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Compact homomorphisms between uniform Fréchet algebras

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Abstract. We study compact homomorphisms between uniform Fréchet algebras, by analyzing the behavior of its spectral adjoint on the underlying spectrum. We prove that every compact homomorphism between uniform Fréchet algebras actually ranges into a uniform Banach algebra, and that its spectral adjoint maps τ-bounded subsets into relatively τ-compact subsets, when τ is the strong or the compact-open topology.

Keywords: uniform Fréchet algebra, compact homomorphism, spectral adjoint, pointwise bounded homomorphism.

Mathematical subject classification: Primary: 46E25; Secondary: 46J10.

1 Introduction

In this article we study compact homomorphisms between abstract uniform Fréchet algebras. Homomorphisms between such algebras behave as composition operators, under a induced map called *spectral adjoint*. In this sense, we investigate such homomorphisms by analysing its behavior on the underlying spectra. For this purpose we introduce an intermediate class of homomorphisms, known as pointwise bounded homomorphisms. We show that every pointwise bounded homomorphism between uniform Fréchet algebras actually ranges into a uniform Banach algebra and is continuous, generalizing a result of [5].

Let A and B be uniform Banach algebras, with spectra M_A and M_B, respectively, and let $T : A \longrightarrow B$ be a homomorphism. In 1997, U. Klein [10] proved that T is compact if, and only if, its spectral adjoint $g_T : M_B \longrightarrow M_A$ maps M_B into a *norm*-relatively compact subset of M_A. In this article, we prove a related

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result for uniform Fréchet algebras. More specifically, when A and B are uniform Fréchet algebras, then M_A (and also M_B) is hemicompact, and then we can write $M_A = \bigcup_{n\in\mathbb{N}} K_n$ (and $M_B = \bigcup_{n\in\mathbb{N}} L_n$), when $(K_n)_{n\in\mathbb{N}}$ (and also $(L_n)_{n\in\mathbb{N}}$) is a fundamental sequence of compact subsets of M_A (respectively M_B). One of our main results shows that if $T : A \longrightarrow B$ is compact, then its spectral adjoint maps each L_n into a relatively τ-compact subset of some K_{n_0}, for all $n \in \mathbb{N}$, when τ is the strong or the compact-open topology.

For the reader who is not familiar with the theory of uniform algebras, we recall some properties of such algebras in Section 2, and give some examples. In Section 3, the results of the article are presented. These results are part of the doctoral thesis of C. Nachtigall, supervised by D.M. Vieira.

2 Preliminaires

Let X be a topological Hausdorff space. We say that X is a **k-space** if a set $A \subset X$ is closed whenever $A \cap K$ is closed in K, for each compact subset K of X. We say that X is **hemicompact** if there is a sequence $(K_n)_{n\in\mathbb{N}}$ of compact subsets of X such that each compact subset of X is contained in some K_n. We refer to [9] for more information about these concepts.

In this section we recall some important properties of uniform Fréchet algebras that will be convenient throughout the article. All the information bellow can be found in [6].

Throughout this paper, we only consider associative and commutative algebras A over the field of complex numbers, having identity denoted by e_A. If A and B are algebras, then a mapping $T : A \longrightarrow B$ is called a *homomorphism* if T is linear and multiplicative.

A **Banach algebra (B-algebra)** is an algebra A, which is a Banach space and the norm satisfies: (i) $\|fg\| \leq \|f\|\|g\|$, for all $f, g \in A$; and (ii) $\|e_A\| = 1$. A **Fréchet algebra (F-algebra)** is a complete topological algebra, whose topology is generated by a sequence $(p_n)_{n\in\mathbb{N}}$ of seminorms on A satisfying: (i) $p_n(fg) \leq p_n(f)p_n(g)$, (ii) $p_n(e_A) = 1$ and (iii) $p_n(f) \leq p_{n+1}(f)$, for all $f, g \in A$ and for all $n \in \mathbb{N}$. This definition of F-algebra is often written as a *locally m-convex unital Fréchet algebra*. It is clear that evey B-algebra is a F-algebra.

