Gabarito Lista 1

Higor Mendes Garcia

07/09/2021

Exercício 1

a)

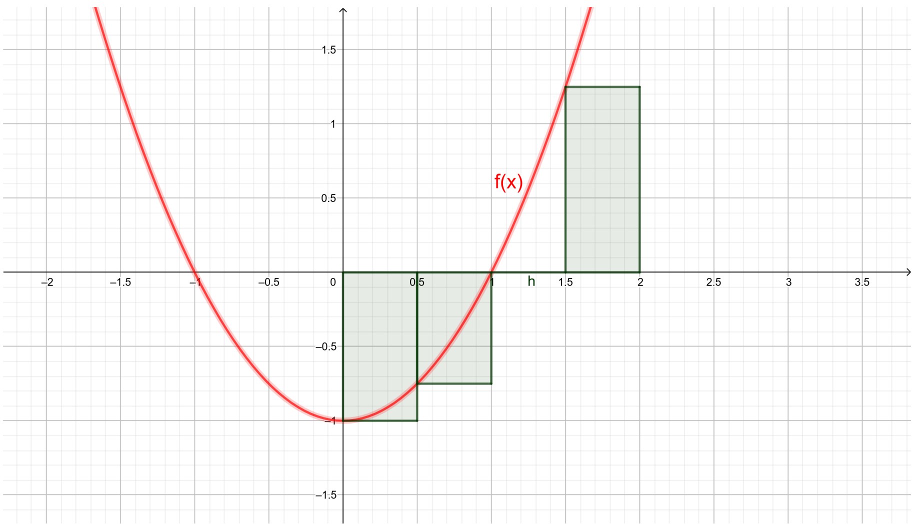

Figure 1: Tomando ξ_i a extremidade esquerda do intervalo.

i	1	2	3	4
ξ_i	0	$\frac{1}{2}$	1	$\frac{3}{2}$
$f(\xi_i)$	-1	$-\frac{3}{4}$	0	$\frac{5}{4}$
$f(\xi_i) \times \Delta x$	$-\frac{1}{2}$	$-\frac{3}{8}$	0	$\frac{5}{8}$

$$\sum_{i=1}^{4} f(\xi_i) \times \Delta x = \left(-\frac{1}{2}\right) + \left(-\frac{3}{8}\right) + 0 + \left(\frac{5}{8}\right) = -\frac{1}{4}$$

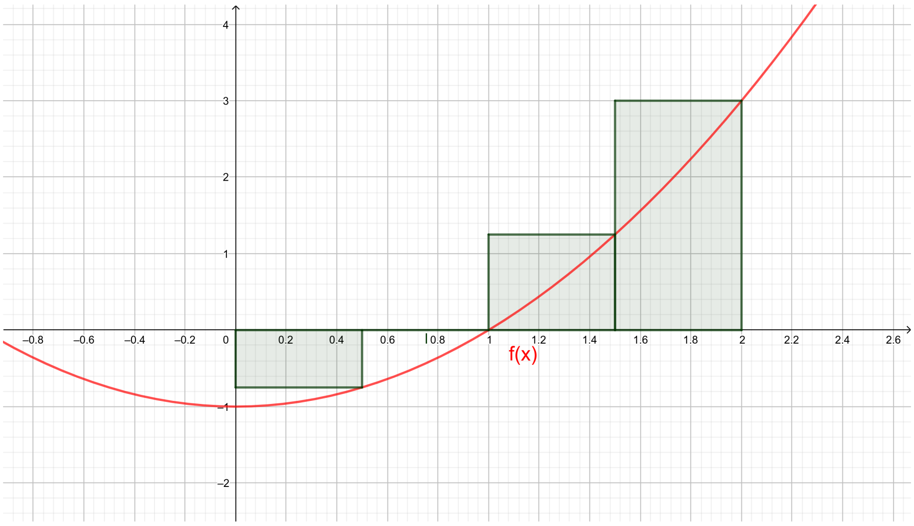

Figure 2: Tomando ξ_i a extremidade direita do intervalo.

i	1	2	3	4
ξ_i	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(\xi_i)$	$-\frac{3}{4}$	0	$\frac{5}{4}$	3
$f(\xi_i) \times \Delta x$	$-\frac{3}{8}$	0	$\frac{5}{8}$	$\frac{3}{2}$

$$\sum_{i=1}^{4} f(\xi_i) \times \Delta x = \left(-\frac{3}{8}\right) + 0 + \left(\frac{5}{8}\right) + \left(\frac{12}{8}\right) = \frac{14}{8}$$

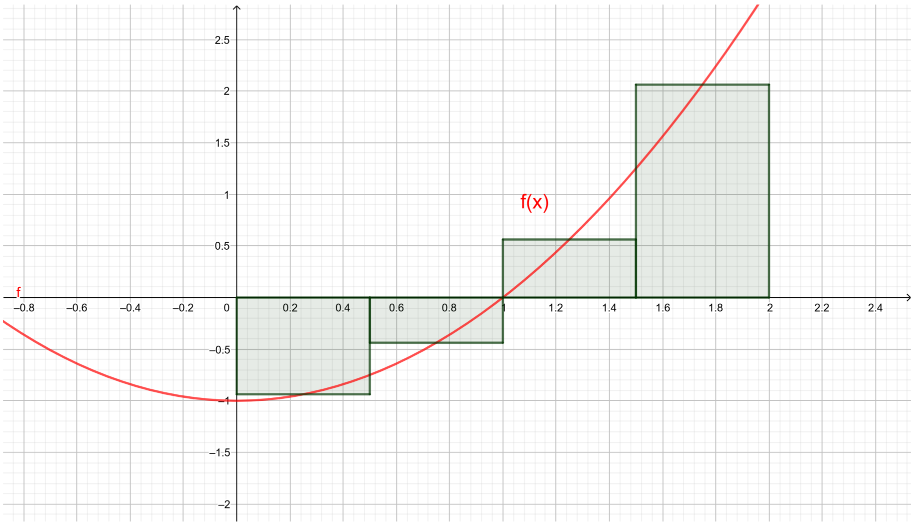

Figure 3: Tomando ξ_i o ponto médio do intervalo.

i	1	2	3	4
ξ_i	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{7}{4}$
$f(\xi_i)$	$-\frac{15}{16}$	$-\frac{14}{32}$	$\frac{18}{32}$	$\frac{33}{16}$
$f(\xi_i)\times\Delta x$	$-\frac{15}{32}$	$-\frac{7}{32}$	$\frac{9}{32}$	$\frac{33}{32}$

$$\sum_{i=1}^{4} f(\xi_i)\times\Delta x = \left(-\frac{15}{32}\right)+\left(-\frac{7}{32}\right)+\left(\frac{9}{32}\right)+\left(\frac{33}{32}\right)=\frac{20}{32}$$

b)

