

Gabarito Lista 4

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Exercício 1

a)

$$\begin{aligned}\sigma(t) &= (2 + \cos(6t), 2 + \sin(6t)) \Rightarrow \sigma'(t) = (-6 \sin(6t), 6 \cos(6t)) \\ \Rightarrow \sigma''(t) &= (-36 \cos(6t), -36 \sin(6t)) \\ \Rightarrow \|\sigma'(t_0)\| &= \sqrt{\left(-6 \sin\left(\frac{2\pi}{3}\right)\right)^2 + \left(6 \cos\left(\frac{2\pi}{3}\right)\right)^2} = 6\end{aligned}$$

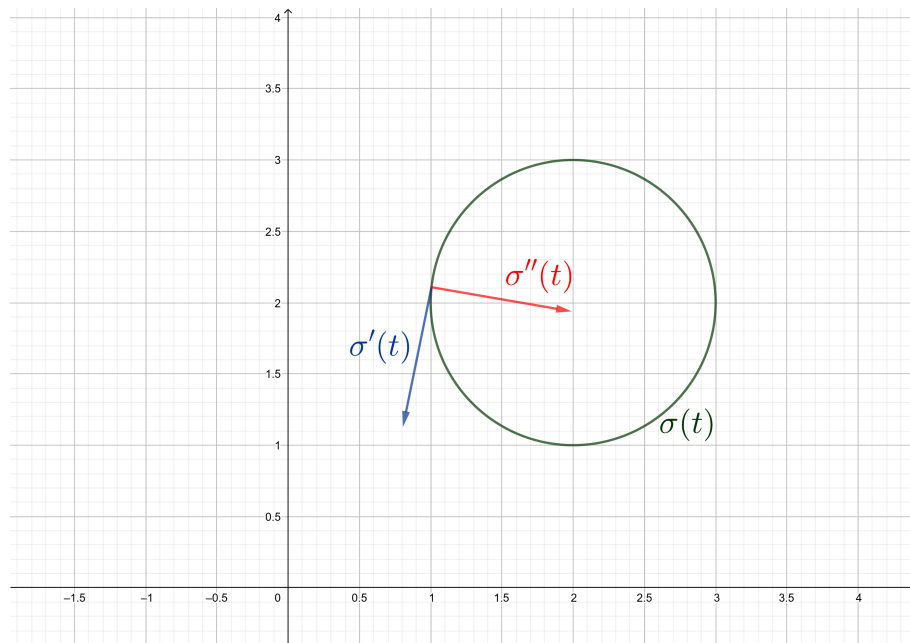


Figure 1: Representação geométrica de $\sigma(t)$, $\sigma'(t)$ e $\sigma''(t)$.

b)

$$\begin{aligned}\sigma(t) &= (\cos(2t), -3\sin(t)) \Rightarrow \sigma'(t) = (-2\sin(2t), -3\cos(t)) \\ \Rightarrow \sigma''(t) &= (-4\cos(2t), 3\sin(t)) \\ \Rightarrow \|\sigma'(t)\| &= \sqrt{(-2\sin(2\pi))^2 + (-3\cos(\pi))^2} = 3\end{aligned}$$