If A is a F-algebra, we define the **spectrum** M_A of A to be the set of all nonzero complex-valued and continuous homomorphisms $\varphi : A \longrightarrow \mathbb{C}$. For each $f \in A$, we set $\widehat{f} : M_A \longrightarrow \mathbb{C}$ by $\widehat{f}(\varphi) = \varphi(f)$, for every $\varphi \in M_A$. The function $\widehat{f}$ is called the **Gelfand transform** of f. We endow M_A with the **Gelfand topology**, the coarsest topology on M_A such that all Gelfand transforms are continuous on M_A. If A is a B-algebra, then M_A is compact, and if A is a

F-algebra, then M_A is hemicompact. We denote $\widehat{A} = \{\widehat{f} : f \in A\}$, and $\widehat{A}$ is a subalgebra of $C(M_A)$. We also define $\Gamma_A : A \longrightarrow \widehat{A}$ by $\Gamma_A(f) = \widehat{f}$. Then Γ_A is a homomorphism of algebras and is called **Gelfand transformation**.

A Banach algebra A is called a **uniform Banach algebra (uB-algebra)**, if its norm satisfies $\|f^2\| = \|f\|^2$, for all $f \in A$. A Fréchet algebra A is called a **uniform Fréchet algebra (uF-algebra)**, if its defining sequence of seminorms $(p_n)_n$ satisfies $p_n(f^2) = p_n(f)^2$, for all $f \in A$ and $n \in \mathbb{N}$. If A is a uB-algebra, then Γ_A is an isometry, and if A is a uF-algebra, then Γ_A is a topological algebra isomorphism. If A is a uF-algebra, then $\widehat{A}$ is a closed subalgebra of $C(M_A)$ that separates points and contains all the constants.

Another important property of uF-algebras is the following. Let A be a uF-algebra. Then there exists a sequence of uB-algebras $(A_n)_{n\in\mathbb{N}}$, such that A is the projective limit of $(A_n)_{n\in\mathbb{N}}$, with a canonical mapping $\pi_n : A \longrightarrow A_n$, which is a continuous homomorphism. If M_{A_n} is the spectrum of A_n, and π'_n denotes the transpose of π_n, then we can write $M_A = \cup_{n\in\mathbb{N}}\pi'_n(M_{A_n})$, and we denote $K_n = \pi'_n(M_{A_n})$, for each $n \in \mathbb{N}$. Then M_A is hemicompact, and each compact subset of M_A is contained in some K_n. (See [6, 3.3.6]).

Let A and B be uF-algebras, and consider $T : A \longrightarrow B$ a continuous homomorphism. The **spectral adjoint of T** is the (continuous) mapping $g_T : M_B \longrightarrow M_A$ given by $g_T(\varphi) = \varphi \circ T$, for all $\varphi \in M_B$. The spectral adjoint g_T induces a continuous homomorphism $\widehat{T} : \widehat{A} \longrightarrow \widehat{B}$ given by $\widehat{T}(\widehat{f}) = \widehat{f} \circ g_T$, for all $\widehat{f} \in \widehat{A}$. Since $\widehat{T}(\widehat{f}) = \widehat{T(f)}$, we have that the following diagram is commutative:

$$\begin{array}{ccc} A & \xrightarrow{T} & B \\ \downarrow{\scriptstyle\Gamma_A} & & \downarrow{\scriptstyle\Gamma_B} \\ \widehat{A} & \xrightarrow{\widehat{T}} & \widehat{B} \end{array} \tag{1}$$

That is, $\widehat{T} \circ \Gamma_A = \Gamma_B \circ T$.

Next we present some examples of uF-algebras.

Examples 2.1.

(a) Let X be a compact Hausdorff topological space. Then $C(X)$, the set of all continuous functions $f : X \longrightarrow \mathbb{C}$ is a uB-algebra, with the *sup* norm. Moreover, A is a uB-algebra if, and only if, there exists a compact Hausdorff topological space X such that A is isometric to a closed pointseparating subalgebra of $C(X)$, see [6, 4.1.1].

(b) Every uB-algebra is a uF-algebra.

(c) Let X be a Hausdorff topological space. If X is a hemicompact k-space, then $C(X)$ is a uF-algebra. Moreover, A is a uF-algebra if, and only if, there exists a hemicompact space X such that A is topologically isomorphic to a complete pointseparating subalgebra of $C(X)$, see [6, 4.1.3, 4.1.4].