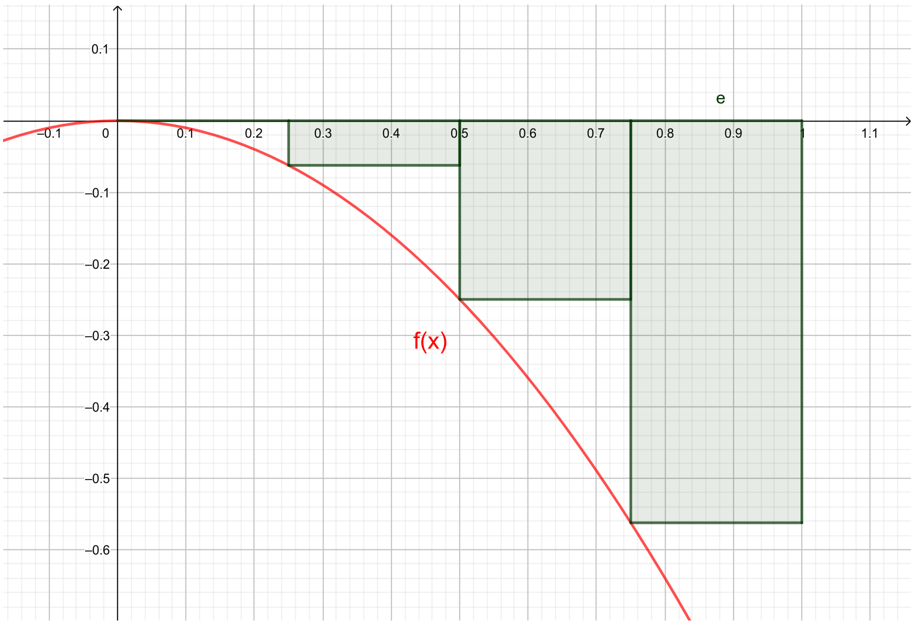

Figure 4: Tomando ξ_i a extremidade esquerda do intervalo.

i	1	2	3	4
ξ_i	0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$
$f(\xi_i)$	0	$-\frac{1}{16}$	$-\frac{1}{4}$	$\frac{9}{16}$
$f(\xi_i) \times \Delta x$	0	$-\frac{1}{64}$	$-\frac{4}{64}$	$-\frac{9}{64}$

$$\sum_{i=1}^{4} f(\xi_i) \times \Delta x = 0 + \left(-\frac{1}{64}\right) + \left(-\frac{4}{64}\right) + \left(-\frac{9}{64}\right) = -\frac{14}{64}$$

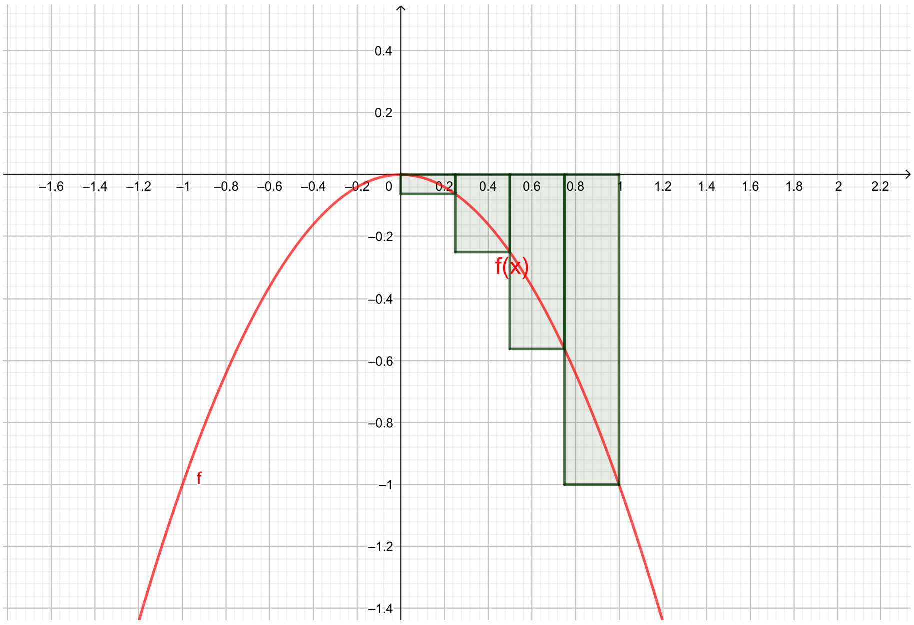

Figure 5: Tomando ξ_i a extremidade direita do intervalo.

i	1	2	3	4
ξ_i	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$f(\xi_i)$	$-\frac{1}{16}$	$-\frac{1}{4}$	$-\frac{9}{16}$	-1
$f(\xi_i) \times \Delta x$	$-\frac{1}{64}$	$-\frac{1}{16}$	$-\frac{9}{64}$	$-\frac{1}{4}$

$$\sum_{i=1}^{4} f(\xi_i) \times \Delta x = \left(-\frac{1}{64}\right) + \left(-\frac{4}{64}\right) + \left(-\frac{9}{64}\right) + \left(-\frac{16}{64}\right) = -\frac{30}{64}$$

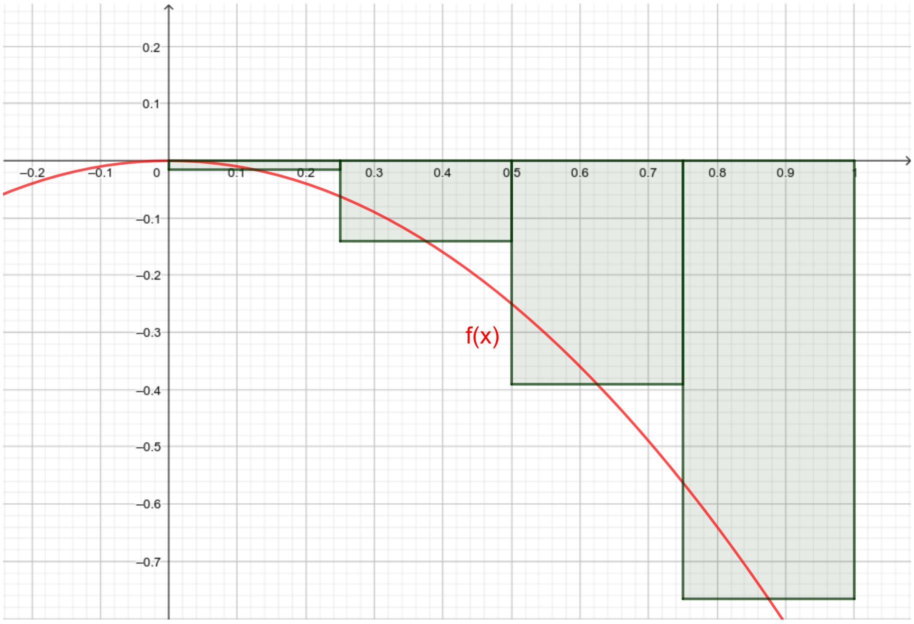

Figure 6: Tomando ξ_i o ponto médio do intervalo.

i	1	2	3	4
ξ_i	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$
$f(\xi_i)$	$-\frac{1}{64}$	$-\frac{9}{64}$	$-\frac{25}{64}$	$-\frac{49}{64}$
$f(\xi_i)\times\Delta x$	$-\frac{1}{256}$	$-\frac{9}{256}$	$-\frac{25}{256}$	$-\frac{49}{256}$

$$\sum_{i=1}^{4} f(\xi_i)\times\Delta x = \left(-\frac{1}{256}\right)+\left(-\frac{9}{256}\right)+\left(-\frac{25}{256}\right)+\left(-\frac{49}{256}\right) = -\frac{21}{64}$$

c)

