 | 1 | 8 | 8 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-----------------------------------------------------+
```

```
uf.union(6, 1);
```



[...]

```java
public class QuickFindUF {
    private int[] id;
    private int count;

    // Initializes an empty union-find data structure with n
    // isolated components 0 through n-1
    public UF(int n) {
        count = n;
        id = new int[n];
        for (int i = 0; i < n; i++) {
            id[i] = i;
        }
    }

    // Return the number of components.
    public int count() {
        return count;
    }

    // Are the two sites p and q in the same component?
    public boolean connected(int p, int q) {
        return find(p) == find(q);
    }

    // Return the component identifier for the component
    // containing site p.
    public int find(int p) {
        return id[p];
    }

    // Merge the component containing site p with the the
    // component containing site q.
```

```
    public void union(int p, int q) {
        int pID = find(p);
        int qID = find(q);

        if (pID == qID) return ;

        for (int i = 0; i < id.length; i++) {
            if (id[i] == pID) id[i] = qID;
        }
        count--;
    }
}
```

```
    Consumo de tempo
    ----------------
    UF():   n
    union(): n
    find(): 1
```

testes

```
    % java Driver < tinyUF.txt
    2 components
    0.002seg
    % java Driver < mediumUF.txt
    3 components
    0.032seg
    % java Driver < largeUF.txt
    :-(
```

Hmm. Em `union()` seria razoáveis alterarmos o menor número possível de posições do vetor `id[]`. Para isso precisamos saber qual conjunto tem o menor número de itens.

Quick-union

A ideia é trocar o indicador `id` do componente por um indicador do "pai" do sítio. Por sua vez, se `p` é um sítio, o pai `p` indica quem é avô (pai[pai[p]]), que indica que é o bisavô (pai[pai[pia[p]]]), que indica o sítio tataravô,...

O **representante** ou **nome** de um componente será o sítio que é o pai de si mesmo. Hmm. Aqui a metáfora fica meio estranha.

É intuitivo representarmos a estrutura através de um conjunto de árvores disjuntas (= **floresta**) onde as raizes das árvores são os sítios `p` tais que `p == pai[p]`.

```
QuickUnionUF uf = new QuickUnion(10);

uf ----+
       |
       V
```

```
+-------------------------------------------------+
| pai (private)                  count: 10 (private)|
|  |                                              |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-------------------------------------------------+

uf.find(3) retorna 3
uf.find(0) retorna 0

uf.union(4, 3);

uf ----+
       |
       V
+-------------------------------------------------+
| pai (private)                  count:  9 (private)|
|  |                                              |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 3 | 3 | 5 | 6 | 7 | 8 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-------------------------------------------------+

uf.find(3) retorna 3
uf.find(4) retorna 3

uf.union(3, 8);

uf ----+
       |
       V
+-------------------------------------------------+
| pai (private)                  count:  8 (private)|
|  |                                              |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 8 | 3 | 5 | 6 | 7 | 8 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
```

```
+--------------------------------------------------+


uf.union(6, 5);

uf ----+
       |
       V
+--------------------------------------------------+
| pai (private)                  count:  7 (private)|
|  |                                               |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 8 | 3 | 5 | 5 | 7 | 8 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                    |
| Métodos: cont(), connected(), find(), union()      |
|                                                    |
+--------------------------------------------------+

uf.union(9, 4);

uf ----+
       |
       V
+--------------------------------------------------+
| pai (private)                  count:  6 (private)|
|  |                                               |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 8 | 3 | 5 | 5 | 7 | 8 | 8 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                    |
| Métodos: cont(), connected(), find(), union()      |
|                                                    |
+--------------------------------------------------+

uf.union(2, 1);

uf ----+
       |
       V
+--------------------------------------------------+
| pai (private)                  count:  5 (private)|
|  |                                               |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 1 | 8 | 3 | 5 | 5 | 7 | 8 | 8 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                    |
```