(d) *uF-algebras of holomorphic functions*. Let U be an open subset of a Banach space E, and let $U_n = \{x \in U \, : \, \|x\| < n \, , \, d_U(x) > \frac{1}{2^n}\}$, for each $n \in \mathbb{N}$. Let $H_b(U)$ be the algebra of all holomorphic functions $f : U \longrightarrow \mathbb{C}$ that are bounded on each U_n, endowed with the topology of the uniform convergence on the sets U_n. Then $H_b(U)$ is a uF-algebra. Since $E' \subset H_b(U)$, $M_{H_b(U)}$ can be naturally projected into E'', the bidual of E, by the mapping $\pi(\varphi) = \varphi|_{E'}$, for all $\varphi \in M_{H_b(U)}$. When U is convex and balanced, then $\pi(M_{H_b(U)}) = \bigcup_{n\in\mathbb{N}} \overline{U_n}^{w^*}$, (see [2, Proposition 18] for instance), where w^* denotes de weak-star topology in E''. Other examples of uF-algebras of holomorphic functions on U are: $H_{wu}(U)$, the algebra of all holomorphic functions $f : U \longrightarrow \mathbb{C}$ that are weakly uniformly continuous on each U_n, endowed with the topology of the uniform convergence on the sets U_n. And if U is an open subset of E', we also have that $H_{w^*u}(U)$, the algebra of all holomorphic functions $f : U \longrightarrow \mathbb{C}$ that are weak*-uniformly continuous on each U_n is a uF-algebra, endowed with the topology the uniform convergence on the sets U_n. The algebras $H_b(U)$, $H_{wu}(U)$ and $H_{w^*u}(U)$ have been studied, for instance, in [1, 2, 4, 5, 11, 14]. If E is separable, then $H_d(U)$, the algebra of all holomorphic functions $f : U \longrightarrow \mathbb{C}$ that are bounded on balls strictly contained in U is a uF-algebra, when endowed with the topology of the uniform convergence over these balls. The algebra $H_d(U)$ is studied in [3, 12].

3 The results

In this section we define an intermediate classe of homomorphisms between uF-algebras that will help us to obtain our results for compact homomorphisms. These homomorphisms are called *pointwise bounded*, and every compact homomorphism is pointwise bounded. We will show that every pointwise bounded homomorphism between uF-algebras is continuous and ranges into a uB-algebra. One of the main results of this section shows that if A and B are uF-algebras and a homomorphism $T : A \longrightarrow B$ is compact, then its spectral adjoint g_T maps τ-bounded subsets into relatively τ-compact subsets, when τ is the strong or the compact-open topology.

We begin with some definitions. We observe that Definition 3.1(b) is inspired by [5], where it is defined when $B = H_b(U)$, cf. Examples 2.1(d).

Definitions 3.1. *Let A and B be uF-algebras spaces and let $T : A \longrightarrow B$ be a homomorphism.*

(a) *We say that T is **compact** if there exists a neighborhood of zero $V \subset A$ such that $T(V)$ is relatively compact in B.*

(b) *We say that T is **pointwise bounded** if there exists a neighborhood of zero $V \subset A$ such that, for each $\psi \in M_B$, the set $\{\psi(T(f)) : f \in V\}$ is bounded in $\mathbb{C}$.*

Remark 3.2. It is not difficult to see that every compact homomorphism is pointwise bounded. An example of a pointwise bounded homomorphism that is not compact will be given after Theorem 3.5.

Next theorem is an important property of pointwise bounded homomorphisms, and it is a generalization of [5, Theorem 2.1]. In [5], the authors show the result when $B = H_b(U)$, cf. Examples 2.1(d).

Theorem 3.3. *Let A and B be uF-algebras, and let $T : A \longrightarrow B$ be a pointwise bounded homomorphism. Then T is continuous and $T(A) \subset D$, where D is a uB-algebra, which is a sub-algebra of B.*

Proof. Let $V \subset A$ be a neighborhood of the origin such that, for each $\psi \in M_B$, there exists $c_\psi > 0$ such that $|\psi(T(f))| \leq c_\psi$, for all $f \in V$. Without loss of generality, we may assume that $V \cdot V \subset V$ and then $V^n \subset V$, $\forall n \in \mathbb{N}$. Then we have that

$$|\psi(T(f))|^n = |\psi(T(f^n))| \leq c_\psi,\ \forall n \in \mathbb{N},\ \forall f \in V.$$

This implies that $|\psi(T(f))| \leq 1$, $\forall \psi \in M_B, \forall f \in V$, or equivalently that $\sup_{\psi \in M_B} |\widehat{T(f)}(\psi)| \leq 1$, $\forall f \in V$. Therefore $\widehat{T}$ is bounded. Since A and B are uF-algebras, we have by (1) that $\widehat{T} = \Gamma_B \circ T \circ (\Gamma_A)^{-1}$ is, in particular, linear and hence $\widehat{T}$ is continuous. By (1) again, it follows that T is continous.