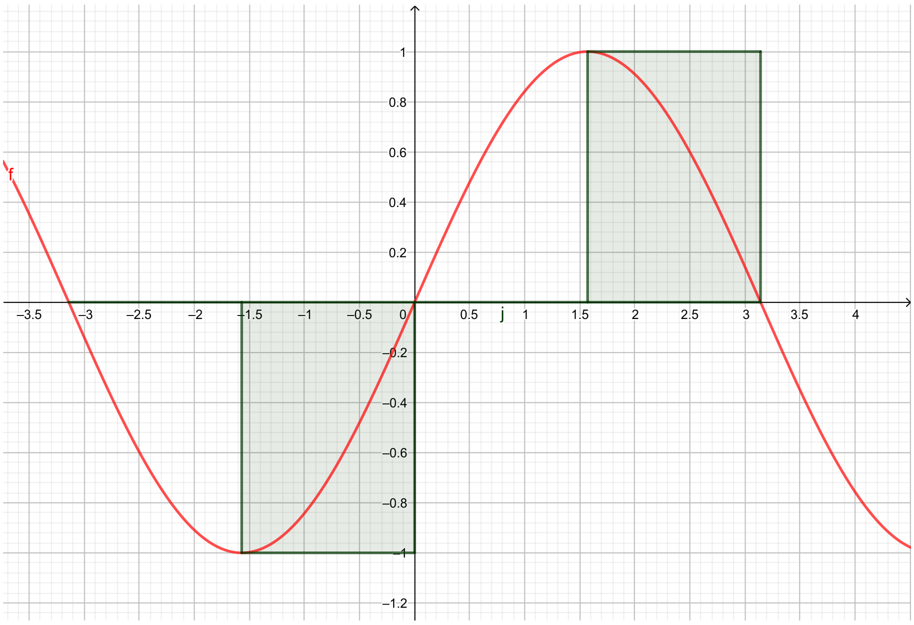

Figure 7: Tomando ξ_i a extremidade esquerda do intervalo.

i	1	2	3	4
ξ_i	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$
$f(\xi_i)$	0	-1	0	1
$f(\xi_i) \times \Delta x$	0	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$

$$\sum_{i=1}^{4} f(\xi_i) \times \Delta x = 0 + \left(-\frac{\pi}{2}\right) + 0\left(\frac{\pi}{2}\right) = 0$$

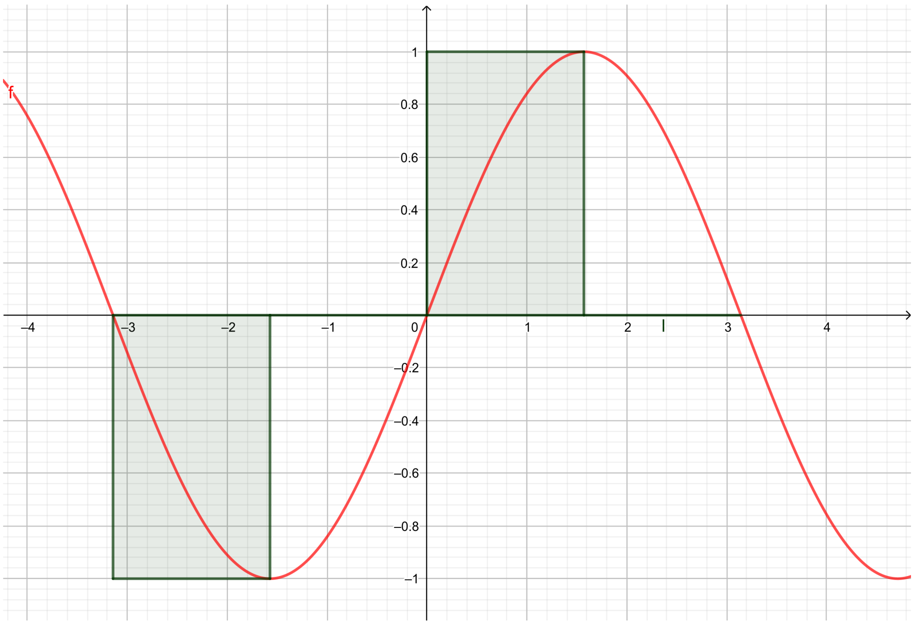

Figure 8: Tomando ξ_i a extremidade direita do intervalo.

i	1	2	3	4
ξ_i	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	0
$f(\xi_i)$	-1	0	1	0
$f(\xi_i) \times \Delta x$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	0

$$\sum_{i=1}^{4} f(\xi_i) \times \Delta x = \left(-\frac{\pi}{2}\right) + 0 + \left(\frac{\pi}{2}\right) + 0 = 0$$

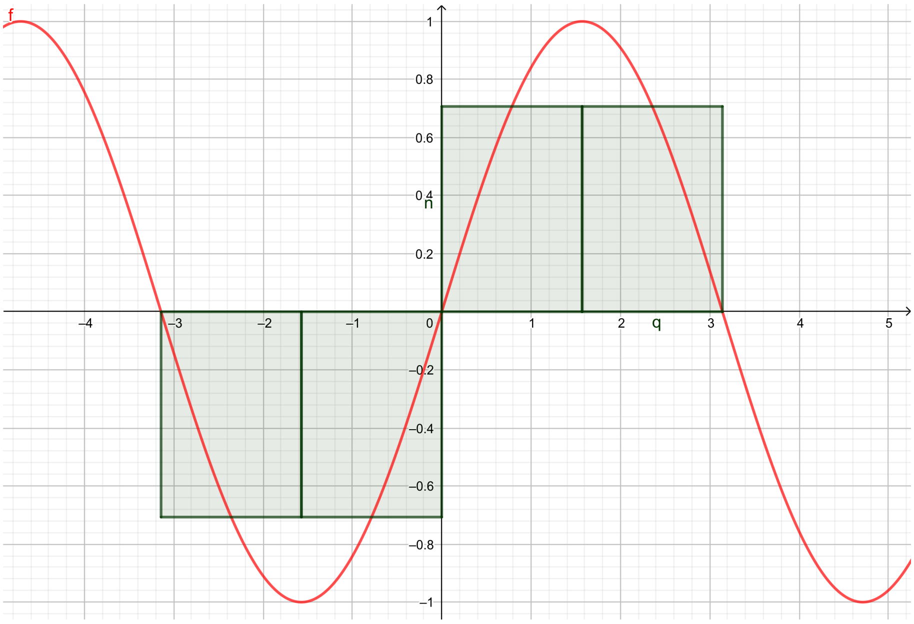

Figure 9: Tomando ξ_i o ponto médio do intervalo.

i	1	2	3	4
ξ_i	$-\frac{3\pi}{4}$	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$
$f(\xi_i)$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$f(\xi_i) \times \Delta x$	$-\frac{\sqrt{2}\pi}{4}$	$-\frac{\sqrt{2}\pi}{4}$	$\frac{\sqrt{2}\pi}{4}$	$-\frac{\sqrt{2}\pi}{4}$

$$\sum_{i=1}^{4} f(\xi_i) \times \Delta x = \left(-\frac{\sqrt{2}\pi}{4}\right) + \left(-\frac{\sqrt{2}\pi}{4}\right) + \left(\frac{\sqrt{2}\pi}{4}\right) + \left(\frac{\sqrt{2}\pi}{4}\right) = 0$$

d)