```
| Métodos: cont(), connected(), find(), union()      |
|                                                    |
+----------------------------------------------------+

uf.union(8, 9);

uf ----+
       |
       V
+----------------------------------------------------+
| pai (private)                  count:  5 (private)|
|  |                                                 |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 1 | 8 | 3 | 5 | 5 | 7 | 8 | 8 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                    |
| Métodos: cont(), connected(), find(), union()      |
|                                                    |
+----------------------------------------------------+

uf.union(5, 0);

uf ----+
       |
       V
+----------------------------------------------------+
| pai (private)                  count:  4 (private)|
|  |                                                 |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 1 | 8 | 3 | 0 | 5 | 7 | 3 | 3 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                    |
| Métodos: cont(), connected(), find(), union()      |
|                                                    |
+----------------------------------------------------+

uf.union(7, 2);

uf ----+
       |
       V
+----------------------------------------------------+
| pai (private)                  count:  3 (private)|
|  |                                                 |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 1 | 8 | 3 | 0 | 5 | 1 | 3 | 3 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
```

```
|                                                      |
| Métodos: cont(), connected(), find(), union()        |
|                                                      |
+------------------------------------------------------+

uf.union(6, 1);

uf ----+
       |
       V
+------------------------------------------------------+
| pai (private)                    count:  2 (private) |
|  |                                                   |
|  |    +---+---+---+---+---+---+---+---+---+---+     |
|  +--> | 1 | 1 | 1 | 8 | 3 | 0 | 1 | 1 | 3 | 3 |     |
|       +---+---+---+---+---+---+---+---+---+---+     |
|         0   1   2   3   4   5   6   7   8   9       |
|                                                      |
| Métodos: cont(), connected(), find(), union()        |
|                                                      |
+------------------------------------------------------+

[...]
```

```java
public class QuickUnionUF {
    private int[] pai;
    private int count;

    // Initializes an empty union-find data structure with n
    // isolated components 0 through n-1
    public UF(int n) {
        count = n;
        pai = new int[n];
        for (int i = 0; i < n; i++) {
            pai[i] = i;
        }
    }

    // Return the number of components.
    public int count() {
        return count;
    }

    // Are the two sites p and q in the same component?
    public boolean connected(int p, int q) {
        return find(p) == find(q);
    }

    // Return the component paientifier for the component
    // containing site p.
```

```
    public int find(int p) {
        while (p != pai[p]) p = pai[p];
        return p;
    }

    // Merge the component containing site p with the the
    // component containing site q.
    public void union(int p, int q) {
        int pRoot = find(p);
        int qRoot = find(q);
        // if (pRoot == qRoot) return ;
        pai[pRoot] = pai[qRoot];
        count--;
    }
}
```

```
    Consumo de tempo
    ----------------
    UF():   n
    union(): n
    find(): n
```

Experimentos

```
    % java Driver < ../testes/tinyUF.txt
    2 components
    0.002seg
    % java Driver < ../testes/mediumUF.txt
    3 components
    0.025seg
    % java Driver < ../testes/largeUF.txt
    :-(
```

Weighted quick-union

Ideia, ligar a raiz da árvore com menos sítios na raiz da árvore com mais sítios. Isso seria a política natural para tornarmos o quick-find mais eficiente.

Estrutura

```
UF uf = new UF(10);

uf ----+
       |
       V
+-------------------------------------------------+
| pai (private)                  count: 0 (private)  |
```

```
|  |                                                    |
|  |    +---+---+---+---+---+---+---+---+---+---+      |
|  +--> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |      |
|       +---+---+---+---+---+---+---+---+---+---+      |
|         0   1   2   3   4   5   6   7   8   9        |
|                                                       |
| sz (private)                                          |
|  |                                                    |
|  |    +---+---+---+---+---+---+---+---+---+---+      |
|  +--> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |      |
|       +---+---+---+---+---+---+---+---+---+---+      |
|         0   1   2   3   4   5   6   7   8   9        |
|                                                       |
| Métodos: cont(), connected(), find(), union()         |
|                                                       |
+-------------------------------------------------------+
```