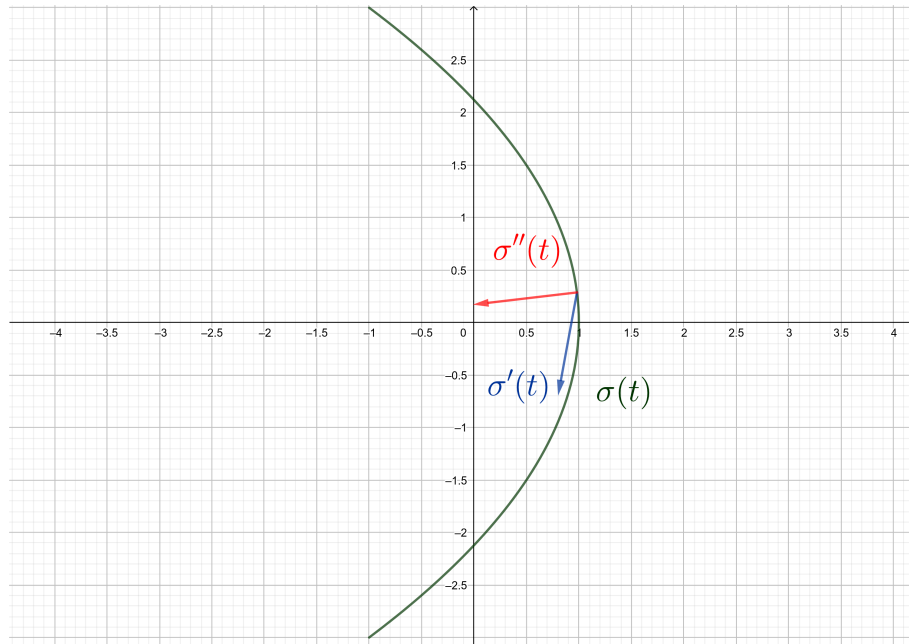


Figure 2: Representação geométrica de $\sigma(t)$, $\sigma'(t)$ e $\sigma''(t)$.

c)

$$\begin{aligned}\sigma(t) &= (e^{2t}, t^2) \Rightarrow \sigma'(t) = (2e^{2t}, 2t) \\ \Rightarrow \sigma''(t) &= (4e^{2t}, 2) \\ \Rightarrow \|\sigma'(t)\| &= \sqrt{(2e^{2 \times 0})^2 + (2 \times 0)^2} = 2\end{aligned}$$

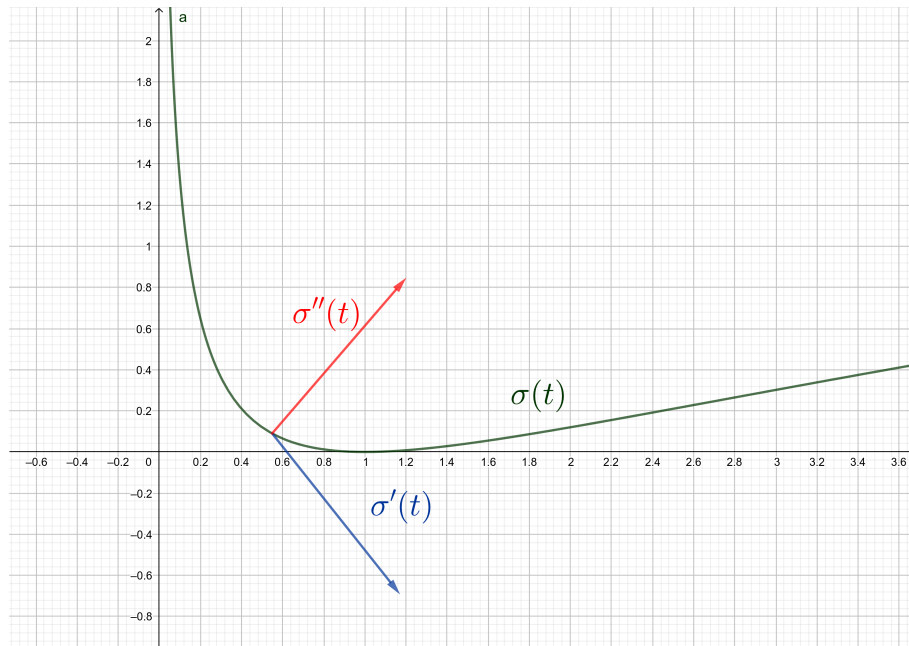


Figure 3: Representação geométrica de $\sigma(t)$, $\sigma'(t)$ e $\sigma''(t)$.

d)

$$\begin{aligned}
 \sigma(t) &= (\cos(t), \sin(t), 2) \Rightarrow \sigma'(t) = (-\sin(t), \cos(t), 0) \\
 &\Rightarrow \sigma''(t) = (\cos(t), \sin(t), 0) \\
 &\Rightarrow \|\sigma'(t)\| = \sqrt{\left(-\sin\left(\frac{\pi}{2}\right)\right)^2 + \left(\cos\left(\frac{\pi}{2}\right)\right)^2} = 1
 \end{aligned}$$