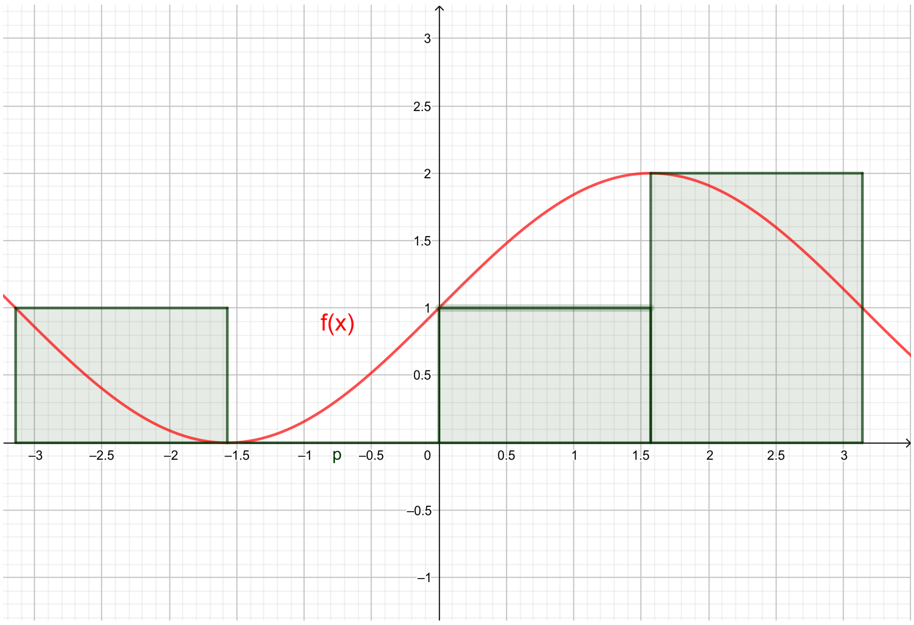

Figure 10: Tomando ξ_i a extremidade esquerda do intervalo.

i	1	2	3	4
ξ_i	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$
$f(\xi_i)$	1	0	1	2
$f(\xi_i)\times\Delta x$	$\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π

$$\sum_{i=1}^{4} f(\xi_i)\times\Delta x = \left(\frac{\pi}{2}\right) + 0 + \left(\frac{\pi}{2}\right) + \pi = 2\pi$$

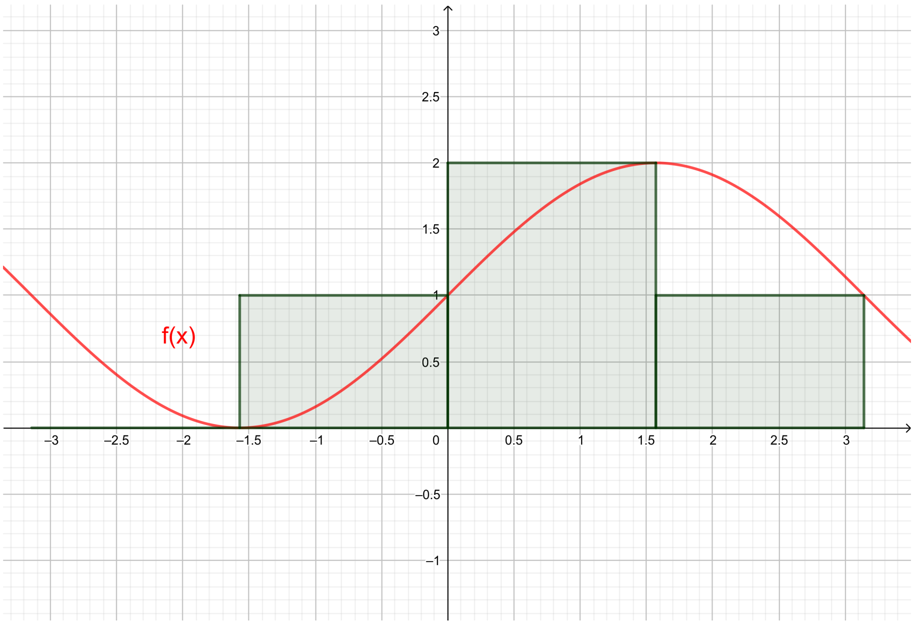

Figure 11: Tomando ξ_i a extremidade direita do intervalo.

i	1	2	3	4
ξ_i	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π
$f(\xi_i)$	0	1	2	1
$f(\xi_i) \times \Delta x$	0	$\frac{\pi}{2}$	π	$\frac{\pi}{2}$

$$\sum_{i=1}^{4} f(\xi_i) \times \Delta x = 0 + \left(\frac{\pi}{2}\right) + \pi + \left(\frac{\pi}{2}\right) = 2\pi$$

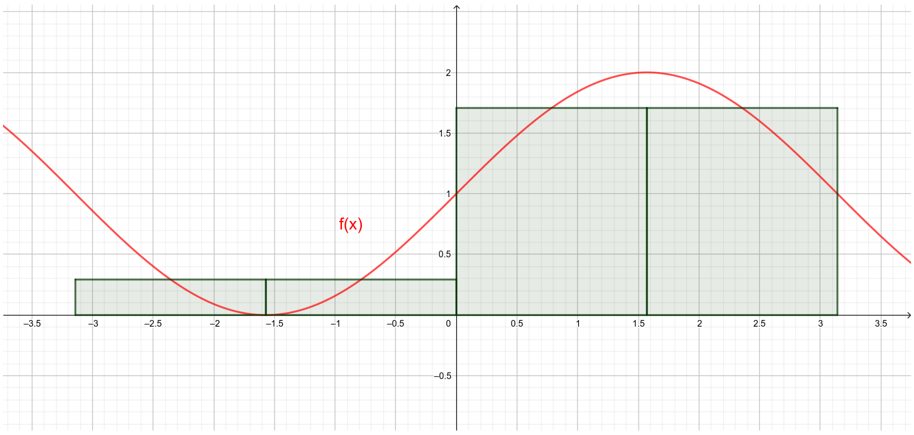

Figure 12: Tomando ξ_i o ponto médio do intervalo.

i	1	2	3	4
ξ_i	$-\frac{3\pi}{4}$	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$
$f(\xi_i)$	$-\frac{\sqrt{2}}{2}+1$	$-\frac{\sqrt{2}}{2}+1$	$\frac{\sqrt{2}}{2}+1$	$\frac{\sqrt{2}}{2}+1$
$f(\xi_i)\times\Delta x$	$-\frac{\pi\sqrt{2}}{4}+\frac{\pi}{2}$	$-\frac{\pi\sqrt{2}}{4}+\frac{\pi}{2}$	$\frac{\pi\sqrt{2}}{4}+\frac{\pi}{2}$	$\frac{\pi\sqrt{2}}{4}+\frac{\pi}{2}$

$$\sum_{i=1}^{4} f(\xi_i)\times\Delta x = \left(-\frac{\pi\sqrt{2}}{4}+\frac{\pi}{2}\right)+\left(-\frac{\pi\sqrt{2}}{4}+\frac{\pi}{2}\right)+\left(\frac{\pi\sqrt{2}}{4}+\frac{\pi}{2}\right)+\left(\frac{\pi\sqrt{2}}{4}+\frac{\pi}{2}\right)=2\pi$$

Exercício 2

a)

$$\int_0^2 x^2\,dx$$

b)

$$\int_{-1}^0 2x^3\,dx$$

c)

$$\int_{-7}^5 (x^2-3x)\,dx$$

d)

$$\int_2^3 \left(\frac{1}{1-x}\right)\,dx$$

e)

$$\int_{-\frac{\pi}{4}}^{0} \sec(x)\,dx$$

Exercício 3

a)