```
WeightedQuickUnionUF uf = new WeightedQuickUnionUF(10);
```

```
uf ----+
       |
       V
+-------------------------------------------------------+
| pai (private)                    count: 10 (private)|
|  |                                                    |
|  |    +---+---+---+---+---+---+---+---+---+---+      |
|  +--> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |      |
|       +---+---+---+---+---+---+---+---+---+---+      |
|         0   1   2   3   4   5   6   7   8   9        |
|                                                       |
| sz (private)                                          |
|  |                                                    |
|  |    +---+---+---+---+---+---+---+---+---+---+      |
|  +--> | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |      |
|       +---+---+---+---+---+---+---+---+---+---+      |
|         0   1   2   3   4   5   6   7   8   9        |
|                                                       |
|                                                       |
| Métodos: cont(), connected(), find(), union()         |
|                                                       |
+-------------------------------------------------------+
```

```
uf.find(3) retorna 3
uf.find(0) retorna 0
```

```
uf.union(4, 3);
```

```
uf ----+
       |
       V
```

```
+-------------------------------------------------+
| pai (private)                  count:  9 (private)|
|  |                                              |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| sz (private)                                        |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 1 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-------------------------------------------------+
```

```
uf.find(3) retorna 3
uf.find(4) retorna 3

uf.union(3, 8);
```

```
uf ----+
       |
       V
+-------------------------------------------------+
| pai (private)                  count:  8 (private)|
|  |                                              |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 4 | 4 | 5 | 6 | 7 | 4 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| sz (private)                                        |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-------------------------------------------------+
```

```
uf.union(6, 5);
```

```
uf ----+
       |
       V
+-------------------------------------------------+
| pai (private)                  count:  7 (private) |
|  |                                               |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 4 | 4 | 6 | 6 | 7 | 4 | 9 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| sz (private)                                        |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 1 | 1 | 1 | 1 | 3 | 1 | 2 | 1 | 1 | 1 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-------------------------------------------------+

uf.union(9, 4);

uf ----+
       |
       V
+-------------------------------------------------+
| pai (private)                  count:  6 (private) |
|  |                                               |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 2 | 4 | 4 | 6 | 6 | 7 | 8 | 4 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| sz (private)                                        |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 1 | 1 | 1 | 1 | 4 | 1 | 2 | 1 | 1 | 1 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-------------------------------------------------+

uf.union(2, 1);
```

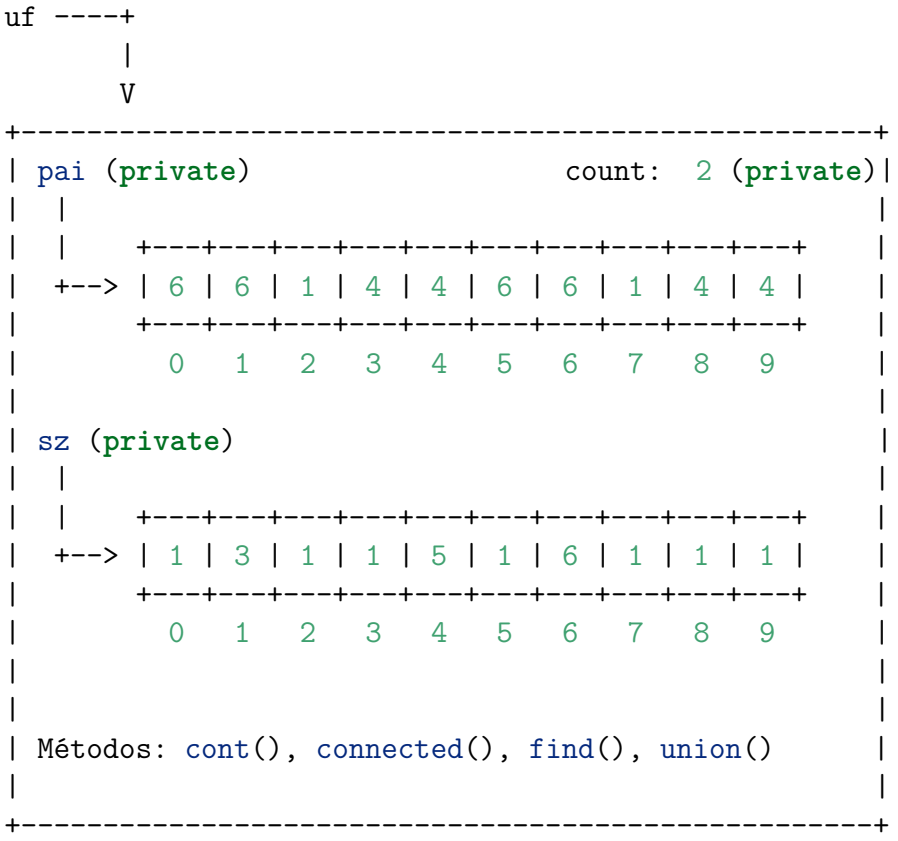
```
uf ----+
       |
       V
+-------------------------------------------------+
| pai (private)                  count:  5 (private)|
|  |                                               |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 1 | 4 | 4 | 6 | 6 | 7 | 8 | 4 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| sz (private)                                        |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 1 | 2 | 1 | 1 | 3 | 1 | 2 | 1 | 1 | 1 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-------------------------------------------------+

