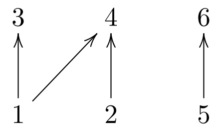
Lista 3

MAT0460/MAT6674 — 2° SEMESTRE DE 2018

Let k be a field and $\mathcal{S}$ be a finite poset.

Exercício 1.
Calculate $J(k\mathcal{S})$.

Exercício 2.
Let S be a poset given by



and consider two matrix representations of $\mathcal{S}$

$$A = \begin{array}{|c|c|c|c|c|c|} \hline 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ \hline \end{array}$$

$$B = \begin{array}{|c|c|c|c|c|c|} \hline 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ \hline \end{array}$$

Show that A represents a decomposable class in $\mathrm{Mat}_{\mathcal{S}}$, whereas B represents indecomposable class. Prove that the endomorphism ring of $F(B)$ in $\mathrm{Mat}_{\mathcal{S}}^{ad}$ is isomorphic k. Construct non-trivial idempotent in endomorphism ring of $F(A)$ in $\mathrm{Mat}_{\mathcal{S}}^{ad}$.

Exercício 3.
Let $\mathcal{S}$ and $\mathcal{T}$ be finite posets. Prove that there is a poset isomorphism $S \cong T$ (i.e., a bijection which preserves order) if and only if the incidence k-algebras kS and kT are isomorphic.

Exercício 4.
Suppose that $\mathcal{S}$ is a poset consisting of three incomparable elements $1, 2, 3$. Describe indecomposable classes in $\mathrm{Mat}_{\mathcal{S}}$.

Exercício 5.
Let $\mathcal{A}$ and $\mathcal{A}'$ be two additive categories with the unique decomposition property and that $H : \mathcal{A} \to \mathcal{A}'$ is a representation equivalence (i.e., H is full, dense and reflects isomorphisms). Prove that any object X in $\mathcal{A}$ is indecomposable iff $H(X)$ is indecomposable in $\mathcal{A}'$.

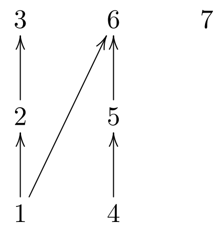
Exercício 6.
Let $\mathcal{S}$ be a poset as in Exercise 2. Show that in this case the reduction functor $H : (\mathrm{Mat}_{\mathcal{S}}^{ad})_0 \to \mathcal{S} - \mathrm{sp}$ is not an equivalence of categories.

Exercício 7.

Let $\mathcal{S} = \{1 \to 2, 3 \to 4\}$ be a poset consisting of two incomparable chains. Construct the diagram of indecomposable objects in $\mathrm{Mat}_{\mathcal{S}}^{ad}$. Prove that the functor $H : (\mathrm{Mat}_{\mathcal{S}}^{ad})_0 \to \mathcal{S} - \mathrm{sp}$ is an equivalence of categories.

Exercício 8.

Let $\mathcal{S}$ be a poset given by



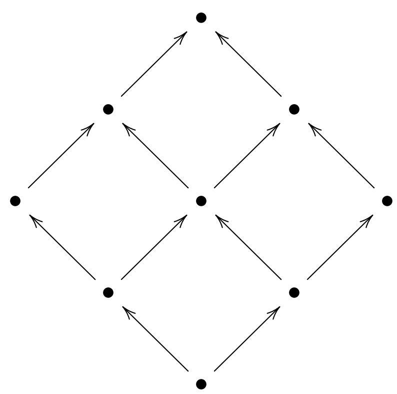
and let

$$A = \left[\begin{array}{cc|c|c|c|c|c|c} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

Construct $\mathcal{S}_7'$ and calculate $\partial_7(V)$, where V is a subspace representations corresponding to A.

Exercício 9.

Prove that poset $\mathcal{S}$ below



is representation-finite and find the number of indecomposable $\mathcal{S}$-spaces up to isomorphism.

Exercício 10.

Let $V, W \in \mathcal{S} - \mathrm{sp}$, and $f : V \to W$ be proper morphism. Prove that $\mathrm{Coker}\, f \in \mathcal{S} - \mathrm{sp}$.

Exercício 11.

Let $V, W \in \mathcal{S} - \mathrm{sp}$, and $f : V \to W$ be a morphism. Prove that f is essential if, and only if, for each $t \in \{\emptyset\} \cup \mathcal{S}$ the induces map

$$V(t^+)/V_t \to W(t^+)/W_t$$

is a bijection.

Exercício 12.
Let $V, W \in \mathcal{S} - \mathrm{sp}$. Prove that $f : V \to W$ is a proper injection if, and only if, for each $t \in \{\emptyset\} \cup \mathcal{S}$ the induces map

$$V(t^{+})/V_t \to W(t^{+})/W_t$$

is an injection.

Exercício 13.
Let $\mathcal{S}$ be a poset as in Exercise 2. Calculate $\int_6 \partial_6$ for subspace representation of $\mathcal{S}$ which corresponds to matrix representations A and B (as in Exercise 2). Describe all subspaces representations V of $\mathcal{S}$, such that $\int_6 \partial_6 V$ is not isomorphic to V.