On the other hand, since V is an absorbing set, for each $f \in A$, there exists $\delta > 0$ such that $\delta f \subset V$. Then $|\widehat{T(f)}(\psi)| \leq \dfrac{1}{\delta}$, for all $\psi \in M_B$. This shows that $\widehat{T}(\widehat{A}) \subset C_b(M_B) \cap \widehat{B}$, where $C_b(M_B)$ denotes the uB-algebra of all continous and bounded functions on M_B, endowed with the *sup* norm on M_B. We observe that $C_b(M_B) \cap \widehat{B}$ is a uB-algebra under this norm. Indeed, if we write $M_B = \cup_{n\in\mathbb{N}} L_n$, and if we take a Cauchy sequence $(\hat{g}_k)$ in $C_b(M_B) \cap \widehat{B}$, then we can find $G \in C_b(M_b)$ and $\hat{g} \in \widehat{B}$ such that

$$\sup_{\psi\in M_B} |\hat{g}_k(\psi) - G(\psi)| \longrightarrow 0 \text{ and } \sup_{\psi\in L_n} |\hat{g}_k(\psi) - \hat{g}(\psi)| \longrightarrow 0, \text{ for each } n \in \mathbb{N}.$$

It turns out that $\sup_{\psi\in L_n} |G(\psi) - \hat{g}(\psi)| = 0$ for each $n \in \mathbb{N}$. Then $G = \hat{g} \in \widehat{B}$ and hence $C_b(M_B) \cap \widehat{B}$ is complete under the *sup* norm.

If we denote by $D = \{g \in B : \hat{g} \in C_b(M_B) \cap \widehat{B}\} = (\Gamma_B)^{-1}(C_b(M_B) \cap \widehat{B})$, then it is not difficult to see that D is a uB-algebra when endowed with the norm $\|g\| = \|\hat{g}\|$, for all $g \in D$, and that $T(A) \subset D$. □

Now we set notation that will be used in the next result. Let A be a uF-algebra and let $M_A = \cup_{n\in\mathbb{N}} K_n$ be its spectrum. For each $n \in \mathbb{N}$, we denote:

$$\left(\widehat{K_n}\right)_A = \{\varphi \in M_A : |\varphi(f)| = |\hat{f}(\varphi)| \le \sup_{K_n} |\hat{f}|, \forall f \in A\}.$$

Theorem 3.4. *Let A be a uF-algebra with spectrum $M_A = \cup_{n\in\mathbb{N}} K_n$, and let B be a uF-algebra. For a continuous homomorphism $T : A \to B$, the following conditions are equivalent:*

(a) *T is pointwise bounded;*

(b) *there exists $n_0 \in \mathbb{N}$ such that $g_T(M_B) \subset \left(\widehat{K_{n_0}}\right)_A$.*

(c) *$g_T(M_B)$ is relatively compact in M_A;*

Proof. $(a) \Rightarrow (b)$ Proceeding as in the proof of Theorem 3.3, we can find a neighborhood of the origin $V \subset A$ such that $|\psi(T(f))| \le 1$, $\forall \psi \in M_B$ and $\forall f \in V$. Without loss of generality, we may assume that $V = \{f \in A : p_{n_0}(f) = \sup_{\varphi\in K_{n_0}} |\varphi(f)| \le a\}$, for some $0 < a < 1$ and some $n_0 \in \mathbb{N}$.

Let us suppose that there exists $\psi \in M_B$ such that $g_T(\psi) \notin \widehat{(K_{n_0})}_A$. Then we can find $f \in A$ and $0 < c < 1$ such that

$$|\psi(T(f))| > 1 > c > \sup_{K_{n_0}} |\hat{f}|.$$

Taking $m \in \mathbb{N}$ such that $c^m < a$, it follows that

$$\sup_{K_{n_0}} |\hat{f}^m| = \left(\sup_{K_{n_0}} |\hat{f}|\right)^m < c^m < a,$$

which shows that $f^m \in V$.

On the other hand, we have that $|\psi(T(f))| > 1$ and hence $|\psi(T(f^m))| > 1$, which is a contradiction. Then (b) follows.

$(b) \Rightarrow (c)$ By [6, 4.3.3.], we have that each $\left(\widehat{K_n}\right)_A$ is compact. Then (c) follows.