uf.union(8, 9);

uf ----+
       |
       V
+-------------------------------------------------+
| pai (private)                  count:  4 (private)|
|  |                                               |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 0 | 1 | 1 | 4 | 4 | 6 | 6 | 7 | 4 | 4 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| sz (private)                                        |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 1 | 2 | 1 | 1 | 4 | 1 | 2 | 1 | 1 | 1 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-------------------------------------------------+

uf.union(5, 0);
```

```
uf ----+
       |
       V
+-------------------------------------------------+
| pai (private)                  count:  3 (private)|
|  |                                               |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 6 | 1 | 1 | 4 | 4 | 6 | 6 | 7 | 4 | 4 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| sz (private)                                        |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 1 | 2 | 1 | 1 | 4 | 1 | 3 | 1 | 1 | 1 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-------------------------------------------------+
```

```
uf.union(7, 2);
```

```
uf ----+
       |
       V
+-------------------------------------------------+
| pai (private)                  count:  2 (private)|
|  |                                               |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 6 | 1 | 1 | 4 | 4 | 6 | 6 | 1 | 4 | 4 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
| sz (private)                                        |
|  |                                                  |
|  |    +---+---+---+---+---+---+---+---+---+---+    |
|  +--> | 1 | 3 | 1 | 1 | 4 | 1 | 3 | 1 | 1 | 1 |    |
|       +---+---+---+---+---+---+---+---+---+---+    |
|         0   1   2   3   4   5   6   7   8   9      |
|                                                     |
|                                                     |
| Métodos: cont(), connected(), find(), union()       |
|                                                     |
+-------------------------------------------------+
```

```
uf.union(6, 1);
```




```java
public class WeightedQuickUnionUF {
    private int[] pai;
    private int[] sz;
    private int count;

    // Initializes an empty union-find data structure with n
    // isolated components 0 through n-1
    public UF(int n) {
        count = n;
        pai = new int[n];
        sz = new int[n];
        for (int i = 0; i < n; i++) {
            pai[i] = i;
            sz[i] = 1;
        }
    }

    // Return the number of components.
    public int count() {
        return count;
    }

    // Are the two sites p and q in the same component?
    public boolean connected(int p, int q) {
        return find(p) == find(q);
    }
```

```
    // Return the component identifier for the component
    // containing site p.
    public int find(int p) {
        while (p != pai[p]) {
            pai[p] = pai[pai[p]]; // diminui o tamanho do caminho a metade
            p = pai[p];
        }
        return p;
    }

    // Merge the component containing site p with the the
    // component containing site q.
    public void union(int p, int q) {
        int pRoot = find(p);
        int qRoot = find(q);
        if (pRoot == qRoot) return ;

        if (sz[pRoot] < sz[qRoot]) {
            pai[pRoot] = pai[qRoot];
            sz[qRoot] += sz[pRoot];
        }
        else {
            pai[qRoot] = pai[pRoot];
            sz[pRoot] += sz[qRoot];
        }
        count--;
    }
}
```