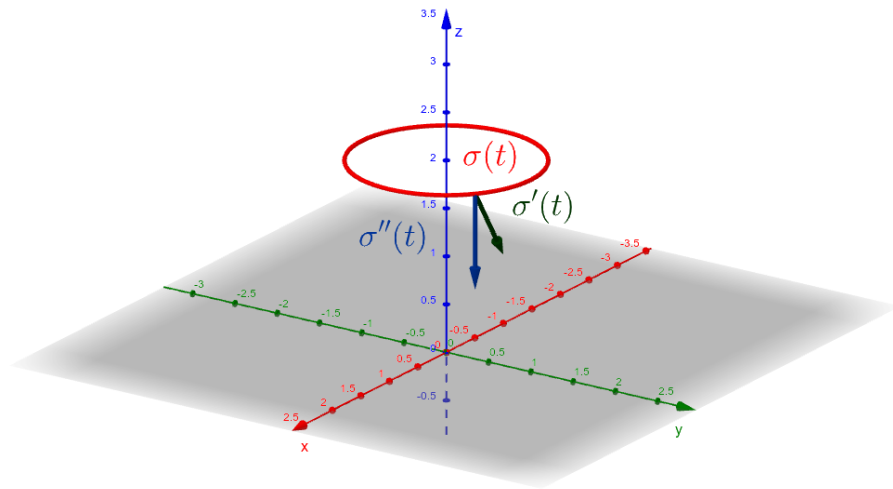


Figure 4: Representação geométrica de $\sigma(t)$, $\sigma'(t)$ e $\sigma''(t)$.

e)

$$\begin{aligned}\sigma(t) &= (1, t - 1, t^2 + 1) \Rightarrow \sigma'(t) = (0, 1, 2t) \\ &\Rightarrow \sigma''(t) = (0, 0, 2) \\ &\Rightarrow \|\sigma'(t)\| = \sqrt{(2 \times 2)^2} = 4\end{aligned}$$

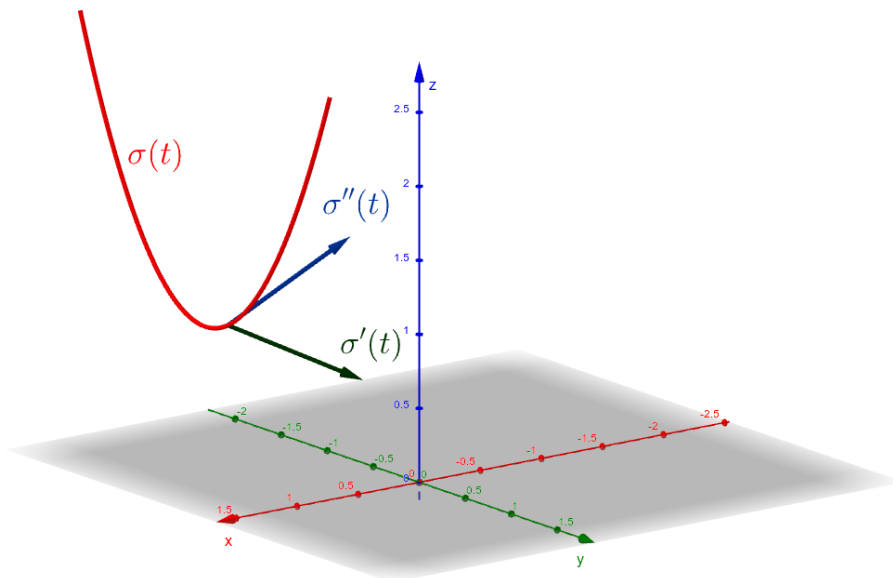


Figure 5: Representação geométrica de $\sigma(t)$, $\sigma'(t)$ e $\sigma''(t)$.

f)

$$\begin{aligned}\sigma(t) &= (3 \cos(2t), 3 \sin(2t), 8t) \Rightarrow \sigma'(t) = (-6 \sin(2t), 6 \cos(2t), 8) \\ \Rightarrow \sigma''(t) &= (-12 \cos(2t), -12 \sin(2t), 0) \\ \Rightarrow \|\sigma'(t)\| &= \sqrt{\left(-6 \sin\left(\frac{\pi}{4}\right)\right)^2 + \left(6 \cos\left(\frac{\pi}{4}\right)\right)^2 + (8)^2} = 10\end{aligned}$$

