

$\mathcal{U}(\mathfrak{h})$ -finite modules and almost-coherent families

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Abstract

In the representation theory of finite-dimensional simple Lie algebras \mathfrak{g} , two categories of modules stand out due to their contrasting nature. The first is the category of weight modules, consisting of \mathfrak{g} -representations where a fixed Cartan subalgebra $\mathfrak{h} \subseteq \mathfrak{g}$ acts semisimply. This category has been extensively studied over the past decades, with a classification of simple modules having finite-dimensional weight spaces obtained by O. Mathieu [1] through the introduction of a special class of modules known as *coherent families*. The second category consists of modules that are freely generated by $\mathcal{U}(\mathfrak{h})$ of finite rank. Recent studies have focused on this category which includes the classification of $\mathcal{U}(\mathfrak{h})$ -free modules of rank one by J. Nilsson in [2] and [3]. Interestingly, these two categories are connected through the *weighting functor* \mathcal{W} , which, as the name suggests, assigns to a \mathfrak{h} -free module M a weight module $\mathcal{W}(M)$. This functor was a key tool in Nilsson's classification of simple $\mathfrak{sp}(2n)$ -modules that are $\mathcal{U}(\mathfrak{h})$ -free of rank one. Building on the results presented in [4], this talk aims to explore these intriguing connections and demonstrate how Nilsson's approach can be extended to the broader category \mathfrak{A} of \mathfrak{g} -modules that are finitely generated by $\mathcal{U}(\mathfrak{h})$. As part of this extension, we introduce the new concept of *almost-coherent families* (a generalization of the standard coherent families) and the notion of almost equivalency, leading to the classification of a subclass of simple modules in \mathfrak{A} .

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References

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