THE CENTRAL POLYNOMIALS OF A STRONGLY LIE NILPOTENT ASSOCIATIVE ALGEBRA

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This is a joint work with Cleber Pereira.

Let F be a field and let A be a unital associative F-algebra. Let $T^{(n)}(A)$ $(n > 1)$ be the two-sided ideal of A generated by the linear span $L^{(n)}(A)$ of all commutators $[a_1, a_2, \dots, a_n]$ $(a_i \in A)$. Define $T^{(1)}(A) = A$. Define the series

$$A = R^{(0)}(A) \supseteq R^{(1)}(A) \supseteq \cdots \supseteq R^{(m)}(A) \supseteq \dots$$

of two sided ideals of A by $R^{(m)}(A) = A[R^{(m-1)}(A), A]A$ $(m \geq 1)$. We say that the algebra A is *Lie nilpotent* of class at most c if $L^{(c+1)}(A) = 0$ and that A is *strongly Lie nilpotent* of class at most c if $R^{(c)}(A) = 0$. Note that $L^{(n+1)} \subseteq R^{(n)}(A)$ so each strongly Lie nilpotent algebra A is Lie nilpotent. However, the converse, in general, is not true.

Let $F\langle X\rangle$ be the free unital associative F-algebra freely generated by a set X. It is known that the vector subspace $C(E)$ of central polynomials of the infinite-dimensional Grassmann algebra E over a field F of characteristic $p > 2$ is not finitely generated as a T-subspace of $F\langle X\rangle$. Since E is Lie nilpotent of class 2, the T-subspace of central polynomials of a Lie nilpotent algebra can be non-finitely generated. The aim of the present talk is to prove that this cannot happen if the algebra is strongly Lie nilpotent. Our main result is as follows: the T-subspace $C(B)$ of central polynomials of a strongly Lie nilpotent unital associative F-algebra B is always finitely generated (as a T-subspace of $F\langle X\rangle$).