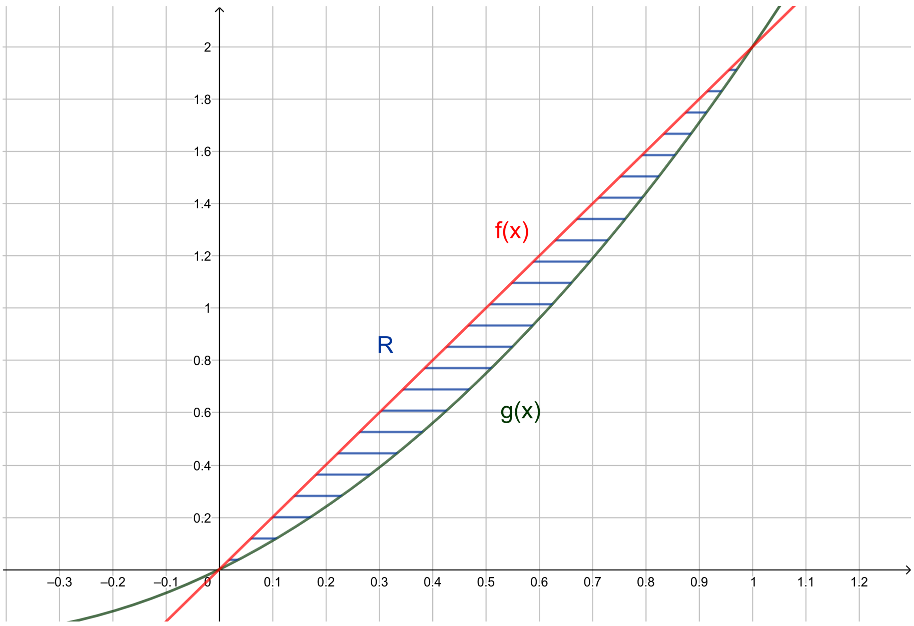

Figure 13: Região (R) delimitada por $f(x)$ e $g(x)$.

Para encontrarmos os limites de integração basta resolvermos a equação:

$$\begin{aligned} 2x &= x^2 + x \\ 2x - x^2 - x &= 0 \\ x - x^2 &= 0 \\ x(1-x) &= 0 \\ &\Rightarrow\; x_1 = 0\,e\,x_2 = 1 \end{aligned}$$

Logo, os limites de integração são [0, 1], logo:

$$\begin{aligned}
A(R) &= \int_0^1 f(x) - g(x)\,dx \\
&= \int_0^1 (2x) - (x^2 + x)\,dx \\
&= \int_0^1 2x - x^2 - x\,dx \\
&= \int_0^1 x - x^2\,dx \\
&= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 \\
&= \frac{1}{6}
\end{aligned}$$

Portanto, a área da região delimitada for $f(x) = 2x$ e $g(x) = x^2 + x$ é $\frac{1}{6}$ unidades de área.

b)

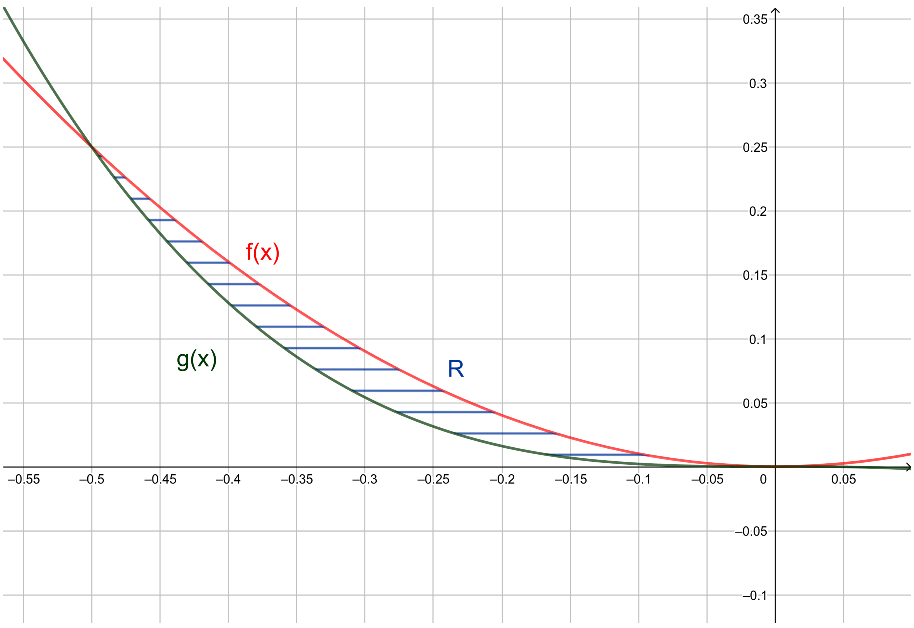

Figure 14: Região (R) delimitada por $f(x)$ e $g(x)$.

Para encontrarmos os limites de integração basta resolvermos a equação:

$$x^2 = -2x^3$$
$$x^2 + 2x^3 = 0$$
$$x^2(1+2x) = 0$$
$$\Rightarrow\; x_1 = 0\, e\, x_2 = -\frac{1}{2}$$

Assim, os limites de integração são $[-\frac{1}{2}, 0]$, logo:

$$\begin{aligned} A(R) &= \int_{-\frac{1}{2}}^{0} f(x) \,-\, g(x)\,dx \\ &= \int_{-\frac{1}{2}}^{0} (x^2) \,-\, (-2x^3)\,dx \\ &= \int_{-\frac{1}{2}}^{0} x^2 + 2x^3\,dx \\ &= \left[\frac{x^3}{3} + \frac{x^4}{2}\right]_{-\frac{1}{2}}^{0} \\ &= \frac{1}{96} \end{aligned}$$

Portanto, a área da região delimitada por $y = x^2$ e $y = -2x^3$ é $\dfrac{1}{96}$ de unidade de área.

c)

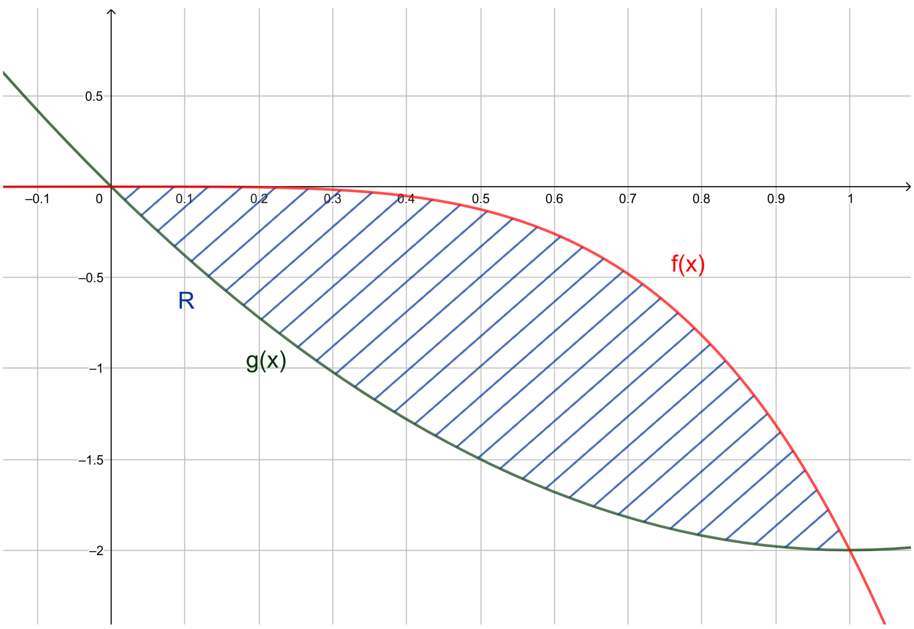

Figure 15: Região (R) delimitada por $f(x)$ e $g(x)$.