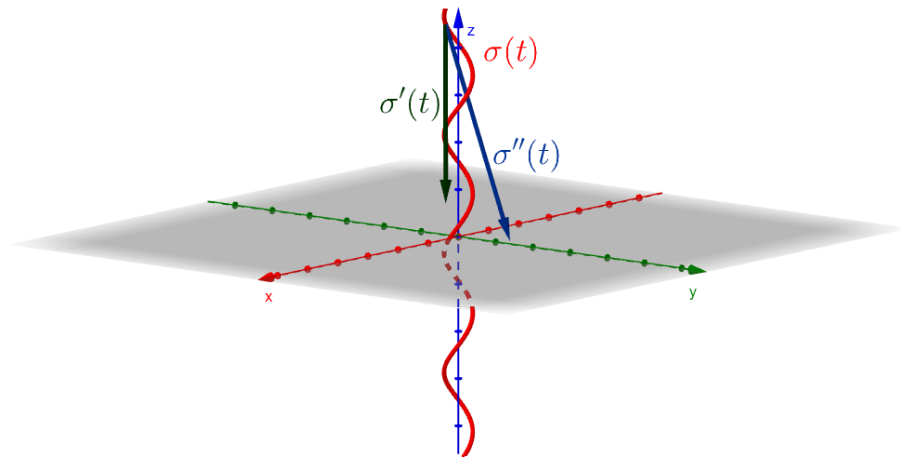


Figure 6: Representação geométrica de $\sigma(t)$, $\sigma'(t)$ e $\sigma''(t)$.

Exercício 2

a)

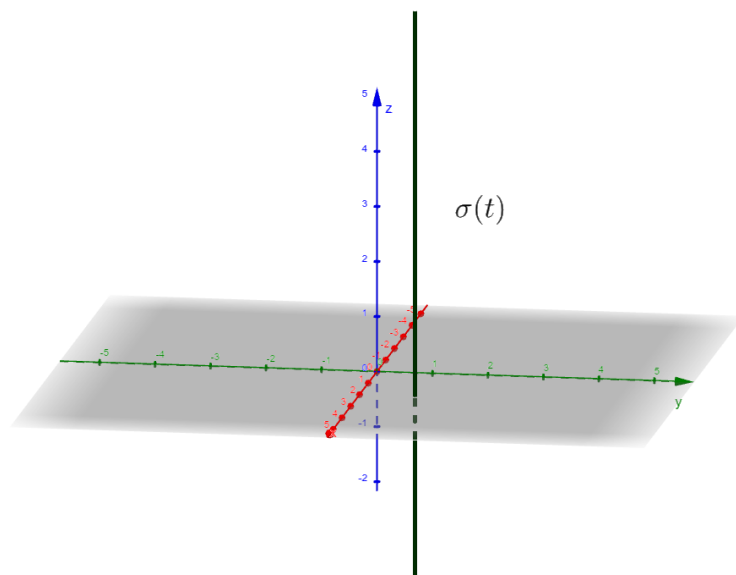


Figure 7: Representação geométrica da curva.

b)

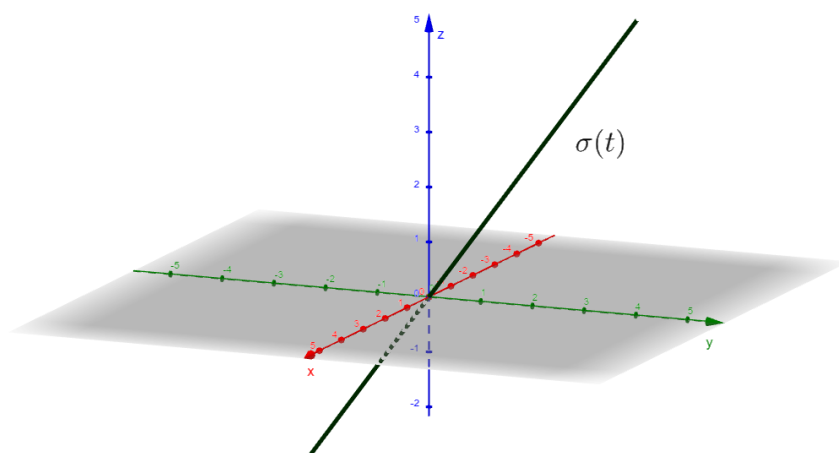


Figure 8: Representação geométrica da curva.

c)

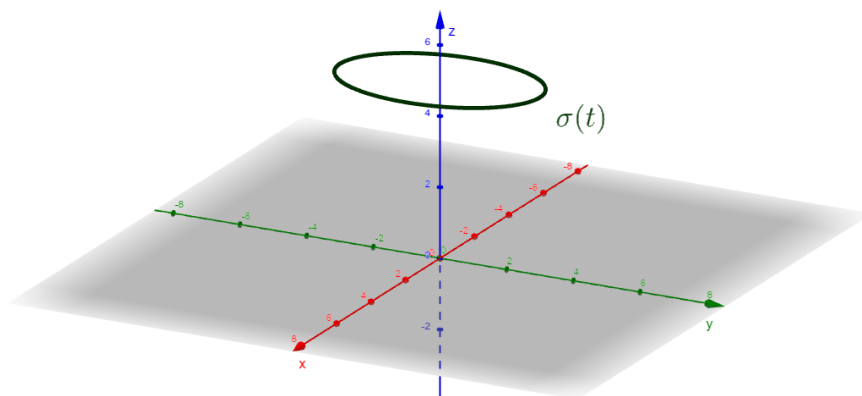


Figure 9: Representação geométrica da curva.