```
    Consumo de tempo
    ----------------
    UF():    n
    union(): lg n
    find():  lg n
```

Para demonstrar que o consumo de tempo de `union()` e `find()` é não superior a $\lg n$, basta verificar que

> Na floresta de árvores disjuntas produzida durante uma sequência de opreções `union()`, toda árvore com altura `h` tem pelo menos 2^h nós.

A demonstração é por indução no número de operações `union()` realizadas.

Inicialmente nenhuma operação `union()` foi realizada e toda árvore tem altura zero e possui um nó. Logo vale a afirmação.

Sejam p e q sítios e considere a operação `union(p, q)`. Se p e q estão em uma mesma árvore não há o que demonstrar. Portanto, podemos supor que a árvore T_p que contém p e árvore T_q que contém q são distintas.

Sejam h_p e n_p a altura e número de nós de T_p e h_q e n_q a altura e número de nós de T_q. Pela hipóse de indução $n_p \geq 2^{h_p}$ e $n_q \geq 2^{h_q}$.

Seja T a árvore de altura h resultante da operação `union(p,q)`. Se $h \leq \max\{h_p, h_q\}$, não há o que demonstrar.

Assim, podemos supor que, digamos, b$n_q \leq n_p$ e e $h = h_p + 1$. Logo,

$$2^h = 2^{h_p+1} = 2^{h_p} + 2^{h_p} \leq n_p + n_p \leq n_p + n_q = n.$$

O que encerra este rascunho de demonstração.

Experimentos

```
% java Driver < ../testes/tinyUF.txt
2 components
0.003seg
% java Driver < ../testes/mediumUF.txt
3 components
0.027seg
% java Driver < ../testes/largeUF.txt
6 components
4.079seg
```

Acrescentando a linha

```
    pai[p] = pai[pai[p]] // diminui a altura pela metade
```

em `find()`.

```
> java Driver < ../testes/tinyUF.txt
2 components
0.003seg
% java Driver < ../testes/mediumUF.txt
3 components
0.025seg
% java Driver < ../testes/largeUF.txt
6 components
3.923seg
```

Weighted quick-union with path compression

```
    Consumo de tempo
    ----------------
    UF():    n
    union(): lg* n, **amortizado**, praticamente constante
    find():  lg* n, **amortizado**, praticamente constante
```

```
public class UF {
    private int[] pai;
    private int[] sz;
    private int count;

    // Initializes an empty union-find data structure with n
    // isolated components 0 through n-1
    public UF(int n) {
        count = n;
```

```
        pai = new int[n];
        sz  = new int[n];
        for (int i = 0; i < n; i++) {
            pai[i] = i;
            sz[i] = 1;
        }
    }

    // Return the number of components.
    public int count() {
        return count;
    }

    // Are the two sites p and q in the same component?
    public boolean connected(int p, int q) {
        return find(p) == find(q);
    }

    // Return the component identifier for the component
    // containing site p.
    // Versão recursiva
    public int find(int p) {
        if (p != pai[p]) {
            pai[p] = find(pai[p]);
        }
        return pai[p];
    }

    // Merge the component containing site p with the the
    // component containing site q.
    public void union(int p, int q) {
        int pRoot = find(p);
        int qRoot = find(q);
        if (pRoot == qRoot) return ;

        if (sz[pRoot] < sz[qRoot]) {
            pai[pRoot] = pai[qRoot];
            sz[pRoot] += sz[qRoot];
-        }
        else {
            pai[qRoot] = pai[pRoot];
            sz[qRoot] += sz[pRoot];
        }
        count--;
    }

}
```

Resumo

	UF()	union()	find()
quick-find	n	n	1
quick-union	n	n	n
weighted quick-union	n	lg n	lg n
com path-compression	n	lg* n	lg* n

n	lg* n
1	0
2	1
4	2
16	3
65536	4
2^65536	5

Parece que na prátiva weighted quick-union e weighted quick-union com path-compression não são muito diferentes.