$(c) \Rightarrow (a)$ Since M_A is hemicompact, there exists $n_0 \in \mathbb{N}$ such that $g_T(M_B) \subset K_{n_0}$. Consider the neighborhood of zero $V = \{f \in A : p_{n_0}(f) < 1\}$. Given $f \in V$ and $\varphi \in M_B$, we have that $|\varphi(T(f))| = |(\varphi \circ T)(f)| = |g_T(\varphi)(f)| \leq \sup_{K_{n_0}} |\hat{f}| < 1$. Therefore $T(V)$ is pointwise bounded. □

Given a uF-algebra A, it is clear that M_A is a subset of A', where A' denotes the topological dual space of the Fréchet space A. For instance, we observe that the weak-star topology $\sigma(A', A)$ of A' induced in M_A coincides with the Gelfand topology, the usual topology considered in M_A. Let τ_0 denote the *compact-open* topology in A', and let τ_β denote the *strong* topology in A'. In the sequel, for $\tau = \tau_0$ or $\tau = \tau_\beta$, when we write (M_A, τ) we are considering the τ-topology induced in M_A by A'.

The next two results, Theorems 3.5 and 3.7 are the main results of this article. They provide some properties of the spectral adjoint $g_T : (M_B, \tau) \longrightarrow (M_A, \tau)$, when $T : A \longrightarrow B$ is a compact homomorphism of uF-algebras, for $\tau = \tau_0$ or $\tau = \tau_\beta$. In Theorem 3.5, we show that g_T maps each τ_0 (or τ_β) bounded subset of M_B into a τ_0 (or τ_β) relatively compact subset of M_A. In Theorem 3.7, we strengthen this result, by showing that $g_T(L_n)$ is a τ_0 (or τ_β) relatively compact subset of some K_{n_0}, for all $n \in \mathbb{N}$.

Theorem 3.5. *Let A and B be uF-algebras, and let $T : A \longrightarrow B$ be a compact homomorphism.*

(a) *If $L \subset M_B$ is τ_0-bounded, then $g_T(L)$ is a τ_0-relatively compact subset of (M_A, τ_0).*

(b) *If $L \subset M_B$ is τ_β-bounded, then $g_T(L)$ is a τ_β-relatively compact subset of (M_A, τ_β).*

Proof. (a) Since $T : A \longrightarrow B$ is compact, it follows by [7, Prop. V.2.1] that its adjoint $T' : (B', \tau_0) \longrightarrow (A', \tau_0)$ is compact. Let V be a τ_0-neighborhood of zero in B' such that $T'(V)$ is a τ_0-relatively compact subset of (A', τ_0). Since L is τ_0-bounded, there exists $\lambda > 0$ such that $L \subset \lambda V$. Then $g_T(L) = T'|_{M_B}(L) \subset \lambda T'(V)$, and then $g_T(L)$ is a τ_0-relatively compact subset of (A', τ_0). On the other hand, T is pointwise bounded by Lemma 3.2, and hence $g_T(M_B)$ is relatively compact in M_A, by Theorem 3.4. Now $\overline{g_T(L)}^{\tau_0} \subset \overline{g_T(L)} \subset \overline{g_T(M_B)} \subset M_A$, where the last closure is considered in the Gelfand topology. We conclude that $\overline{g_T(L)}^{\tau_0}$ is a τ_0-compact subset of (M_A, τ_0).

(b) To show the statement for $\tau = \tau_\beta$, the same proof of (a) applies, by using [7, Exerc. IV.2.4] instead of [7, Prop. V.2.1]. □

Example 3.6. Let $E = T'$ be the original Tsirelson's space ([13]). Then $M_{H_b(B_E)} = \delta(B_E)$, where B_E denotes the open unit ball in E and δ denotes the evaluation mapping. (See, for instance, [11]). If we consider $T : H_b(B_E) \longrightarrow H_b(B_E)$ given by $T(f)(x) = f(\frac{x}{2})$, for all $f \in H_b(B_E)$ and for all $x \in B_E$, then $g_T : \delta(B_E) \longrightarrow \delta(B_E)$ is given by $g(\delta(x)) = \delta(\frac{x}{2})$. Then it follows that T is pointwise bounded by Theorem 3.4 but not compact by Theorem 3.5.