d)

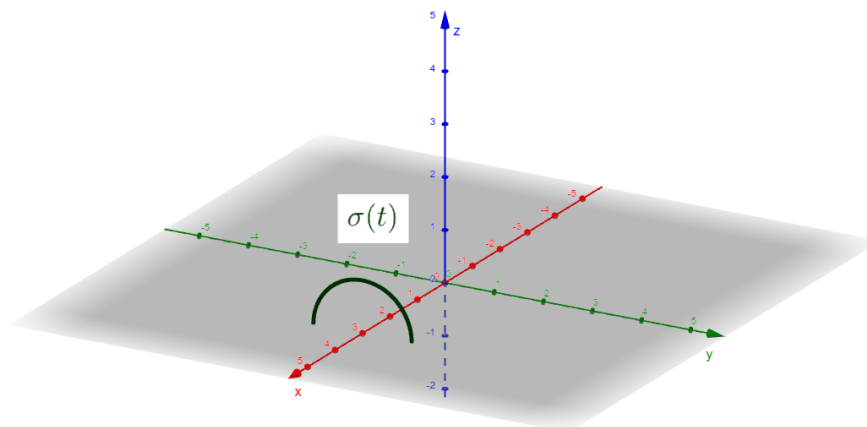


Figure 10: Representação geométrica da curva.

e)

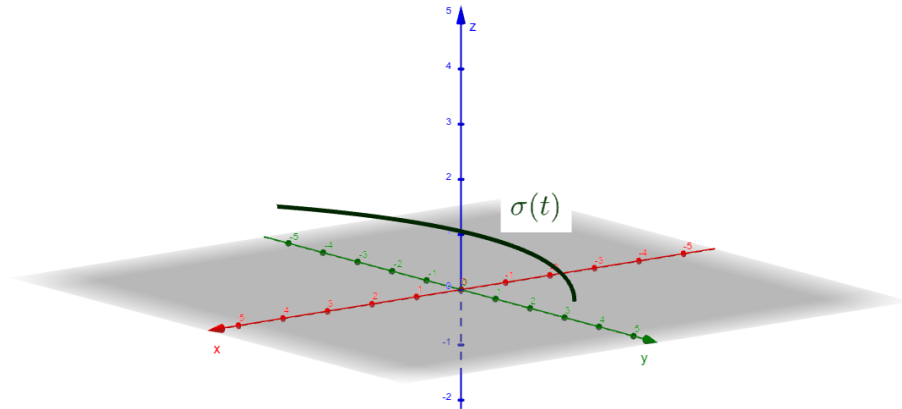


Figure 11: Representação geométrica da curva.

Exercício 3

a)

$$2x - 3y = 6 \Rightarrow 3y = 6 - 2x \Rightarrow y = 2 - \frac{2x}{3}$$

$$x = t \Rightarrow y = 2 - \frac{2t}{3} \Rightarrow \left(t, 2 - \frac{2t}{3}\right), t \in \mathbb{R}.$$

b)

$$x^2 = 4y \Rightarrow y = \frac{x^2}{4} \Rightarrow \left(t, \frac{t^2}{4}\right), t \in \mathbb{R}.$$

c)

Uma parametrização possível para essa curva é:

$$(x - a)^2 + (y - b)^2 = r^2 \Rightarrow (a + r \cos(t), b + r \sin(t)), t \in \mathbb{R}$$

d)

Uma parametrização possível para essa curva é:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow (a \cos(t), b \sin(t)), 0 \leq t \leq \pi$$

e)

Considere as funções $\cosh(t) = \frac{e^t + e^{-t}}{2}$ e $\sinh(t) = \frac{e^t - e^{-t}}{2}$, então uma parametrização para o ramo positivo da hipérbole será:

$$(\cosh(t), \sinh(t)), t \in \mathbb{R}.$$

Outra parametrização possível para essa curva seria:

$$(a \sec(t), b \tan(t)), -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

f)

Basta encontrar o vetor diretor da reta fazendo $A - B$ e fixar ele em algum dos dois pontos:

$$(3t - 1, 3t, 2 + t), 0 \leq t \leq 1.$$