Para encontrarmos os limites de integração basta resolvermos a equação:

$$\begin{aligned} -2x^4 &= 2x^2 - 4x \\ 2x^2 + 2x^4 - 4x &= 0 \\ 2x(x^3 + x - 2) &= 0 \\ 2x(x^3 - x + 2x - 2) &= 0 \\ 2x(x(x^2 - 1) + 2(x - 1)) &= 0 \\ &\Rightarrow\; x_1 = 0 \, e \, x_2 = 1 \end{aligned}$$

Logo, os limites de integração são [0, 1], então:

$$\begin{aligned} A(R) &= \left| \int_0^1 g(x)\,dx \right| - \left| \int_0^1 f(x)\,dx \right| \\ &= \left| \int_0^1 (2x^2 - 4x)\,dx \right| - \left| \int_0^1 (-2x^4)\,dx \right| \\ &= \left| \left[\frac{2x^3}{3} - 2x^2 \right]_0^1 \right| - \left| \left[\frac{-2x^5}{5} \right]_0^1 \right| \\ &= |\frac{2}{3} - 2| - \left| -\frac{2}{5} \right| \\ &= \frac{14}{15} \end{aligned}$$

Portanto, a região limitada por $g(x)$ e $f(x)$ possui $\dfrac{14}{15}$ de unidade de área.

d)

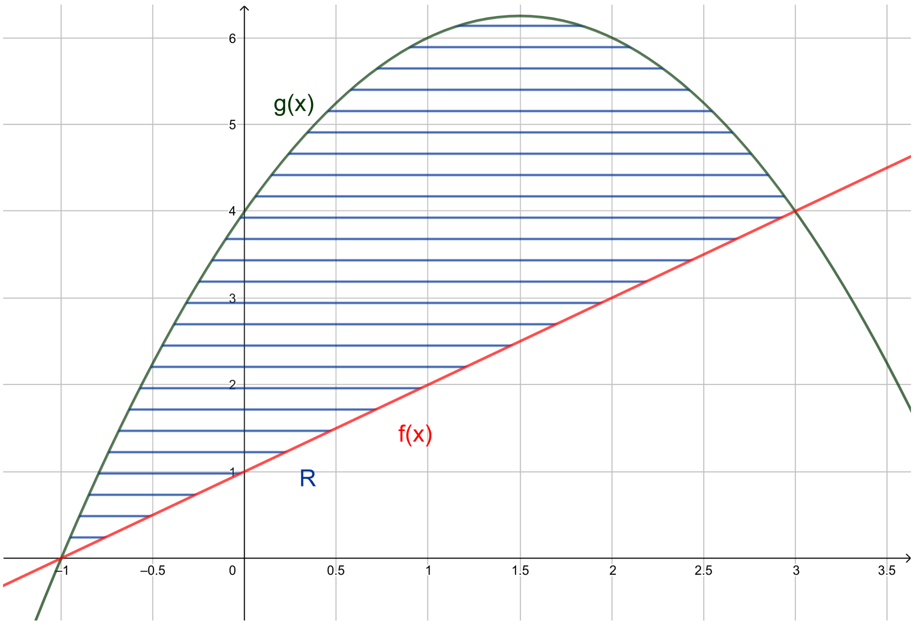

Figure 16: Região (R) delimitada por $f(x)$ e $g(x)$.

Para encontrarmos os limites de integração basta resolvermos a equação:

$$\begin{aligned} x+1 &= 4+3x-x^2 \\ 3+2x-x^2 &= 0 \\ x^2-3-2x &= 0 \\ x^2+x-3x-3 &= 0 \\ x(x+1)-3(x+1) &= 0 \\ &\Rightarrow\ x_1=-1\, e\, x_2=3 \end{aligned}$$

Logo, os extremos de integração são $[-1,\, 3]$, então:

$$\begin{aligned}
A(R) &= \int_{-1}^{3} g(x) - f(x)\, dx \\
&= \int_{-1}^{3} (4 + 3x - x^2) - (x + 1)\, dx \\
&= \int_{-1}^{3} (3 + 2x - x^2)\, dx \\
&= \left[3x + x^2 - \frac{x^3}{3}\right]_{-1}^{3} \\
&= \left(3 \times 3 + 3^3 - \frac{3^3}{3}\right) - \left(3 \times (-1) + (-1)^2 - \frac{-1^3}{3}\right) \\
&= \frac{32}{3}
\end{aligned}$$

Portanto, a região limitada por $f(x) = x + 1$ e $g(x) = 4 + 3x - x^2$ possui $\frac{32}{3}$ unidades de área.

Exercício 4

a)

Comecemos calculando $f'(x)$:

$$\begin{aligned}
f(x) = \frac{e^x + e^{-x}}{2} \Rightarrow f'(x) &= \frac{d}{dx}\left(\frac{e^x + e^{-x}}{2}\right) \\
&= \frac{1}{2} \times \frac{d}{dx}(e^x + e^{-x}) \\
&= \frac{1}{2} \times (e^x - e^{-x}) \\
&= \frac{e^{2x} - 1}{2e^x}
\end{aligned}$$

Assim, temos que:

$$
\begin{aligned}
l(f(x)) &= \int_0^1 \sqrt{1+(f'(x))^2}\,dx \\
&= \int_0^1 \sqrt{1+\left(\frac{e^{2x}-1}{2e^x}\right)^2}\,dx \\
&= \int_0^1 \sqrt{1+\left(\frac{(e^{2x}-1)^2}{4e^{2x}}\right)}\,dx \\
&= \int_0^1 \sqrt{\left(\frac{4e^{2x}+(e^{2x}-1)^2}{4e^{2x}}\right)}\,dx \\
&= \int_0^1 \sqrt{\left(\frac{e^{4x}+2e^{2x}+1}{4e^{2x}}\right)}\,dx \\
&= \int_0^1 \left(\frac{\sqrt{(e^{2x}+1)^2}}{2e^x}\right)\,dx \\
&= \int_0^1 \frac{e^{2x}+1}{2e^x}\,dx \\
&= \frac{1}{2}\times\int_0^1 \frac{e^{2x}+1}{e^x}\,dx \\
&= \frac{1}{2}\times\int_0^1 e^x+\frac{1}{e^x}\,dx \\
&= \frac{1}{2}\times\left[e^x-\frac{1}{e^x}\right]_0^1 \\
&= \frac{e}{2}-\frac{1}{2e}
\end{aligned}
$$

b)

Comecemos calculando $f'(x)$:

$$
\begin{aligned}
f(x) = 1-\ln(\sin(x)) \Rightarrow f'(x) &= \frac{d}{dt}(1-\ln(\sin(x))) \\
&= \frac{d}{dx}(1)-\frac{d}{dx}(\ln(\sin(x))) \\
&= 0-\left(\left.\frac{d}{dg}\right|_{g=\sin(x)}(\ln(g))\times\frac{d}{dx}(\sin(x))\right) \\
&= -\cot(x)
\end{aligned}
$$

Assim temos que:

$$
\begin{aligned}
l(f(x)) &= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sqrt{1 + (f'(x))^2}\, dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sqrt{1 + (\cot^2(x))}\, dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sqrt{1 + \frac{\cos^2(x)}{\sin^2(x)}}\, dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sqrt{1 + \frac{1 - \sin^2(x)}{\sin^2(x)}}\, dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sqrt{1 + \frac{1}{\sin^2(x)} - 1}\, dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sqrt{\frac{1}{\sin^2(x)}}\, dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{|\sin(x)|}\, dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{\sin(x)}\, dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{\sin(x)} \times \frac{\sin(x)}{\sin(x)}\, dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin(x)}{\sin^2(x)}\, dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin(x)}{1 - \cos^2(x)}\, dx \\
&= |_{t=\cos(x)} \int -\frac{t}{1 - t^2}\, dt \\
&= |_{t=\cos(x)} \frac{1}{2} \times \ln\left(\left|\frac{t-1}{t+1}\right|\right) \\
&= |_{t=\cos(x)} \frac{1}{2} \times \ln\left(\left|\frac{\cos(x)-1}{\cos(x)+1}\right|\right) \\
&= \frac{1}{2} \times \ln\left(\left|-\tan\left(\frac{x}{2}\right)^2\right|\right) \\
&= \frac{1}{2} \times \ln\left(\tan^2\left(\frac{x}{2}\right)\right) \\
&= \left[\ln\left(\tan\left(\frac{x}{2}\right)\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} \\
&= \ln\left(2\tan\left(\frac{\pi}{8}\right) + \tan\left(\frac{\pi}{8}\right) \times \sqrt{3}\right)
\end{aligned}
$$

c)