Theorem 3.7. *Let A and B be uF-algebras, with spectra $M_A = \cup_{n\in\mathbb{N}} K_n$ and $M_B = \cup_{n\in\mathbb{N}} L_n$, respectively. Let $T : A \longrightarrow B$ be a compact homomorphism. Then there exists $n_0 \in \mathbb{N}$ such that:*

(a) *$g_T(L_n)$ is a τ_0-relatively compact subset of (K_{n_0}, τ_0), for all $n \in \mathbb{N}$;*

(b) *$g_T(L_n)$ is a τ_β-relatively compact subset of (K_{n_0}, τ_β), for all $n \in \mathbb{N}$.*

Proof. (a) We know that B is a projective limit of a sequence of uB-algebras $(B_n)_{n\in\mathbb{N}}$ [6, 3.3.7 and 4.1.5], with a canonical mapping $\pi_n : B \longrightarrow B_n$, such that $L_n = \pi_n'(M_{B_n})$. Since π_n is continuous, we have that its adjoint $\pi_n' : (B_n', \tau_0) \longrightarrow (B', \tau_0)$ is continuous, see [8, Prop. 3.12.7]. And since M_{B_n} is a subset of the unit ball of the Banach space B_n', we get that L_n is τ_0-bounded. By Theorem 3.5(a), it follows that $g_T(L_n)$ is a τ_0-relatively compact subset of (M_A, τ_0). Since T is pointwise bounded by Lemma 3.2, it follows by Theorem 3.4(c), that there exists $n_0 \in \mathbb{N}$ such that $g_T(M_B) \subset K_{n_0}$. Now: $\overline{g_T(L_n)}^{\tau_0} \subset \overline{g_T(L_n)} \subset K_{n_0}$, and the statement follows.

(b) To show the statement for $\tau = \tau_\beta$, the same proof of (a) applies, by using [8, Corollary of Prop. 3.12.3] instead of [8, Prop. 3.12.7], and by using Theorem 3.5(b) instead of Theorem 3.5(a). □

In [10], the author shows that if A and B are uB-algebras, then $T : A \longrightarrow B$ is compact if, and only if, g_T maps M_B into a *norm*-relatively compact subset of M_A. If A is a uF-algebra and B is a uB-algebra, then M_B is τ_β-bounded. So, in this case, if $T : A \longrightarrow B$ is a compact homomorphism, then it follows by Theorem 3.5 that $g(M_B)$ is a τ_β-relatively compact subset of (M_A, τ_β). But we don't know if this is a sufficient condition for T to be compact.

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References

[1] P. Burlandy and L.A. Moraes. *The spectrum of an algebra of weakly continuous holomorphic mappings*. Indag. Math. (N.S.), **11**(4) (2000), 525–532.

[2] D. Carando, D. García and M. Maestre. *Homomorphisms and composition operators on algebras of analytic functions of bounded type*. Adv. Math., **197**(2) (2005), 607–629.

[3] S. Dineen and M. Venkova. *Extending bounded type holomorphic mappings on a Banach space*. J. Math. Anal. Appl., **297** (2004), 645–658.

[4] D. García, M.L. Lourenço, L.A. Moraes and O.W. Paques. *The spectra of some algebras of analytic mappings*. Indag. Math. (N.S.), **10**(3) (1999), 393–406.

[5] P. Galindo, L. Lourenço and L. Moraes. *Compact and weakly homomorphisms on Fréchet algebras of holomorphic functions*. Math. Nachr., **236** (2002), 109–118.

[6] H. Goldmann. *Uniform Fréchet Álgebras*. North-Holland Math. Stud., 162. North-Holland, Amsterdam, (1990).

[7] A. Grothendieck. *Topological Vector Spaces*. Gordon and Breach Science Publishers, New York, (1973).

[8] J. Horváth. *Topological Vector Spaces and Distributions*, Vol. I, Addison-Wesley, Reading-Massachussets, (1966).

[9] J.L. Kelley. *General Topology*. Graduate Texts in Math. 27, Springer-Verlag, New York-Berlin, (1975).

[10] U. Klein. *Kompakte multiplikative operatoren auf uniformen algebren*. Mitt. Math. Sem. Giessen, **232** (1997), 1–120.

[11] J. Mujica. *Ideals of holomorphic functions on Tsirelson's space*. Arch. Math., **76** (2001), 292–298.

[12] J. Mujica and D.M. Vieira. *Weakly continuous holomorphic functions on pseudoconvex domains in Banach spaces*. Rev. Mat. Complut., **23** (2010), 435–452.

[13] B. Tsirelson. *Not every Banach space contains an imbedding of l_p or c_0*. Functional Anal. Appl., **8** (1974), 138–141.

[14] D.M. Vieira. *Spectra of algebras of holomorphic functions of bounded type*. Indag. Mathem. N.S., **18**(2) (2007), 269–279.

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