Comecemos calculando $f'(x)$:

$$f(x) = \ln(x) \Rightarrow f'(x) = \frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$$

Assim temos:

$$\begin{aligned}
l(f(x)) &= \int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + (f'(x))^2}\, dx \\
&= \int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + \left(\frac{1}{x}\right)^2}\, dx \\
&= \int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + \frac{1}{x^2}}\, dx \\
&= \int_{\sqrt{3}}^{\sqrt{8}} \frac{1}{x}\sqrt{x^2+1}\, dx \\
&= |_{u=\sqrt{x^2+1}} \int \left(1 + \frac{1}{u^2-1}\right) du \\
&= |_{u=\sqrt{x^2+1}}\ [u] + \int \left(\frac{1}{u^2-1}\right) du \\
&= |_{u=\sqrt{x^2+1}} \left[\sqrt{x^2+1}\right]_{\sqrt{3}}^{\sqrt{8}} + \int \left(\frac{1}{u^2-1}\right) du \\
&= |_{u=\sqrt{x^2+1}}\ 1 + \int \left(\frac{1}{2(u-1)} - \frac{1}{2(u+1)}\right) du \\
&= |_{u=\sqrt{x^2+1}}\ 1 + \int \left(\frac{1}{2(u-1)}\right) du - \int \left(\frac{1}{2(u+1)}\right) du \\
&= |_{u=\sqrt{x^2+1}}\ 1 + \frac{1}{2} \times \ln\left(|u-1|\right) - \frac{1}{2} \times \ln\left(|u+1|\right) \\
&= 1 + \left[\frac{1}{2} \times \ln(|\sqrt{x^2+1} - 1|) - \frac{1}{2} \times \ln(|\sqrt{x^2+1} + 1|)\right]_{\sqrt{3}}^{\sqrt{8}} \\
&= 1 + \frac{\ln(\sqrt{9}-1)}{2} - \frac{\ln(\sqrt{4}-1)}{2} - \frac{\ln(\sqrt{9}+1)}{2} + \frac{\ln(\sqrt{4}+1)}{2}
\end{aligned}$$

d)

Comecemos calculando $f'(x)$:

$$\begin{aligned} f(x) = \sqrt{x^3} \Rightarrow f'(x) &= \frac{d}{dx}(x\sqrt{x}) \\ &= \frac{d}{dt}(x)\sqrt{x} + x\frac{d}{dx}(\sqrt{x}) \\ &= 1\sqrt{x} + x\frac{1}{2\sqrt{x}} \\ &= \frac{3\sqrt{x}}{2} \end{aligned}$$

Assim:

$$\begin{aligned} l(f(x)) &= \int_0^4 \sqrt{1 + \left(\frac{3\sqrt{x}}{2}\right)^2}\, dx \\ &= \int_0^4 \sqrt{1 + \frac{9x}{4}}\, dx \\ &= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx \\ &= \int_0^4 \frac{\sqrt{4+9x}}{2}\, dx \\ &= \frac{1}{2} \times \int_0^4 \sqrt{4+9x}\, dx \\ &= |_{t^2=4+9x}\ \frac{1}{2} \times \int_{\sqrt{4}}^{\sqrt{40}} \frac{2t^2}{9}\, dt \\ &= \frac{1}{18} \times \int_{\sqrt{4}}^{\sqrt{40}} 2t^2\, dt \\ &= \frac{1}{18} \times \left[\frac{2t^3}{3}\right]_{\sqrt{4}}^{\sqrt{40}} \\ &= \frac{80\sqrt{10} - 8}{27} \end{aligned}$$

Exercício 5

a)

$$
\begin{aligned}
\gamma(t) &= (t^3, t^2), 1 \leq t \leq 3 \\
&\Rightarrow \gamma'(t) = (3t^2, 2t) \\
&\Rightarrow ||\gamma'(t)|| = \sqrt{(3t^2)^2 + (2t)^2} \\
&= t\sqrt{9t^2+4}
\end{aligned}
$$

Desta forma:

$$
\begin{aligned}
\int_1^3 ||\gamma'(t)||\,dt &= \int_1^3 t\sqrt{9t^2+4}\,dt \\
&= |_{u=9t^2+4} \int \frac{1}{18}\sqrt{u}\,du \\
&= |_{u=9t^2+4} \frac{1}{18} \times \frac{2u\sqrt{u}}{3} \\
&= \left[\frac{(9t^2+4)\sqrt{9t^2+4}}{27}\right]_1^3 \\
&= \frac{85\sqrt{85} - 13\sqrt{13}}{27}
\end{aligned}
$$

b)

$$
\begin{aligned}
\gamma(t) &= (2(t - \sin(t), 2(1-\cos(t)), 0 \leq t \leq \pi \\
&\Rightarrow \gamma'(t) = (2 - 2\cos(t), 2\sin(t)) \\
&\Rightarrow ||\gamma'(t)|| = 2\sqrt{2 - 2\cos(t)}
\end{aligned}
$$

Assim

$$\begin{aligned}
\int_0^{\pi} ||\gamma'(t)||\, dt &= \int_0^{\pi} 2\sqrt{2-2\cos(t)}\, dt \\
&= 2 \times \int_0^{\pi} \sqrt{2-2\cos(t)}\, dt \\
&= |_{u=tan(\frac{t}{2})}\ 2 \times \int_0^{\infty} \frac{4u}{(u^2+1)^{\frac{3}{2}}}\, du \\
&= 8 \times \int_0^{\infty} \frac{u}{(u^2+1)^{\frac{3}{2}}}\, du \\
&= |_{v=\sqrt{u^2+1}}\ 8 \times \int_1^{\infty} \frac{1}{v^2}\, dv \\
&= 8
\end{aligned}$$

c)

$$\begin{aligned}
\gamma(t) &= (-\sin(t), \cos(t)), 0 \leq t \leq \pi \\
&\Rightarrow \gamma'(t) = (-\cos(t), -\sin(t)) \\
&\Rightarrow ||\gamma'(t)|| = 1
\end{aligned}$$

Assim

$$\int_0^{2\pi} ||\gamma'(t)||\, dt = \int_0^{2\pi} 1\, dt = 2\pi$$

d)

$$\begin{aligned}
\gamma(t) &= (e^t\cos(t), e^t\sin(t)), 1 \leq t \leq 2 \\
&\Rightarrow \gamma'(t) = (e^t\cos(t) - e^t\sin(t), e^t\cos(t) + e^t\sin(t)) \\
&\Rightarrow ||\gamma'(t)|| = \sqrt{2}e^t
\end{aligned}$$

Logo

$$\int_1^2 ||\gamma'(t)||\, dt = \int_1^2 \sqrt{2}e^t\, dt = \sqrt{2}\int_1^2 e^t\, dt = \sqrt{2} \times [e^x]_1^2 = \sqrt{2}e^2 - \sqrt{2}e$$

Exercício 6

a)

Do enunciado temos $f(x) = x + 1$ e os limites de integração $[0, 2]$, então:

$$V(S) = \pi \times \int_0^2 (x+1)^2 \, dx = \pi \left[\frac{x^3}{3} + x^2 + x \right]_0^2 = \frac{26\pi}{3}$$

b)

Do enunciado temos $f(x) = x^2 - x^3$ e os limites de integração $[0, 1]$, então:

$$V(S) = \pi \times \int_0^1 (x^2 - x^3)^2 \, dx = \pi \left[\frac{x^7}{7} - \frac{x^6}{3} + \frac{x^5}{5} \right]_0^1 = \frac{\pi}{105}$$

c)

Do enunciado temos $f(x) = x^3$ e os limites de integração $[-1, 1]$, então:

$$V(S) = \pi \times \int_{-1}^1 (x^3)^2 \, dx = \pi \times \int_{-1}^1 x^6 \, dx = \pi \times \left[\frac{x^7}{7} \right]_{-1}^1 = \frac{2\pi}{7}$$

d)

Do enunciado temos $f(x) = \cos(x)$ e os limites de integração $[0, \frac{\pi}{4}]$, então:

$$V(S) = \pi \times \int_0^{\frac{\pi}{4}} (cos(x))^2 \, dx = \pi \times \left[\frac{x}{2} + \frac{\sin(2x)}{4} \right]_0^{\frac{\pi}{4}} = \frac{\pi^2}{8} + \frac{\pi}{4}$$

Exercício 7

a)

Do enunciado temos $f(x) = \sin(x)$ e os limites de integração $[0, \pi]$, assim:

$$\begin{aligned} A(S) &= \int_0^\pi 2\pi \sin(x) \sqrt{1 + \cos^2(x)} \, dx \\ &= |_{u=\cos(x)} \; 2\pi \times \left(- \int_1^{-1} \sqrt{u^2 + 1} \, du \right) \\ &= |_{u=\tan(\theta)} \; 2\pi \times \left(\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^3(\theta) \, d\theta \right) \\ &= 2\pi \times (\sqrt{2} + \ln(1 + \sqrt{2})) \end{aligned}$$

Observe que podemos calcular $\int \sec^3(\theta)\,d\theta$ por partes como se segue:

$$\int u\,dv = uv - \int v\,du$$

$$\begin{cases} u = \sec(\theta) \Rightarrow du = \sec(\theta)\tan(\theta)\,d\theta \\ v = \int \sec^2(\theta)\,d\theta \Rightarrow dv = \sec^2(\theta)\,d\theta \end{cases}$$

$$\begin{aligned}
\int \sec^3(\theta)\,d\theta &= \sec(\theta)\tan(\theta) - \int \sec(\theta)\tan^2(\theta)\,d\theta \\
\tan^2(\theta) = \sec^2(\theta) - 1 &\Rightarrow \sec(\theta)\tan(\theta) - \int \sec(\theta)(\sec^2(\theta) - 1)\,d\theta \\
&= \sec(\theta)\tan(\theta) - \int \sec^3(\theta) - \sec(\theta)\,d\theta \\
&= \sec(\theta)\tan(\theta) - \int \sec^3(\theta)\,d\theta - \int \sec(\theta)\,d\theta \\
&= \sec(\theta)\tan(\theta) - \int \sec^3(\theta)\,d\theta - \ln|\sec(\theta) + \tan(\theta)| \\
&\Rightarrow 2 \times \int \sec^3(\theta) = \sec(\theta)\tan(\theta) - \ln|\sec(\theta) + \tan(\theta)| \\
&\Rightarrow \int \sec^3(\theta) = \frac{\sec(\theta)\tan(\theta)}{2} - \frac{\ln|\sec(\theta) + \tan(\theta)|}{2} + \mathrm{C}, \mathrm{C} \in \mathbb{R}.
\end{aligned}$$

b)

Do enunciado temos $f(x) = x^2$ e os limites de integração $[0, \frac{1}{2}]$, assim:

$$
\begin{aligned}
A(S) &= \int_0^{\frac{1}{2}} 2\pi x^2 \sqrt{1 + (2x)^2}\, dx \\
&= 2\pi \times \int_0^{\frac{1}{2}} x^2 \sqrt{1 + 4x^2}\, dx \\
&= |_{u=\sqrt{4x^2+1}-2x}\ 2\pi \times \int_1^{\sqrt{2}-1} \left(-\frac{u^3}{128} + \frac{2u^4 - 1}{128u^5} \right) du \\
&= 2\pi \times \int_1^{\sqrt{2}-1} \left(-\frac{u^3}{128} + \frac{2u^4}{128u^5} - \frac{1}{128u^5} \right) du \\
&= 2\pi \times \left(-\frac{1}{128} \int_1^{\sqrt{2}-1} \left(u^3 - \frac{2}{u} + \frac{1}{u^5} \right) du \right) \\
&= 2\pi \times \left(-\frac{1}{128} \left[\frac{u^4}{4} + 2\ln(|u|) - \frac{1}{4u^4} \right]_1^{\sqrt{2}-1} \right) \\
&= 2\pi \times \left(\frac{3\sqrt{2}}{64} - \frac{\ln(1 + \sqrt{2}}{64} \right) = \frac{3\pi\sqrt{2}}{32} - \frac{\pi \ln(1 + \sqrt{2})}{32}
\end{aligned}
$$

c)

Do enunciado temos $f(x) = \sqrt{x}$ e os limites de integração $[1, 4]$, logo:

$$
\begin{aligned}
A(S) &= \int_1^4 2\pi\sqrt{x} \sqrt{1 + \left(\frac{1}{2\sqrt{x}} \right)^2}\, dx \\
&= 2\pi \times \int_1^4 \frac{\sqrt{4x + 1}}{2}\, dx \\
&= 2\pi \times \frac{1}{2} \times \int_1^4 \sqrt{4x + 1}\, dx \\
&= |_{u^2=\sqrt{4x+1}}\ \ \pi \times \int_{\sqrt{5}}^{\sqrt{17}} u^2\, du \\
&= \frac{\pi}{2} \times \left[\frac{u^3}{3} \right]_{\sqrt{5}}^{\sqrt{17}} = \frac{\pi}{6} \times \left(17\sqrt{17} - 5\sqrt{5} \right)
\end{aligned}
$$

d)

Do enunciado temos $f(x) = \dfrac{e^x + e^{-x}}{2}$ e os limites de integração $[-1, 1]$, então:

$$\begin{aligned}
A(S) &= \int_{-1}^{1} 2\pi \left(\frac{e^x + e^{-x}}{2}\right) \sqrt{1 + \left(\frac{e^{2x} - 1}{2e^x}\right)^2}\, dx \\
&= 2\pi \times \int_{-1}^{1} \left(\frac{e^{4x} + 2e^{2x} + 1}{4e^{2x}}\right) dx \\
&= 2\pi \times \frac{1}{4} \times \int_{-1}^{1} \left(\frac{e^{4x}}{e^{2x}} + \frac{2e^{2x}}{e^{2x}} + \frac{1}{e^{2x}}\right) dx \\
&= \frac{\pi}{2} \times \int_{-1}^{1} \left(e^{2x} + 2 + \frac{1}{e^{2x}}\right) dx \\
&= \frac{\pi}{2} \times \left[\frac{e^{2x}}{2} + 2x - \frac{1}{2e^{2x}}\right]_{-1}^{1} \\
&= \frac{\pi}{2} \times (e^2 + 4 - e^{-2})
\end{aligned}$$